CIVIL ENGINEERING

A TEXT-BOOK FOR A SHORT COURSE

BY

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PREFACE.

This text-book is prepared especially for the cadets of the United States Military Academy, who are being fitted for duty in the United States Army. It is a revision of the text-books written by my predecessors, Colonel Wheeler in 1884, and Colonel Fiebeger in 1904, for exactly the same purpose. It follows their methods as far as possible; and, as the principles of Civil Engineering have not changed, this text in many pages uses exactly the texts and figures of Colonels Wheeler and Fiebeger. Likewise, as in all text-books, information is obtained from books by other authors; and effort is made at the end of each chapter to show the principal sources of this information.

Every army officer must have a fair working knowledge of the principles and practice of engineering; therefore, in this book the cadet is taught enough of all branches of civil engineering so that he may in future understand the practical engineering requirements of his profession. For the engineering work of the army requiring great technical knowledge, this text-book is supplemented by additional courses given in army schools to officers of the engineering branches of the army.

W. A. MITCHELL.

West Point, New York, May 2, 1924.

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INTRODUCTION.

1. **Engineering** is defined to be *"the science and art of utilizing the forces and materials of nature."*

It is divided, for purposes of instruction at the Military Academy, into two principal branches, *Civil* and *Military Engineering.*

Military engineering embraces the planning and construction of fortifications and structures used in military operations. It is not treated in this text.

Civil engineering comprises the designing and building of works intended to add to the comfort of man, or to improve the country either by beautifying it or increasing its prosperity. In this branch, the constructions are divided into two classes, according as the parts of which they are made are to be relatively at rest or in motion. In the former case they are known as *structures,* and in the latter as *machines.*

2. It is usual to limit the term *civil engineering* to the planning and construction of works of the first class, and to use the term *mechanical engineering* when the works considered are machines.

It is also usual to subdivide civil engineering into classes, according to the prominence given to some one or more of its parts when applied in practice, as *topographical engineering, hydraulic engineering,* etc. Notwithstanding this separation into branches and subdivisions, there are certain general principles common to them all.

3. The object of the following pages is to give in regular order these elementary principles, common to all branches of engineering, which the student should learn so that he may understand the nature of the engineers' profession, and know how to apply these principles in practice.

The subject is covered in the following manner:

Part I: Mechanics of Engineering.
Part II: Building Materials.
Part III: Component Parts of a Structure.
Part IV: Engineering Structures.

4. In the Mechanics of Civil Engineering there are chapters on the following:

Forces.
Tension; Compression; Shear; Torsion; Flexure.
Beams.
Moving Loads.
Pressure and Flow of Water.

In these chapters are included the principles of engineering which are constantly needed throughout the book. These principles must be thoroughly understood in order to study intelligently the applications of them which are given later.

5. Building Materials are discussed in chapters on the following:

Timber.
Metals.
Natural and Artificial Stone.
Testing of Materials.

Endeavor is made to bring out the characteristics of materials which make them useful in engineering constructions.

6. The Component Parts of a Structure are discussed in chapters on the following:

Framing.
Masonry.
Columns.
Foundations.
Metal Work.

The component parts must be properly designed and constructed so that the structure may be satisfactory. Therefore, each chapter is descriptive of the use of the component part in the structure as a whole.

7. Engineering Structures comprise a broad field and can only be considered very briefly in this course. Only the most important are discussed. The chapters cover to a certain extent the following:

Buildings (including roofs).
Bridges.
Dams and Retaining Walls.
Water Supply.
Sewers and Sewage Disposal.
Railways.
Roads and Pavements.
City Engineering.
Construction Equipment.
Specifications.

The engineering principles and the practical field results are combined in the discussion, so that neither shall unduly overshadow the other.

8. In all discussions of engineering principles, effort is made to present the idea in the same manner in each case, viz.:

1st. Discuss the general idea, if necessary.
2nd. State the object of the deduction from which a principle is to be determined.
3rd. Prove the principle and state it.
4th. Point out the actual application of the principles in engineering practice.
5th. Give a problem, illustrating the principle.
6th. Solve the problem.

In studying certain parts of the book, it will be advantageous to remember the above manner of presentation.

9. In addition to the problem illustrating each principle, there are many problems at the end of each chapter, which will be useful in acquiring greater familiarity with the subject matter of the chapter.

10. **Symbols.**—The same symbols are used throughout the text. These symbols are the same as used in all the work of the Department of Engineering at the United States Military Academy, and follow a general system, viz.:

1st. The basic (central) letter names the most important idea to be conveyed, thus: b = breadth, F = force, s = stress.
2nd. Several basic letters with the same idea of equal importance are designated by primes, seconds, etc., thus: F', F'', F''', F^{iv}.
3rd. The subscripts to the basic letters explain the character of the important ideas as conveyed by the basic letters, thus: M_f = moment of flexure, s_c = unit stress of compression. If two explanations of the basic letter are necessary, both are placed as subscripts to the letter, thus: a_{ts} = allowable unit tensile stress of steel.
4th. As far as practicable, small letters are used to designate unit stresses, and capitals to designate total stresses.

11. It will be necessary for the student to become familiar with these symbols as given below; they are not defined in each discussion throughout the text:

A = area of cross-section.
a = allowable (safe) stress in pounds per square inch, called allowable unit stress.
a_c, a_s, a_t, etc. = allowable (safe) unit stresses of compression, shear, tension, etc.
b = breadth.
C = constant.
D = diameter, depth of beam.
d = sign of differentiation.

E = modulus (measure) of elasticity.
E_t, E_c, E_{tr} = modulus (measure) of tensile, compressive, torsional, elasticity.
F = force.
f = force in pounds per square inch, called unit force.
g = acceleration due to gravity.
H = head of water.
h = height.
I = moment of inertia of the cross-section of a beam about its neutral axis.
I_p = polar moment of inertia.
$\frac{I}{y}$ = section modulus of a beam.
L, l = length, distance.
λ = unit elongation or shortening.
M_f = bending moment; moment of flexure.
M_{fm} = maximum bending moment.
M_{tr} = torsional moment.
μ = coefficient of friction.
N = number of revolutions per minute.
O = origin of co-ordinates.
OX = axis of X co-ordinates.
OY = axis of Y co-ordinates.
OZ = axis of Z co-ordinates.
p = pressure.
Q = quantity of discharge of water, or other fluid.
r = radius of circle or radius of gyration.
ρ = radius of curvature.
R = reaction.
s = stress in pounds per square inch, called unit stress.
s_c, s_s, s_t = unit stresses of compression, shear, tension, etc.
s_f, s_{tr} = unit stresses of flexure and torsion in surface fiber of a beam.
$(s_f)_1$, $(s_{tr})_1$ = unit stresses of flexure and torsion at unit distance from neutral axis of a beam.
s_{fm}, s_{sm}, s_{tm}, etc. = maximum unit stresses of flexure, shear, tension, etc.
S_h = horizontal shear.
S_v = vertical shear.
S_{hm}, S_{vm} = maximum horizontal and vertical shear.
t = thickness.
u = ultimate (breaking) stress in pounds per square inch, called ultimate unit stress.
u_c, u_s, u_t, etc. = ultimate (breaking) unit stresses of compression, shear, tension, etc.
V = velocity.
v = volume.
W = weight of load.
w = weight uniformly distributed per unit of measure.
y_m = maximum deflection of a beam.
α, β, γ and φ = angles.
$\int$ = sign of integration.
Δ = increment.

PART I.

MECHANICS OF ENGINEERING.

It is necessary to acquire a knowledge of the basic mathematical principles of engineering construction. Before the development of these principles, construction was based on practical experience, and little progress was made; as more and more of the principles became known, useless work and useless materials were eliminated. Although, at the present time, a foreman on unimportant work in the field can follow rules of thumb learned by experience, even this experience in the last analysis is based on constructions designed by mathematical principles and as the result of practical experiments made by men acquainted with the theory.

Theory and practice are both necessary for best results. In this text, theory is first considered; and it is later seen, in the discussion of the engineering constructions, that the theory is applied at almost every step of the field work.

CHAPTER I.

FORCES.

101. An **engineering structure**, as a building, bridge, or dam, is a combination of rigid or resistant bodies, so formed and connected that it will withstand the action of external forces to which it may be exposed, and still preserve its form. The bodies composing the structure are called *pieces*, and the surfaces where they touch and are connected are called *joints*.

A **machine** differs from a structure in that the bodies are so formed and connected that some of their parts can move upon each other.

102. In planning and building a structure, the engineer should be governed by the following conditions:

The structure should possess the *necessary strength;* should last the required time; must be reasonable in cost; and must be suitable for the purpose. In other words, the engineer in projecting and executing a work should duly consider the elements of *strength, durability, economy, and suitability.*

103. The designing and building of a structure comprise three distinct operations, as follows:

a. The conception of the project or plan;

b. Putting this on paper, so it can be understood;

c. The execution of the work.

The conception of the project or plan requires a perfect acquaintance with the locality where the structure is to be placed, the ends or objects to be attained by it, and the kind and quantity of materials that can be supplied at that point for its construction.

The putting of the plan on paper requires that the designer know something of drawing, as it is only by drawings and models accompanied by descriptive memoirs, with estimates of cost, that the arrangement and disposition of the various parts, and the expense of a proposed work, can be understood by others. The drawings are respectively called the *plan*, *elevation*, and *cross-section*, according to the views they represent. A symmetrical structure requires but few drawings; one not symmetrical, or having different fronts, will require a greater number. These, to be understood, must be accompanied by written specifications explaining fully all the parts. The estimate of cost is based upon the cost of the materials, the price of labor, and the time required to finish the work.

The execution of the work may be divided into three operations:
(1) Laying out the work in the field;
(2) Putting together of the materials into parts;
(3) Combining of these parts in the structure.

These three operations in the execution of the work require:
(a) A knowledge of surveying, leveling, and other operations incident to laying out the work;
(b) A knowledge of the physical properties of the materials used;
(c) A knowledge of the art of forming them into the shapes required;
(d) The ability to join materials and shapes together to best satisfy the conditions that are to be imposed upon the structure.

EQUILIBRIUM OF FORCES.

104. As the pieces of an engineering structure do not move upon each other, they must be in equilibrium. If the structure is in equilibrium and another force acts upon it, then this force must be transmitted throughout the structure and resistances must be developed so that the forces are again in equilibrium; otherwise the structure will move. If the structure does not move, it is in equilibrium and is a properly arranged structure; if it should move, it would break and would cease to be a properly arranged structure.

105. **Free Bodies.**—It is sometimes convenient to consider a portion of an engineering structure as a "free body"; that is, the portion in question is considered as having been separated from the rest of the structure, while continuing to be acted upon by the same forces as when it formed an integral part of the complete struc-

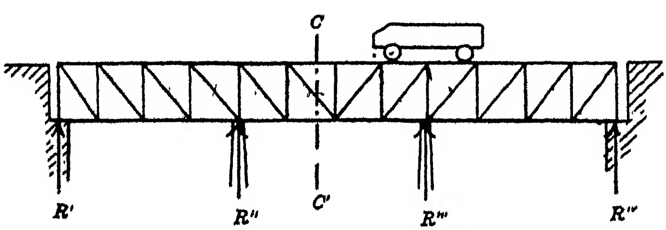


Fig. 105 A

ture. Since, by definition, an engineering structure, or any part of it, must be at rest, the forces acting upon the "free body" must form a system in equilibrium. For example, if we conceive the highway bridge shown in Fig. 105A to have been cut in half by the

plane CC′, and the left half removed, the remaining portion (Fig. 105 B) must be in equilibrium under the forces acting upon it,

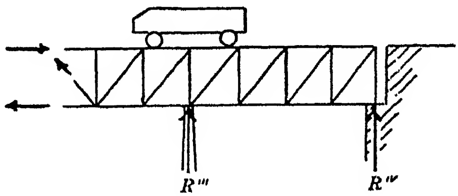


Fig. 105 B

which are, the stresses in the three members cut, the reactions R‴ and R^{iv}, the weight of this portion of the bridge, and the weight of the truck shown.

Conditions of Equilibrium.

106. In order that there may be equilibrium in any structure, it must have no linear or rotary motion. To produce this condition, it is necessary that the forces fulfill two general conditions, viz.: that the algebraic sum of the components of all the forces in any direction must be equal to zero, and the algebraic sum of the moments of the forces around any axis must be equal to zero.

107. In considering the forces that act upon a structure, it is generally assumed that the forces which act on each elementary part, and sometimes on the entire structure, form a co-planar system; that is, the action lines of all the forces lie in the same plane. Actually they do not all act in the same plane, but the assumption is satisfactory and the forces are substantially correct as assumed. For example, if the walls of a building are L feet long, weight W pounds, and are supported by A square feet of foundation; it is usually satisfactory to consider W/L for each linear foot of foundation as though each unit of weight acted in the same plane.

108. In engineering design, it is customary to separate the forces into vertical and horizontal components. This makes two fixed conditions out of the general condition of equilibrium "that the algebraic sum of the components of all of the forces in any direction must be equal to zero." Therefore, these general conditions of equilibrium of co-planar forces are, in the usual case, expressed as three conditions, thus:

a. The algebraic sum of the vertical components of the forces must be equal to zero.

b. The algebraic sum of the horizontal components of the forces must be equal to zero.

c. The algebraic sum of the moments of the forces, about any axis perpendicular to the plane of the forces, must be equal to zero.

Equations of Equilibrium.

109. In order to prevent confusion of signs and moments, it is a rule in this text that:

a. Components acting upward or to the right are considered positive; those downward or to the left, negative.

b. Clockwise moments are considered positive; counter-clockwise moments, negative.

110. Following the above rule, the three conditions of equilibrium are expressed by the following equations, as illustrated by Fig. 110A, in which O is the center of moments:

$$F' \sin \alpha + F'' \sin \beta + F''' \sin \gamma + F^{\text{iv}} \sin \varphi + \text{etc.} = 0 \qquad (110\text{A})$$

$$F' \cos \alpha + F'' \cos \beta + F''' \cos \gamma + F^{\text{iv}} \cos \varphi + \text{etc.} = 0 \qquad (110\text{B})$$

$$F' l' \sin \alpha + F'' l'' \sin \beta + F''' l''' \sin \gamma + F^{\text{iv}} l^{\text{iv}} \sin \varphi + \text{etc.} = 0 \quad (110\text{C})$$

In each of these equations, F', F'', etc., are the intensities of the forces; α, β, γ, etc., fix the directions; and l', l'', etc., fix the distance from the center of moments to the action line of each force measured on a horizontal line through the center of moments, thus fixing at least a point on the action line of each force. The three for each force fix the force definitely.

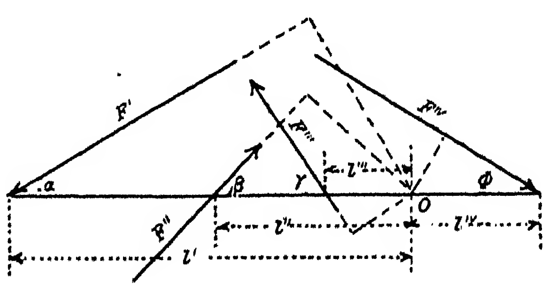


Fig. 110 A

Application of Equations of Equilibrium.

111. Since only three unknown quantities can be determined from three equations, it is evident that in a determinate problem all of the other quantities must be known. Also, since l', l'', etc., enter but a single equation, only one of these distances can be unknown.

It is not, however, always necessary to use the three general equations in complete form. As shown below, certain ones become identical in some cases, their terms reduce to zero in others. The general equations are, nevertheless, the basis of all computations on engineering structures.

112. **Concurrent Forces.**—If the action lines of the forces intersect at a common point, and if the center of moments be taken at this point, the first member of equation 110C reduces to zero for all values of the forces; or it reduces to equation 110A if the center of moments is taken at another point on the horizontal line through the point of intersection of the action lines. Therefore, the equations of equilibrium become only two in number. From this we see that, with concurrent forces, it is possible to determine only two unknown quantities, and neither of these can be the distance to the action line of a force.

113. **Parallel Forces.**—If the action lines of the forces are parallel, $\alpha = \beta = \gamma = \varphi$, and the equations become

$$F' \sin \alpha + F'' \sin \alpha + F''' \sin \alpha + F^{iv} \sin \alpha + \text{etc.} = 0 \qquad (113A)$$
$$F' \cos \alpha + F'' \cos \alpha + F''' \cos \alpha + F^{iv} \cos \alpha + \text{etc.} = 0 \qquad (113B)$$
$$F' l' \sin \alpha + F'' l'' \sin \alpha + F''' l''' \sin \alpha + F^{iv} l^{iv} \sin \alpha + \text{etc.} = 0 \qquad (113C)$$

Dividing by *sin* α and *cos* α, we obtain

$$F' + F'' + F''' + F^{iv} + \text{etc.} = 0 \qquad (113D)$$
$$F' + F'' + F''' + F^{iv} + \text{etc.} = 0 \qquad (113E)$$
$$F' l' + F'' l'' + F''' l''' + F^{iv} l^{iv} + \text{etc.} = 0 \qquad (113F)$$

The first two equations are identical, leaving only two independent equations. From this we see that, with parallel forces, it is possible to determine only two unknown quantities, and only one of these can be the distance to the action line of a force.

114. **Non-concurrent and non-parallel forces.**—If the action lines are non-concurrent and non-parallel, then the three equations of equilibrium are wholly independent; and it is possible to determine three unknown quantities, provided that only one of the unknowns is the distance to the action line of a force.

115. **Resultants.**—If any system of forces is in equilibrium, the resultant of any selected forces must be in equilibrium with the

remaining forces. From this, we conclude that any force must be equal and opposite to the resultant of the others—that the resultant of any two forces must be equal and opposite to the resultant of the remaining forces, etc. This is correct in a concurrent system where one force can be equal and opposite to the resultant of other forces; but in a parallel system, and often in a non-concurrent system, it is found that the resultant of certain forces may form a couple; therefore, in such systems, this couple must be equal and opposite to another couple, and cannot be equal and opposite to a single force.

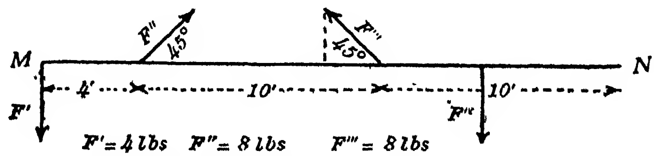


Fig. 116 A

116. **Problem.**—(Fig. 116A). A rigid bar MN is at rest under the action of the four forces F', F'', F''', F^{iv}. By use of the equations of condition for equilibrium determine the intensity and point of application of the force F^{iv}.

Solution.—Using the equations

$$F' \sin \alpha + F'' \sin \beta + F''' \sin \gamma + F^{\text{iv}} \sin \varphi + \text{etc.} = 0 \qquad (109A)$$

$$F' \cos \alpha + F'' \cos \beta + F''' \cos \gamma + F^{\text{iv}} \cos \varphi + \text{etc.} = 0 \qquad (109B)$$

$$F' l' \sin \alpha + F'' l'' \sin \beta + F''' l''' \sin \gamma + F^{\text{iv}} l^{\text{iv}} \sin \varphi + \text{etc.} = 0 \qquad (109C)$$

Taking moments around M, and making proper substitutions, we have

$$-4 + 8 \sin 45° + 8 \sin 45° - F^{\text{iv}} = 0$$

$$8 \cos 45° - 8 \cos 45° = 0$$

$$4 \times 0 - 4 \times 8 \sin 45° - 14 \times 8 \sin 45° + F^{\text{iv}} l^{\text{iv}} = 0$$

Combining and solving, we have

$$F^{\text{iv}} = 7.3 \text{ pounds}$$

$$l^{\text{iv}} = 14' \text{ from } M$$

MOLECULAR ACTION.

117. Every solid body is formed of molecules, grouped together by certain laws. Each molecule is so related to those surrounding it that its position cannot be changed except by the application of an extraneous force.

If a solid, which is not allowed to move from its place, be acted upon by an extraneous force, the equilibrium of the internal forces acting between the molecules will be disturbed and variations caused in the distances between the molecules and in the intensities of the forces that bind them together.

By these variations, an equilibrium between the external and internal forces is effected, and an alteration of the form of the solid

is caused. This alteration of form is called a *strain*, and the force by which the molecules resist it is called a *stress*.

External forces, therefore, acting upon solids not free to move cause strains and develop stresses in the bodies so placed.

Classification of Stresses.

118. If a solid body like a beam (Figs. 118A, B, C, D, E) be firmly fastened at one end so that it will not move, and then be acted upon by an extraneous force, this beam will be subjected to a strain. Stress will be developed in the beam to resist the strain and to establish an equilibrium between the external and internal forces.

Let us assume that *AB* and *CD* are two adjacent sections of the beam perpendicular to the axis. Suppose these sections to take all the positions they can have with respect to each other.

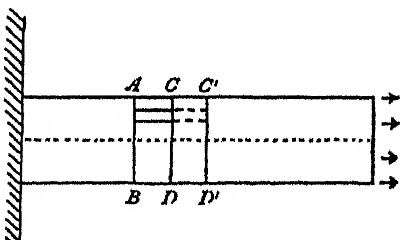


Fig. 118 A

a. Let the action of the straining force be such that the section *CD*, while remaining parallel to *AB*, shall move away from it. (Fig. 118A.) This can be done only by lengthening the fibers connecting the two sections; and since the sections remain parallel, by lengthening all of them an equal amount.

This lengthening of the fibers is called a *strain of tension*, the resistance offered by the fiber is called a *tensile stress*, and the external force producing the strain, a *tensile force*.

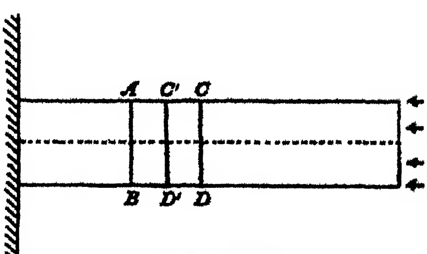


Fig. 118 B

b. If the force acting on the section *CD* is such as to make it approach *AB* but still be parallel to it (Fig. 118B), the fibers would be shortened. The strain is one of *compression;* the stress, *compressive;* and the straining force, a *compressive force.*

c. Suppose the section *CD,* under the action of the force, to move past *AB,* while the planes of the sections remain parallel. This action would require the fibers to be distorted as shown in Fig. 118C, in which the fiber *ab* takes a new position as *ab′*. This distortion is called a *shearing* strain; the stress, a *shearing* one, or simply a *shear;* and the straining force, a *shearing force.*

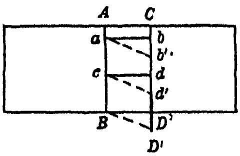


Fig. 118 C

d. The section *CD* may be made to revolve around *FE* as an axis. The fiber *mn* will then become distorted to position *mn′* (Fig. 118D). This distortion is called a strain of *torsion;* the stress, *torsional;* and the force one of *twisting.*

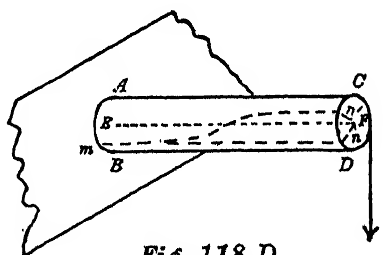


Fig. 118 D

e. Suppose the section *CD,* under the action of the force, had taken a position as *C′D′*, as shown (Fig. 118E). This position could not have been assumed unless the fibers were deflected; that is, the fibers above the axis of rotation lengthened, and those below it

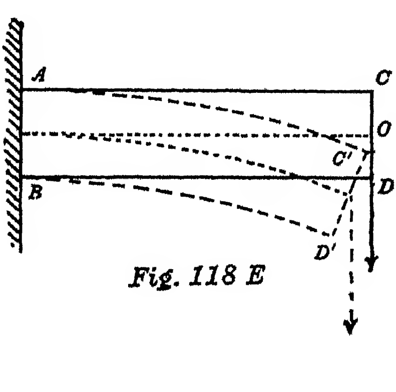


Fig. 118 E

shortened. The distortion of the fiber in this case is called a *cross strain;* the stress, a *bending* one, and the straining force, a *bending force,* or *force of flexure.*

119. **Examples.**—Weights, either permanently or temporarily applied, form the extraneous forces that ordinarily strain a structure. The stresses developed are as follows:

a. Tensile: as the stress developed in a vertical rod, chain, etc., fastened at the upper end and sustaining a weight at the lower end.

b. Compressive: as the stress developed in a pillar when a load is placed on its top.

c. Shearing: as the stress developed in the rivets holding two plates together, by a force which tends to slide one plate along the other.

d. Torsional: as the stress developed in the shaft of a windlass when lifting a weight.

e. Bending: as the stress developed in a beam supported at its ends, and with a weight placed upon it between the supports. It is to be noted that the force causing bending stress may also cause one or more of the other stresses at the same time.

120. **Laws of Elasticity.**—From a great number of experiments, made on a great variety of materials, it has been found that practically,

1st. All bodies are elastic.

2nd. Within certain limits, they may be considered as perfectly elastic.

3rd. Within the field of perfect elasticity, the amount of deformation is directly proportional to the force that produces it.

4th. Beyond the limit of elasticity, the amount of deformation is not proportional to the force that produces it.

121. **Limit of Elasticity.**—If the force applied to a body is small enough, that is, within the field of perfect elasticity, the body will return to its original dimensions upon removal of the force. If the force is large enough, that is, beyond the field of perfect elasticity, the body will not return to its original dimensions upon removal of the force. It will be somewhat changed in dimensions; this change is called a *permanent set.* The limit of elasticity is the unit

adopted for measure to the end of the field of perfect elasticity; therefore, we have the definition:

The limit of elasticity of any material is the greatest unit force which may be applied to, or the greatest unit stress which may be developed in, the material without producing a permanent set.

122. **Modulus of Elasticity.**—For a body of any material, the strain or distortion will be different if acted upon by different forces. Even with the same unit force, the strain or distortion will be different for different materials. Therefore, there must be a measure of the strain or distortion of different materials. This measure is called the Modulus or Coefficient of Elasticity, which is expressed by the definition:

The Modulus of Elasticity of any material is the quantity obtained by dividing the unit force or unit stress, not exceeding its limit of elasticity, by the corresponding unit strain or distortion.*

From the definition, it is seen that the Modulus of Elasticity is simply a constant for each material in accordance with its third law of elasticity (par. 120). Expressing this law by an equation, we have

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

in which E is the Modulus of Elasticity, F the total force, A the area of cross-section, l the distortion, and L the total length.

This equation may be expressed in the form

$$\frac{F}{A} = E\frac{l}{L} \tag{122B}$$

which is of the same form as the equation of a right line,

$$y = m\,x.$$

E has the same position in the equation as the coefficient m; hence it is often called the coefficient of elasticity.

123. **Ultimate Strength.**—If the force on the body is increased until the material ruptures (breaks or tears apart), the ultimate

* Modulus is a latin word meaning *measure*.

strength of the material has been reached. Expressed as a definition, we have:

> *The ultimate strength of any material is the unit force applied to, or the unit stress developed in, the material at the moment of rupture.*

Since the ultimate strength of a material can be accurately measured (and the limit of elasticity cannot), it is usually taken as the basis of calculations in civil engineering. In other branches of engineering, however, the elastic limit is often used as a basis for calculations.

DESIGNING.

124. In planning a structure, three general problems are to be considered:

a. The general type of the structure.
b. The nature and magnitude of the forces which are to act upon it.
c. The proper distribution and size of its various parts, so that they shall successfully resist the action of these forces.

125. The nature and magnitude of the forces must be determined after consideration of the purpose of the structure, the location, and the forces which it will have to sustain in order to carry out the purpose for which it is intended. For example, if the structure is to be a bridge, the problem will require consideration of the loads that will come upon it, the wind forces, the foundation, action of floods, etc.

126. The determining of the proper distribution and size of the various parts is simply a series of problems of computing the form and dimensions of each elementary part. Cost of material and deterioration are considered. The standard sizes as commercially manufactured should preferably be used, because it is not possible to buy at a reasonable price the exactly calculated sizes.

127. **Factor of Safety.**—In calculations, allowance must be made for unknown weaknesses in the material and for excessive loading. The allowance is called *factor of safety,* and is the ratio of the ultimate to the allowable unit stress. It is employed to determine the allowable unit stress when the tables give only the ultimate or breaking unit stress.

The factor of safety depends on the uniformity of structure of the material and the character of the force which must be resisted.

The table below gives values of such factors for ordinary building materials:

	Quiescent Force: Buildings	Varying Force: Bridges	Shock: Machines
Timber and cast iron........	6	9	12
Steel and wrought iron......	4	6	8

Engineering handbooks give the allowable unit stresses as well as the ultimate unit stresses. If not given directly for any particular material, the ultimate stress is obtained by experiment or calculating from data in hand-books and divided by the factor of safety in order to determine the allowable unit stress to be used in the engineering structure contemplated. For example, if a piece of steel to be used in a building has an ultimate unit stress of 60,000 pounds, the allowable unit stress is 60,000/4 pounds, or 15,000 pounds.

PROBLEMS.

P. 101. (Fig. P. 101.) By use of the equations of condition for equilibrium, determine completely the force which will produce equilibrium with the forces shown in the figure.

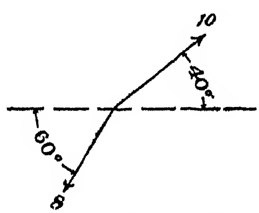


Fig. P. 101.

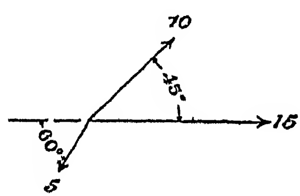


Fig. P. 102.

P. 102. (Fig. P. 102.) By use of the equations of condition for equilibrium, determine completely the force which will produce equilibrium with the forces shown in the figure.

P. 103. (Fig. P. 103.) A semi-circular disk is acted upon by forces as shown in the figure. By use of the equations of condition for equilibrium, determine the directions and intensities of the parallel forces, which if applied at A and B, will hold the disk at rest.

$C = 6$ pounds,
$D = 3$ pounds,
$E = 4$ pounds,
$F = 5$ pounds.

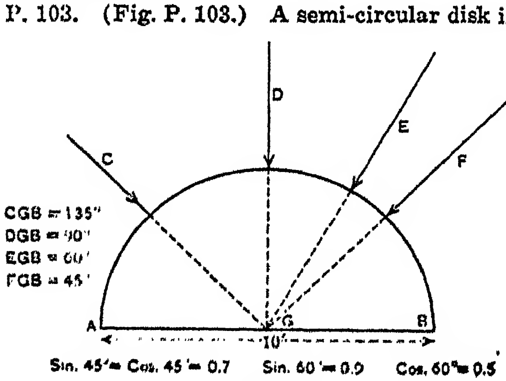


Fig. P. 103.

P. 104. (Fig. P. 104.) A beam AB is at rest. By use of the equations of condition for equilibrium, determine the intensities of the forces A and B.

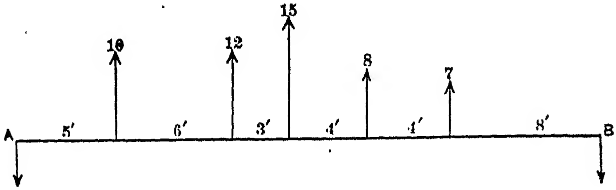


Fig. P. 104.

P. 105. (Fig. P. 105.) By use of the equations of condition for equilibrium, determine the forces which if applied at A and B, will produce equilibrium.

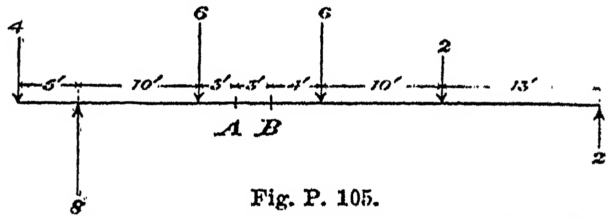


Fig. P. 105.

P. 106. (Fig. P. 106.) By use of the equations of condition for equilibrium, determine the vertical forces X and Y, which will produce equilibrium in the figure.

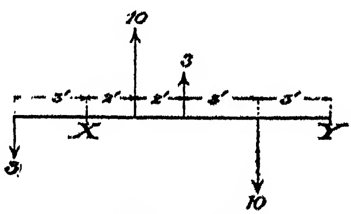


Fig. P. 106.

P. 107. (Fig. P. 107.) A triangle is at rest under the action of five forces. The three given forces are shown in the figure, and intersect the sides of the triangle midway between base and vertex. The two unknown forces are parallel and applied at A and B. Determine their direction and intensities by use of the equations of condition for equilibrium.

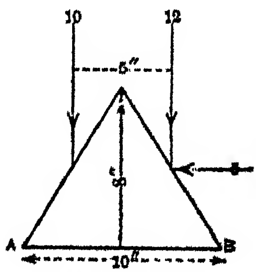


Fig. P. 107.

P. 108. (Fig. P. 108.) A beam acted upon by four forces as shown in the figure is at rest. By use of the equations of condition for equilibrium, determine the intensities of the forces x, y, and z. $C = 10$.

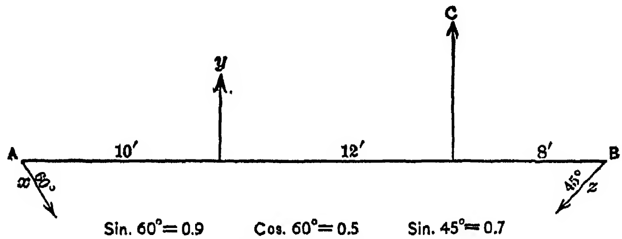


Fig. P. 108.

P. 109. If the modulus of elasticity of cast-iron is 15,000,000 pounds per square inch, how much will a bar 2″ by 3″ by 6′ stretch under a tension of 5,000 pounds?

P. 110. Two bars, one of steel, 4″ by 1″ in rectangular cross-section, and one of brass, 4″ by ⅜″ in rectangular cross-section, are rigidly joined to each other, side by side. The combined bar is then subjected to a load of 24 tons of 2,000 pounds each. Find the load carried by each bar. Neglect weight of bars. E for brass equals 9,000,000 pounds per square inch. E for steel equals 30,000,000 pounds per square inch.

CHAPTER II.

TENSION, COMPRESSION, SHEARING, TORSION, FLEXURE.

TENSION.

201. **Tensile Stress.**—If a steel rod is fixed at one end and pulled with a force, F, at the other, it is found by experiment that

a. All the fibers are elongated.

b. The elongation is proportional to the unit force applied, if the elastic limit is not exceeded.

202. It is assumed that the material is homogeneous, and that the stress is therefore uniformly distributed over each area of cross-section. Expressing this in an equation, we have

$$s_t = \frac{F}{A} \qquad (202A)$$

That is, *the unit tensile stress equals the unit force applied.*

(Note: No consideration is given to change in area of cross-section due to application of the force.)

203. **Limit of Tensile Elasticity.**—If the rod is made of *structural* steel and the force pulling it is gradually increased, the rod will stretch proportionally until the force is about 35,000 pounds per square inch. Up to that unit force, the rod will return to its original dimensions if the force is removed. This is the limit of tensile elasticity. If the force is increased beyond about 35,000 pounds per square inch, the rod will not thereafter stretch proportionally, nor will it return to its original dimensions if the force is removed; it will have received a permanent set.

> *The limit of tensile elasticity of any material is the greatest unit tensile force which may be applied to, or the greatest unit tensile stress which may be developed in, a rod of the material without producing a permanent set.*

204. **Modulus of Tensile Elasticity.**—From paragraph 122, we

have a definition of the modulus of elasticity and the equation

$$E = \frac{\dfrac{F}{A}}{\dfrac{l}{L}} \tag{122A}$$

Applying this to tension, we have $E_t = \dfrac{\dfrac{F}{A}}{\dfrac{l}{L}}$ (204A)

That is, *the modulus of tensile elasticity of any material is the quantity obtained by dividing any unit tensile force or stress, within the limit of elasticity, by the corresponding unit elongation.*

205. **Problem.**—Find the modulus of tensile elasticity of a round bar of wrought iron, $1\frac{1}{4}''$ in diameter and 16′ long, which elongates $\frac{1}{8}''$ under a tensile force of 21,000 pounds.

Solution.—Using equation 204A:

$$E_t = \frac{\dfrac{F}{A}}{\dfrac{l}{L}} \tag{204A}$$

Substituting the values from the statement of the problem, we have

$$E_t = \frac{\dfrac{21{,}000}{\pi(\frac{5}{8})^2}}{\dfrac{\frac{1}{8}}{16 \times 12}} = 26{,}284{,}500 \text{ pounds per square inch.}$$

206. Ultimate Strength in Tension.—If the force pulling the steel rod is increased until it reaches about 70,000 pounds per square inch, the rod will be torn apart, thus indicating that the ultimate tensile strength of steel is about 70,000 pounds.

The ultimate strength in tension of any material is the unit tensile force applied to, or the unit tensile stress developed in, the material at the moment of rupture.

207. Stress-strain Curve.—If, on a piece of cross-section paper, we lay off as ordinates the successive unit tensile forces applied to a rod, and lay off as abscissas the corresponding elongations, we

obtain a stress-strain curve for that material (Fig. 207A). A testing machine may be arranged to plot this curve automatically.

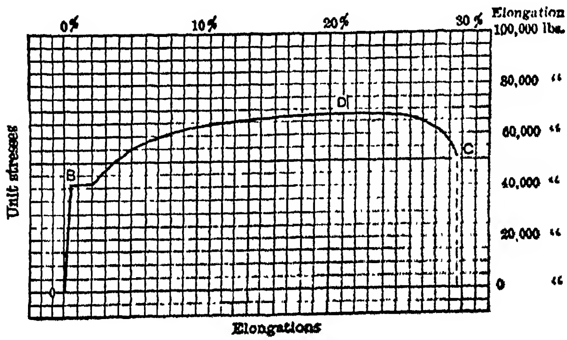


Fig. 207 A

Inspection of this curve shows:

a. It is a right line up to the point *B*, which is the limit of elasticity.

b. At this point, *B*, the elongations suddenly increase, due to the yielding of the material when it passes beyond its limit of elasticity. This is called the *yield point*.

c. Thereafter, increased force is necessary to rupture the piece. This takes place at the point, *D*, which is the point of rupture.

d. The piece does not tear apart immediately at rupture; it still lengthens, and force must still be applied to tear it apart. However, the ultimate strength of rupture is the force at the point, *D*, as thereafter a smaller force can produce rupture.

208. **Allowable Unit Tensile Stress.**—The allowable unit tensile stress is the greatest unit tensile stress to which it is safe to subject a material. It must be less than the ultimate strength in tension, and for building materials, it is less than the limit of tensile elasticity. It is obtained by dividing the ultimate strength in tension by a suitable factor of safety.

209. The table below gives general values of the constants above described, in pounds per square inch, for ordinary building materials. However, they vary considerably for different varieties of same material.

Constants	Wood	Stone	Cast Iron	Wrought Iron	Steel
Limit of tensile elasticity..................	3000	....	6,000	26,000	35,000
Modulus of tensile elasticity E_t...........	1,000,000	1,000,000	15,000,000	27,000,000	30,000,000
Ultimate strength in tension u_t..........	10,000	500	18,000	48,000	70,000
Allowable unit stress in tension a_t.........	1,000	50	3,000	12,000	16,000

Application.

210. The ordinary problem of design in tension consists in determining the area of cross-section to safely resist a given tensile force. We use equation 202A, thus:

$$s_t = \frac{F}{A} \tag{202A}$$

Substitute the allowable value, a_t, for the general value, s_t, thus:

$$a_t = \frac{F}{A} \tag{210A}$$

Make the proper substitutions and determine the area of cross-section necessary.

211. **Problem.**—Find the size of a round wrought-iron rod to carry safely a tensile stress of 100,000 pounds.

Solution.—Using equation (210A):

$$a_t = \frac{F}{A} \tag{210A}$$

Making proper substitutions, we have

$$12{,}000 = \frac{100{,}000}{\pi r^2} \tag{211A}$$

Solving, we obtain

$$r = 1.628'' \tag{211B}$$

Work Performed by Tensile Force.

212. The force applied to lengthen a rod may be either a *gradually applied force* or a *suddenly applied force*. A gradually applied force is one which gradually increases from zero to a maximum; a suddenly applied force is one whose intensity is constant.

If a weight of one pound is suddenly placed in the scale-pan of a spring balance, it will cause the scale-pan to temporarily descend for l'' inches, but it will finally come to rest at say l' inches. If this weight of one pound is applied gradually, as by pouring sand grain by grain into the scale-pan, it will cause the scale-pan to descend l' inches, but no further. To make it descend l'' inches, more sand must be poured in.

213. **Work of Gradually Applied Force.**—The expression for the work done by a gradually applied force, which is a varying force, is

$$Work = \int F\,dl \tag{213A}$$

in which F = varying intensity of the force;
l = path of force.

Taking equation 122A:

$$E = \frac{\dfrac{F}{A}}{\dfrac{l}{L}} \qquad \text{or} \qquad F = \frac{EAl}{L} \tag{122A}$$

Substituting this value of F in equation 213A, we have

$$Work = \int EA\frac{l}{L}\,dl = \frac{EA}{L}\int l\,dl \tag{213B}$$

Integrating for values between l' and 0, we have

$$\textit{Work of a gradually applied force} = \frac{EAl'^2}{2L} = \frac{F'l'}{2} \tag{213C}$$

in which F' is the intensity at l'.

214. **Work of Suddenly Applied Force.**—Taking the most comprehensive case, that of a weight suddenly applied at the end of a rod suspended vertically, and disregarding weight of the rod, we have

$$\text{Work} = \text{Force} \times \text{Path} + \text{Kinetic Energy} = Wl + \frac{WV^2}{2g} \tag{214A}$$

in which,

W = intensity of the force (weight)
l = path of the force at any instant

When the rod has reached its greatest temporary elongation, V becomes zero, and we have the equation for the maximum work done, namely

$$\text{Total work} = Wl \tag{214B}$$

Applying finite values, F'' and l'', to the general equation 214B, we have

$$\text{Total work of suddenly applied force} = F''l'' \tag{214C}$$

in which F'' is the intensity of the suddenly applied force throughout the length l''.

To obtain an expression for the work of a suddenly applied force in terms of the modulus of elasticity, area of cross-section, elongation, and total length, let l_1'' be the maximum elongation which will be obtained if the force F'' is gradually applied, this being likewise the permanent elongation if the force F'' is suddenly applied. Then we have, by substituting these finite values in equation 122A,

$$E = \frac{\frac{F''}{A}}{\frac{l_1''}{L}} \qquad \text{or} \qquad F'' = \frac{EAl_1''}{L} \tag{214D}$$

Substituting this value of F'' in equation 214C, we obtain

$$\text{Total work of a suddenly applied force} = F''\,l'' = \frac{EAl_1''\,l''}{L} \tag{214E}$$

in which F'' is the intensity of the suddenly applied force which will produce a temporary elongation of l'' and a permanent elongation of l_1''.

215. Relation Between Work of Gradually and Suddenly Applied Forces.—To determine the relation between the work performed by the two methods of applying the weight, we take equations 213C and 214C, make the $l' = l''$, thus making the work of one equal to the work of the other, and find the relation between F' and F''. We then obtain

$$\frac{F'\,l}{2} = F''\,l \tag{215A}$$

$$\frac{F'}{2} = F'' \tag{215B}$$

From this we see that

1. The work performed by a suddenly applied tensile force, within the limit of elasticity, is equal to that performed by a gradually applied force of double its intensity.

2. A suddenly applied force will, within the limit of

elasticity, produce a temporary strain as great as the permanent strain produced by a gradually applied force of double its intensity.

3. Since within the limit of elasticity the unit strain varies directly with the unit force, it follows that if a suddenly applied unit force is not to produce a permanent set, it must not exceed one-half the limit of elasticity.

216. **Problem.**—A certain tensile force, suddenly applied, expends 50 foot-pounds of work in elongating a horizontal steel bar 35′ long. Determine the elongation of the bar in inches. $A = \frac{1}{2}$ square inch. $E = 30{,}000{,}000$ pounds.

Solution.—Using equation 214E and remembering that $l_1'' = l''/_2$, we have

$$\text{Work of a suddenly applied force} = \frac{EAl''^2/_2}{L}$$

Substituting proper values, we have

$$12 \times 50 = \frac{30{,}000{,}000 \times \frac{1}{2} \times l''^2}{12 \times 35 \times 2}$$

Solving, we obtain

$$l'' = 0.18''$$

217. **Graphical Representation of Work Done.**—In Fig. 207A, the area between the stress-strain curve, the axis of elongation, and the ordinates of the curve is a measure of the expression $\int F dl$; that is, it is a measure of the work done. The work of stretching the rod to its elastic limit is represented by the area included up to the ordinate of the point B, and is quite small; the work of tearing the rod apart is represented by the area included up to the ordinate of the point C, and is quite large.

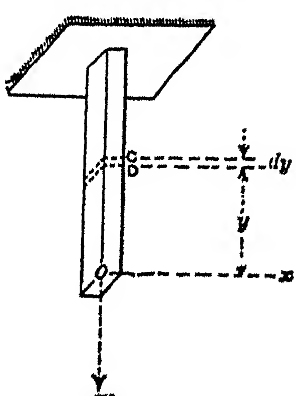


Fig. 218 A

218. **Elongation of a Rod of Uniform Cross-section.**—Let it be required to determine the elongation of the rod due to its own weight, within the limit of perfect elasticity. In Fig. 218A,

Let $DC = dy =$ length of an elementary portion of the rod in inches;
$w =$ weight of a cubic inch of the rod in pounds;
$A =$ area of cross-section in square inches;
$L =$ original length of the rod in inches;
$l =$ elongation of the rod in inches;
$dl =$ elongation of the elementary portion in inches;
$F =$ tensile force in pounds on the area of cross-section at D.

Then, $Ay =$ volume of rod between O and D;
$Awy =$ weight of rod between O and D;
$F = Awy =$ force applied to cross-section D.

Taking equation 122A for the coefficient of tensile elasticity,

$$E_t = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

and substituting the above values, we have

$$E_t = \frac{\frac{Awy}{A}}{\frac{dl}{dy}} \tag{218A}$$

Transposing, we have for the elongation of the elementary portion of the rod at D

$$dl = \frac{Awydy}{AE_t} \tag{218B}$$

To find the elongation of the rod due to its own weight, we integrate between the limits $y = 0$ and $y = L$. This gives

$$l = \frac{\frac{1}{2}wL^2A}{AE_t} = \frac{wL^2}{2E_t} \tag{218C}$$

219. To find the elongation of a rod due to a weight equal to its own weight suspended at its lower end, neglecting the weight of the rod itself, we have

$$wAL = \textit{weight of rod} \tag{219A}$$

Hence, $F = wAL$ = tensile force at cross-section D, and every other cross-section.

Substituting this value of F in equation 112A, we have

$$E_t = \frac{\dfrac{wAL}{A}}{\dfrac{l}{L}} \qquad (219B)$$

$$l = \frac{wL^2}{E_t} \qquad (219C)$$

220. Comparing equations 218C and 219C, we see that *the elongation of a rod due to its own weight is one-half the elongation caused by suspending from its lower end a weight equal to that of the rod, and neglecting the weight of the rod itself.*

The above discussions (paragraph 212-220) are equally applicable to work performed by compressive force, and shortening of a vertical column under its own or a superimposed weight, within the limit of elasticity.

221. **Problem.**—A wrought-iron bar is 10′ long and $\frac{1}{6}$ of a square inch in cross-section. Find the weight which, suspended from the lower end of the bar, will, together with the weight of the bar, produce an elongation of $\frac{1}{2}$″. Weight of wrought-iron = 480 pounds per cubic foot (5/18 pounds per cubic inch).

Solution.—Equation 122A gives elongation due to suspended weight:

$$E_t = \frac{\dfrac{F}{A}}{\dfrac{l}{L}} \text{ or } l = \frac{FL}{E_tA} \qquad (122A)$$

Equation 218C gives elongation due to own weight:

$$l = \frac{wL^2}{2E_t} \qquad (218C)$$

The total elongation is the sum of these two, thus:

$$\text{Total elongation} = \frac{FL}{E_tA} + \frac{wL^2}{2E_t} \qquad (221A)$$

Substituting proper values, we have

$$\frac{1}{2} = \frac{F \times 120}{27{,}000{,}000 \times \frac{1}{6}} + \frac{\frac{5}{18} \times 120 \times 120}{2 \times 27{,}000{,}000}$$

Solving, we obtain

$$F = 18{,}747\tfrac{2}{9} \text{ pounds}$$

COMPRESSION.

222. **Compressive Stress.**—If a steel rod is fixed at one end and a compressive force applied at the other, it is found by experiment that

a. All the fibers are shortened.

b. The shortening is proportional to the unit force applied, if the elastic limit is not exceeded.

(Note.—These laws apply only if the length of the rod does not exceed 10 times its least dimension of cross-section, as the stress of bending must then be considered. This is discussed in Chapter XIV on Columns.)

It is assumed that the material is homogeneous, and that the stress is therefore uniformly distributed over each area of cross-section. Expressing this in an equation, we have

$$s_c = \frac{F}{A} \tag{222A}$$

That is, *the unit compressive stress equals the unit force applied.*

223. **Limit of Compressive Elasticity.**—The limit of compressive elasticity is determined in a manner similar to the limit of tensile elasticity (paragraph 203). Its definition is:

> *The limit of compressive elasticity of any material is the greatest unit compressive force which may be applied to, or the greatest compressive stress which may be developed in, a short rod of the material without producing a permanent set.*

224. **Modulus of Compressive Elasticity.**—From paragraph 122, we have a definition of the modulus of elasticity and the equation

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

Applying this to compression, we have $E_c = \dfrac{\frac{F}{A}}{\frac{l}{L}}$ (224A)

That is, *the modulus of compressive elasticity of any material is the force or stress obtained by dividing any unit compressive*

force or stress, within the limit of elasticity, by the corresponding unit shortening.

225. **Ultimate Strength in Compression.**—If the compressive force on a *structural* steel rod is increased until it reaches about 50,000 pounds per square inch, the rod will be crushed (ruptured), thus indicating that the ultimate crushing strength is 50,000 pounds. The rod must be so short that it is crushed before bending. If of steel, the rod will crush first if its diameter is not less than one-tenth of its length; if of wood, its diameter must be not less than one-fifth of its length.

The ultimate strength of crushing of any material is the unit compressive force applied to, or the unit compressive stress developed in, the material at the moment of rupture.

226. **Stress-strain Curve.**—A curve for compression will be of the same general character as that for tension (Fig. 207A). For certain materials, the limit of elasticity and the ultimate strength for tension and compression are so nearly equal that it is quite usual to employ the same values for both.

227. **Allowable Unit Compressive Stress.**—The allowable unit compressive stress is the greatest unit compressive stress to which it is safe to subject a material. It must be less than the ultimate strength, and, for building materials, it is usually less than the limit of elasticity. It is obtained by dividing the ultimate strength by a suitable factor of safety.

228. The table below gives general values of the constants above described in pounds per square inch for ordinary building materials.

Constants	Wood	Stone	Cast Iron	Wr'g't Iron	Steel
Limit of Compressive elasticity	3,000	....	6,000	26,000	35,000
Modulus of Compressive elasticity E_c	1,000,000	1,000,000	15,000,000	27,000,000	30,000,000
Ultimate strength in compression u_c	5,000	5,000	80,000	48,000	50,000
Allowable unit stress in compression a_c	800	600	10,000	10,000	16,000

Application.

229. The ordinary problem of design in compression consists in determining the area in cross-section to safely resist a given force

in compression. We use equation 222A, with allowable unit stress in compression, thus:

$$a_c = \frac{F}{A} \tag{229A}$$

Make the proper substitutions, and determine the area of cross-section.

230. **Problem.**—What is the greatest weight in compression which can be carried by a cube of building stone which measures 10″ on an edge?

Solution.—Using equation 229A:

$$a_c = \frac{F}{A} \qquad \text{(229A}$$

Making proper substitutions, we have

$$600 = \frac{F}{10 \times 10} \tag{230A}$$

Solving, we obtain

$$F = 60{,}000 \text{ pounds} \tag{230B}$$

SHEARING.

231. If two steel plates are riveted together, and a force is then applied to slide one along the other, it is found by experiment, that

a. The fibers of the shank of each rivet are equally distorted laterally.

b. The distortion is proportional to the unit force applied, if the elastic limit is not exceeded.

It is assumed that the material is homogeneous, and that the stress is therefore uniformily distributed over each area of cross-section. Expressing this in an equation, we have

$$s_s = \frac{F}{A} \tag{231A}$$

That is, *the unit shearing stress equals the unit force applied.*

232. **Limit of Shearing Elasticity.**—It is difficult to measure accurately the distortions produced in the rivets as the force shears them; but most effective measurements place at 30,000 pounds per square inch the limit beyond which a shearing force will produce a permanent set in rivets made of steel.

The limit of shearing elasticity of any material is the greatest unit shearing force which may be applied to, or

the greatest unit shearing stress which may be developed in, the material without producing a permanent set.

233. **Modulus of Shearing Elasticity.**—From paragraph 122, we have a definition of the modulus of elasticity and the equation

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

Applying this to shearing, we have $$E_s = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{233A}$$

That is, *the modulus of shearing elasticity of any material is the force or stress obtained by dividing any unit shearing force or stress, within the limit of elasticity, by the corresponding unit distortion.*

As with the limit of shearing elasticity, this cannot be exactly measured; but the most effective results indicate that it is about 9,000,000 pounds per square inch for steel.

234. **Ultimate Strength in Shearing.**—If the shearing force is increased until it reaches about 70,000 pounds per square inch, steel rivets will be ruptured along planes perpendicular to their axes, thus indicating that the ultimate shearing strength is about 70,000 pounds.

The ultimate strength in shearing of any material is the unit shearing force applied to, or the unit shearing stress developed in, the material at the moment of rupture.

235. **Allowable Unit Shearing Stress.**—The allowable unit shearing stress is the greatest unit shearing stress to which it is safe to subject a material. It is obtained by dividing the ultimate strength in shearing by a suitable factor of safety. The table below gives general values of the constants above described, in pounds per square inch, for ordinary building materials.

Constants	Wood with grain	Wood across grain	Cast Iron	Wrought Iron	Steel
Ultimate strength in shearing u_s	600	3,000	20,000	40,000	70,000
Allowable unit shearing stress a_s	150	1,000	3,000	9,000	12,000

Application.

236. The ordinary problem of design in shearing consists in determining the area in cross-section to safely resist a given force. We use equation 231A, with allowable unit stress in shearing, thus:

$$a_s = \frac{F}{A} \tag{236A}$$

Make the proper substitutions, and determine the area of cross-section.

237. **Problem.**—The weight of an eight-wheel freight car with load is 100,000 pounds. What should be the diameter of the steel axles for safety against shearing?

Solution.—Using equation (236A):

$$a_s = \frac{F}{A} \tag{236A}$$

Making proper substitutions, we have

$$12{,}000 = \frac{100{,}000/8}{\pi d^2/4} \tag{237A}$$

Solving, we obtain $d = 1''.151$

238. **Shearing Stress in Plane Perpendicular to Force.**—Careful observation of the deformations produced by shearing forces, show that there is a shearing stress developed in planes perpendicular to the action line of the shearing force.

Let it be required to find and compare the unit shearing stresses in the vertical and horizontal planes, due to a vertical shearing force, as F in Fig. 238A. Let $ABCDEFGH$ be an elementary cube

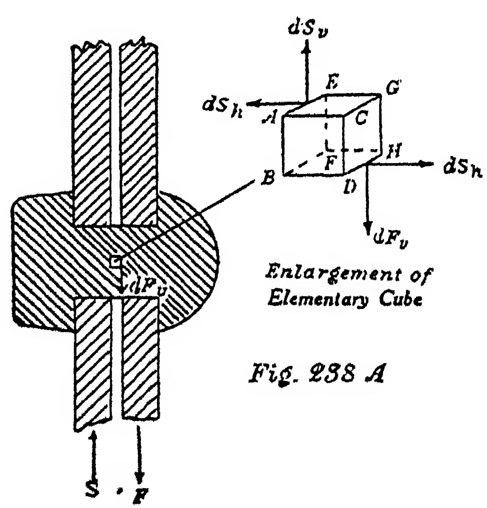


Fig. 238 A

in the interior of the rivet subjected to shear. The faces *ABFE* and *CDHG* are parallel to the applied shearing force.

$dt = AB$, the depth of the cube.
$db = AE$, the breadth of the cube.
$dl = AC$, the length of the cube.
$dF_v =$ elementary shearing force applied on face *CDHG*.
$dS_v =$ elementary shearing stress developed in face *ABFE*.

Since the stress developed must equal the force causing it, we have

$dF_v = dS_v$, the two forces forming a couple. (238A)

The effort of the force dF_v is to move the face on which it acts, in the direction of the force, and to increase the angles of the elementary cube at *B* and *C*, and decrease the angles at *A* and *D*. This would cause the planes *AEGC* and *BDHF* to slide on the corresponding faces of the cubes above and below. As the cube does not so slide, it must be held by some force. Since the vertical forces on the elementary cube form a couple (equation 238A), the horizontal forces must also form an equal and opposite couple. Equating the two, we have

$$dF_v dl = dS_h dt \qquad (238B)$$

Remembering that (equation 231A)

$$s_s = \frac{F}{A} \quad \text{or} \quad F = s_s A \qquad (231A)$$

We have, $dF_v = s_{sv}\ area\ CDHG = s_{sv}\ db\ dt$ (238C)

And, $dS_h = s_{sh}\ area\ BDHF = s_{sh}\ dl\ db$ (238D)

Substituting in equation 238B we have,

$$s_{sv}\ db\ dt\ dl = s_{sh}\ dt\ dl\ db \qquad (238E)$$

Hence $s_{sv} = s_{sh}$ (238F)

That is, *the unit shearing stresses on the vertical and horizontal faces of the elementary cube are equal.*

TORSION.

239. **Torsional Moment.**—If a force, *F* (Fig.239A), is applied to a shaft, so that its action line lies in a plane perpendicular to the axis of the shaft but does not intersect this axis, it will produce torsional stress in the shaft unless the shaft can rotate freely upon its axis. The moment produced by this force is called the torsional moment (M_{tr}), and equals the intensity of the force times its lever arm to the axis.

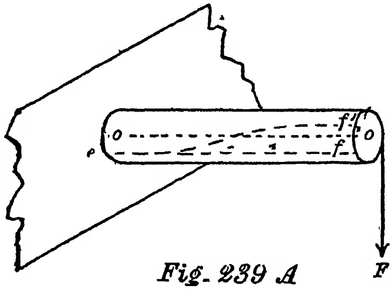


Fig. 239 A

240. The torsional moment is transmitted from one section to another, until it reaches the fixed end. Assuming the shaft to be cut in half and taking the outer half as a free body, it will be necessary to replace the stress at the inner half by an equal and opposite force, otherwise the outer half would rotate. If one torsional force is applied at one point of the half cut off, and another at another point of it, then at the section cut the force to establish equilibrium must be equal and opposite to the resultant of the other two. If the section is cut between the two forces, only the outer force need be considered.

Hence, *the torsional moment at any cross-section of a shaft is equal to the algebraic sum of the torsional moments acting on one side of the section.*

241. **Torsional Stress.**—Careful observations of experiments with torsional forces lead to the following hypotheses:

a. Each section between the outer section and the fixed section, is rotated about the axis of the shaft.

b. The amount of rotation of any section will vary with the force applied and with the distance of this section from the fixed section.

c. The amount of rotation of any point in a section varies with its distance from the axis of the shaft.

242. As shown in Fig. 239A, the rotation is gradually decreased from the outer end where it is a maximum to the fixed section where it is zero; that is, the intermediate sections transmit and adjust the force and rotation from the outer section to the equal stress and zero rotation at the fixed section. The fiber *ef* becomes a helix; in the outer section, there is an angle through which the fiber is rotated, *fof′*, called the angle of torsion; and the arc *ff′* is called the arc of torsion.

243. Let it be required to find the unit torsional stress at a distance *r* from the axis.

From hypothesis 241*c*, based on experiment, the amount of this distortion is proportional to the distance from the center. From this, since within the elastic limit the stress developed is porportional to the distortion, we know that the *stress in any fiber will vary directly with its distance from the axis of rotation.*

Now, since $(s_{tr})_1$ = unit torsional stress in a fiber at unit distance from the axis,

then, $(s_{tr})_1\ r$ = unit torsional stress in a fiber at a distance r from the axis.

Assuming $dy\ dz$ = elementary area of cross-section,

then, $(s_{tr})_1\ r\ dy\ dz$ = total torsional stress in an elementary area at distance r from the axis.

$(s_{tr})_1\ r^2\ dy\ dz$ = torsional moment of an elementary area at distance r from the axis.

$(s_{tr})_1 \int \int r^2\ dy\ dz$ = torsional moment of all the stresses about the axis.

Since the torsional moment of the stresses equals the torsional moment of the force, we have

$$(s_{tr})_1 \int \int r^2 dy\ dz = M_{tr} \qquad (243A)$$

The expression $\int \int r^2\ dy\ dz$ is given in Mechanics as the moment of inertia of a plane area about an axis through its center and normal to its plane, and is called the polar moment of inertia (I_p).

Substituting, we have

$$(s_{tr})_1 I_p = M_{tr} \qquad (243B)$$

Hence

$$(s_{tr})_1 = \frac{M_{tr}}{I_p} \qquad (243C)$$

Multiplying both members by r, we have

$$(s_{rt})_1 r = \frac{M_{tr}\, r}{I_p} \qquad (243D)$$

That is, *the unit torsional stress at a distance r from the axis is equal to the product of the torsional moment by the distance r, divided by the polar moment of inertia of the cross-section.*

244. Since the stress cannot be uniformly distributed over the area of cross-section, there can be no unit area over which the stress is uniform, as is the case in tension, compression, and shearing. The unit stress at a distance r from the axis is, therefore, the stress on a hypothetical unit area, each element of which is subjected to the same stress as the elementary area $dy\ dz$ which is at a distance r from the axis.

Applying this to the surface fiber, where the stress (s_{tr}) is greatest, we have, assuming r as the distance to the surface fiber,

$$(s_{tr})_1 r = \text{stress at surface fiber} = s_{tr}$$

Substituting in equation 243D, we have

$$s_{tr} = \frac{M_{tr}\, r}{I_p} \tag{244A}$$

That is, *the unit torsional stress in the surface fiber is equal to the product of the torsional moment by the distance from the axis to the surface fiber, divided by the polar moment of inertia of the cross-section.*

245. Limit of Torsional Elasticity.—If the force, F, in Fig. 239A is gradually increased, it will eventually reach a value which will produce a permanent set in the surface fiber. This limiting value is the limit of elasticity in torsion or the limit of torsional elasticity of the material.

The limit of torsional elasticity of a material is the greatest unit torsional stress which can be developed in the surface fiber of a shaft of the material without producing a permanent set.

246. Modulus of Torsional Elasticity.—From paragraph 122, we have a definition of the modulus of elasticity and the equation

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

Applying this to torsion, we find that F/A is not applicable to torsion, because the force cannot be applied to a unit area. However, the stress on the surface fiber (s_{tr}) is the determining factor and this is substituted for F/A. Thus, we have

$$E_{tr} = \frac{s_{tr}}{\frac{l}{L}} \tag{246A}$$

or

$$s_{tr} = E_{tr}\frac{l}{L} \tag{246B}$$

in which l = distortion of surface fiber = ff' in Fig. 239A.
L = length of every fiber = ef in Fig. 239A.
s_{tr} = unit torsional stress in the surface fiber.

Hence, the modulus of torsional elasticity of any material is the quantity obtained by dividing the unit torsional stress on the surface fiber of a shaft, by the unit distortion of that fiber, within the limit of elasticity.

247. **Ultimate Strength in Torsion.**—If the force be increased until the surface fiber of the shaft ruptures, the corresponding value of stress in the surface fiber will be the ultimate strength in torsion of the material of the shaft. *The ultimate strength in torsion is the unit stress on the extreme fiber of a shaft of the material, at the instant of rupture.*

248. **Allowable Unit Torsional Stress.**—The allowable unit torsional stress is the greatest unit torsional stress to which it is safe to subject the material. This stress is reached first in the surface fiber; and it is not safe to increase this stress even though the stresses on the inner fibers have not reached the limit of safety. It is evident that the material will be unsafe if any of its fibers are stressed beyond safety.

249. The approximate values of the constants of torsion for ordinary building materials are given in the following table:

Constants	Wood	Cast Iron	Wrought Iron	Steel
Modulus of elasticity........	.....	6,000,000	10,000,000	12,000,000
Ultimate strength in torsion..	1,500	25,000	50,000	60,000
Allowable unit stress.......	375	4,000	9,000	10,000

Application.

250. The problem of designing a piece to resist torsional stress is usually that of determining the diameter of a shaft to resist a given torsional moment.

We use equation 244A with safe unit stress in torsion, and with r equal to the maximum distance to the surface fiber.

Thus $$a_{tr} = \frac{M_{tr} r}{I_p} \qquad (250A)$$

251. **Problem.**—The screw of a screw jack is of steel and has a diameter of 2″; it is operated by the direct application of a bar. If a man can gradually apply a force of 300 pounds at the end of the bar, how long may the bar be made and the screw still be safe against failure by torsion. $I_p = \frac{\pi r^4}{2}$

Solution.—Using the equation (250A):

$$a_{tr} = \frac{M_{tr}r}{I_p} \tag{250A}$$

Making proper substitutions, we have

$$10{,}000 = \frac{300 \times l \times 1}{\dfrac{\pi \times (1)^4}{2}}$$

Solving, we obtain

$$l = 52.36''$$

252. **Power Transmitted by Shafts.**—In Fig. 252A, let E be the pulley run by a belt to the engine, and let M be the pulley from which a belt leads to the machines. It is assumed that neither belt slips.

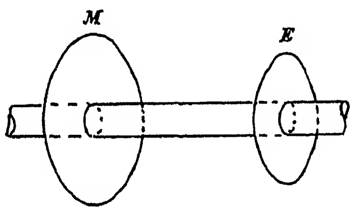


Fig. 252 A

Let the uniform pull on belt E in pounds be represented by F, and the radius of the wheel in inches by r_w. At each revolution of the wheel the path of the force F is $2\pi r_w$ and the work performed by the force F is $2\pi r_w F$. If the shaft makes N revolutions in one minute, the work per minute is $2\pi r_w FN$. This is reduced to horsepower by dividing by $33{,}000 \times 12$, a horse-power being 33,000 foot-pounds per minute.

Hence

$$\text{Horse-power} = \frac{2\pi r_w FN}{33{,}000 \times 12} \tag{252A}$$

To determine the size of a shaft to transmit a certain horse-power, we must have this equation in terms of stress in surface fiber of the shaft. We know that $Fr_w = M_{tr}$, and from equation 244A, $M_{tr} = \dfrac{s_{tr}I_p}{r}$, where r is the distance from the center to the surface fiber (radius of the shaft).

Hence,
$$Fr_w = M_{tr} = \frac{s_{tr} I_p}{r}$$

Substituting, we obtain

$$\text{Horse-power} = \frac{2\pi N\, s_{tr}\, I_p}{33{,}000 \times 12 \times r} \tag{252B}$$

From mechanics, we know that I_p, for a circle $= \frac{1}{2}\pi r^4$
Substituting, we obtain

$$\text{Horse-power} = \frac{s_{tr}\pi^2 r^3 N}{396{,}000} = \frac{9.87 s_{tr} r^3 N}{396{,}000} = \frac{s_{tr} r^3 N}{40{,}000} \text{ approximately.} \tag{252C}$$

253. **Problem.**—Calculate the horse-power that a round, wrought iron shaft 8″ in diameter and making 150 R. P. M. will transmit with a factor of safety of 6.

Solution.—Using equation 252C:

$$\text{H. P.} = \frac{s_{tr}\, r^3 N}{40{,}000}$$

Substituting, and remembering that $s_{tr} = \frac{u_{tr}}{6} = \frac{50{,}000}{6}$, we have

$$\text{H. P.} = \frac{50{,}000 \times 4 \times 4 \times 4 \times 150}{6 \times 40{,}000} = 2{,}000$$

BENDING OR FLEXURE.

254. **Bending Moment.**—If a force, F (Fig. 254A), is applied to the end of a beam fixed at the other end, and its action line is perpendicular to the axis of the beam, it will produce bending stress in the beam. The moment produced by this force at any section is

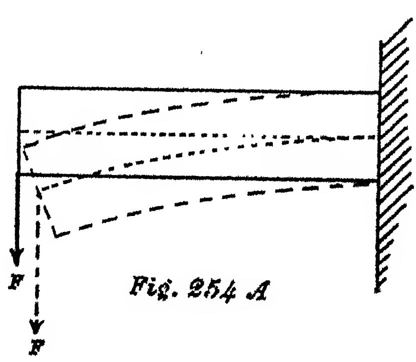


Fig. 254 A

called the bending moment or moment of flexure M_f, and equals the intensity of the force times the lever arm from the point of application to the section.

(Note: — In paragraph 110*b*, it is stated that clockwise moments are considered positive; counter-clockwise moments, negative. *Bending moments are however considered positive when they produce compression in the upper fibers of a beam and tension in the lower fibers.* The two systems of notation will agree if the center of moments is to the right of the forces. Therefore, in calculations for bending moments, it is usually preferable to work from the left end of the beam, as by working from the left end of a beam it is possible to designate forces acting upward as positive and forces acting downward as negative, multiply them by their lever arms without regard to sign, and obtain the bending moment with its proper sign.)

255. Let it be required to find the bending moment at any section of a beam, under the action of several forces.

Taking the beam in Fig. 255A as in a state of rest when acted upon by the four bending forces, and taking *c* as the section at which the bending moment is desired, then we have for the total bending moment:

Working from the left, $F' \times ac - F'' \times bc =$ bending moment at section *c*.

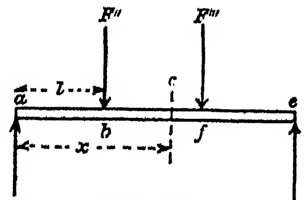


Fig. 255 A

Working from the right, $F^{iv} \times ec - F''' \times fc$ = bending moment at *c*.

As the two are the same, we have

$$M_f = F' \times ac - F'' \times bc = F^{iv} \times ec - F''' \times fc \qquad (255A)$$

That is, *the bending moment at any section of a beam is the algebraic sum of the bending moments of all bending forces acting on either side of the section;* or expressed in another way, it is *the algebraic sum of the products of all forces on either side of the section, each with its proper sign, by their respective lever arms to the section.*

256. Therefore, to determine the bending moment at any section of a beam, two steps are necessary:

a. Find the intensities and action lines of all of the forces on one side (preferably left) of the section. To find these forces, employ the equations of equilibrium as may be necessary.

b. Multiply each force (without changing sign if on the left) by its lever arm to the section, and add algebraically the bending moments so obtained.

As stated above, it is usually simpler to use the forces on the left of the section, because the forces (with their proper signs) multiplied by their lever arms give the bending moments with the proper signs. However, if it is easier to find the forces on the right of the section, it is not difficult to obtain the bending moment by paying proper attention to the signs.

257. **Problem.**—A cantilever beam is loaded as shown in Fig. 257A. Find bending moments at sections *b*, *c*, *d*.

Solution.—From the principle in paragraph 255, we have

At *b*, $M_f = -100\ (8-5) = -300$ ft. lbs. (257A)

At *c*, $M_f = -100\ (12-5) - 200\ (12-10) = -1100$ ft. lbs. (257B)

At *d*, $M_f = -100\,(16-5) - 200\,(16-10) - 300\,(16-15) = -2600$ ft. lbs. (257C)

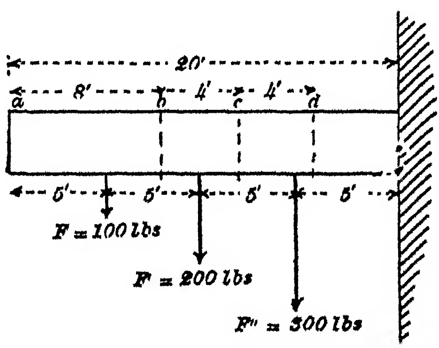


Fig. 257 A

258. **Bending Stress (Stress of Flexure).**—Careful observations of experiments with bending forces acting perpendicular to the axis of a beam lead to the following hypotheses, with reference to the bending effect, only:

a. That the fibers on the convex side of the beam are extended, and those on the opposite side are shortened.

b. That there is a neutral surface between the compressed and extended fibers in which the fibers are neither compressed nor ex-

tended. The intersection of this neutral surface by the plane of cross-section is called the *neutral axis* of the section.

c. That the cross-sections of the beam normal to the fibers before bending will remain normal to them after bending.

d. That rupture will take place either by compression, or by tension, of the fibers on the surface of the piece when the stress is equal to the ultimate strength of compression or tension.

From the above hypotheses, we deduce that:

e. The strains of the fibers caused by the bending force are either compressive or tensile (from *a*).

f. The strains of the fibers are proportional to their distances from the neutral surface (from *b* and *c*).

As shown in Fig. 254A, the bending is gradually decreased from the outer end where it is a maximum to the fixed section where it is zero: that is, the intermediate sections, as in torsion, are moved by the applied force, but the distance through which they move varies from a maximum at the outer end to zero at the fixed end, and the internal couple which the material exerts to hold itself together varies from zero to a maximum.

259. Let it be required to find the unit bending stress (stress of flexure) in the surface fiber of a section of a beam.

In Fig. 259A, take the portion of the beam to the left of the section *cc* as a "free body." Then since it must be at rest, we must have the force, $+ F_v$, to balance the given force, $- F_v$; and we must have the two horizontal forces, $+ S_h$ and $- S_h$, representing the developed tensile and compressive stresses, to balance the couple formed by $+ F_v$ *and* $- F_v$.

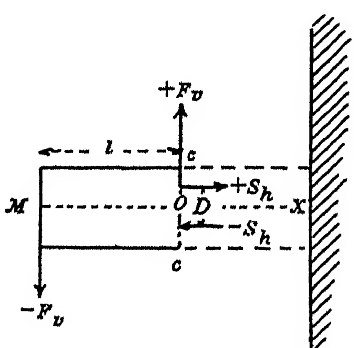


Fig. 259 A

The moments of these two couples are equal, and the moment due to the forces $+ F_v$ and $- F_v$ equals the bending moment at the

section. Expressed in an equation we have, taking the center of moments at O,

$$S_h \frac{D}{2} + S_h \frac{D}{2} = S_h D = -F_v l = M_f \qquad (259A)$$

$S_h D$ is the expression for the moment of stress on the entire area of cross-section. But the greatest stress is at the surface fiber: therefore, the next step is to obtain an expression for the unit stress at the surface fiber.

First, from 258*f*, the amount of distortion is proportional to the distance from the neutral surface. From this, since within the elastic limit the stress is proportional to the distortion, we know that the stress on any fiber will vary directly with its distance from the neutral surface.

Let $(s_f)_1$ = unit bending stress in a fiber at unit distance from the neutral surface.

Then $(s_f)_1 y$ = unit bending stress in a fiber at distance y from the neutral surface.

Assume $dy\, dz$ = elementary area of cross-section.

Then $(s_f)_1 y\, dy\, dz$ = total bending stress in an elementary area at distance y from the neutral surface.

$(s_f)_1 y^2\, dy\, dz$ = bending moment of an elementary area at distance y from the neutral surface.

$(s_f)_1 \int \int y^2\, dy\, dz$ = bending moment of all the stresses with respect to the neutral axis, if integrated between proper limits.

Since the bending moment of the stresses equals the bending moment of the force, we have

$$(s_f)_1 \int \int y^2\, dy\, dz = S_h d = M_f \qquad (259B)$$

The expression $\int \int y^2\, dy\, dz$ is given in mechanics as the moment of inertia of a plane area about the axis in its own plane, corresponding to the neutral axis in this case. It is represented by I.

Substituting, we have $(s_f)_1 I = M_f \qquad (259C)$

Hence
$$(s_f)_1 = \frac{M_f}{I} \qquad (259D)$$

That is, *the unit bending stress at a unit distance from the neutral surface is equal to the bending moment at the section divided by the moment of inertia of the section.*

260. As in torsion, the stress cannot be uniformly distributed over the area of the section and there can be no unit area over which

Assume $\delta y \delta z$ = elementary area of cross-section.

the stress is uniform, as in tension, compression, and shearing. The unit bending stress at a distance y from the neutral surface is, therefore, the stress on a hypothetical unit area, each element of which is subject to the same stress as the elementary area $dydz$ which is at a distance y from the neutral surface.

Applying this to the surface fiber, where the stress (s_f) is greatest, we have, assuming y as the distance to the surface fiber,

$$(s_f)_1\, y = \text{stress at surface fiber} = s_f$$

Substituting in equation 259D, we have

$$(s_f)_1\, y = s_f = \frac{M_f y}{I} = \frac{M_f}{\frac{I}{y}} \qquad (260A)$$

That is, *the unit bending stress in the surface fiber of a beam at any cross-section is equal to the product of the bending moment at the section by the distance from the neutral surface to the surface fiber, divided by the moment of inertia of the cross-section.*

The expression I/y is called the *section modulus.* Therefore the principle expressed by equation 260A may be stated in another manner, thus: *The unit bending stress in the surface fiber of a beam at any cross-section is equal to the bending moment at the section divided by the section modulus of the section.*

261. The values of I, y, and I/y for ordinary forms of cross-section are tabulated in engineering manuals. Some of these are given in the following table:

Forms of cross-section	Dimensions	I	y	Section Modulus $\frac{I}{y}$
Rectangle	breadth b; depth D	$\frac{bD^3}{12}$	$\frac{D}{2}$	$\frac{bD^2}{6}$
Square with a vertical diagonal	side of square D	$\frac{D^4}{12}$	$D\sqrt{\frac{1}{2}}$	$\frac{D^3}{8.5}$
Triangle with horizontal base	base b; altitude D	$\frac{bD^3}{36}$	$\frac{2}{3}d$	$\frac{bD^2}{24}$
Circle	diameter D	$\frac{\pi D^4}{64}$	$\frac{D}{2}$	$\frac{D^3}{10+}$
Ellipse	axes b and D; b = neutral axis	$\frac{\pi bD^3}{64}$	$\frac{D}{2}$	$\frac{bD^2}{10+}$

262. **Limit of Bending Elasticity.**—If the force, F, Fig. 259A, is gradually increased, it will eventually reach a value which will produce a permanent set in the surface fiber of the cross-section considered. This limiting value of the stress in the outer fiber is the limit of elasticity of the material in bending or flexure. Actually, it is a tensile or compressive stress and its limiting value is usually taken as that of the material in tension, under the assumption that the stress on the surface fiber could be measured over a unit area.

The limit of bending elasticity of a material is the greatest unit bending stress which can be developed in the surface fiber of a beam of the material without producing a permanent set.

263. **Modulus of Bending Elasticity.**—From paragraph 122, we have a definition of the modulus of elasticity and the equation

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

But the expression F/A is not applicable to flexure since the force cannot be applied to a unit area. However, the stress on the surface fiber (s_f) is the determining factor and this is substituted for F/A. Thus we have

$$E_f = \frac{s_f}{\frac{l}{L}} \tag{263A}$$

or

$$s_f = E_f \frac{l}{L} \tag{236B}$$

Hence, *the modulus of bending elasticity of any material is the quantity obtained by dividing the unit bending stress in the surface fiber of a beam, by the unit distortion of that fiber, within the limit of elasticity.*

264. **Ultimate Strength in Bending or Flexure.**—If the force be increased until the surface fiber of the beam ruptures at the cross-section considered, the corresponding value of the stress in the surface fiber will be the ultimate strength in bending of the material of the beam. *The ultimate strength of bending or flexure of a material is the unit bending stress in the extreme fiber of a beam of the material at the instant of rupture by a bending force.*

265. **Allowable Unit Bending Stress.**—The allowable unit bending stress is the greatest unit bending stress to which it is safe to subject the material. This stress is reached first in the surface fiber; and, as in torsion, it is not safe to increase the stress even though the inner fibers have not reached their safe bending stress. The surface fiber is the determining fiber, and when its unit fiber stress has reached the safe bending stress, the beam has reached the limit of its safety.

266. The values of the constants of bending for ordinary building materials are given approximately in the following tables:

Constants	Wood	Cast Iron	Wrought Iron	Steel
Modulus of bending elasticity	1,000,000	15,000,000	27,000,000	30,000,000
Ultimate strength in bending	6,000	36,000	48,000	60,000
Safe unit stress in bending	1,000	6,000	12,000	16,000

Application.

267. The problem of designing a piece to resist bending stress consists in determining the dimensions of a piece which can safely resist a given bending force.
We use equation 260A, with allowable unit bending stress, thus

$$a_f = \frac{M_f}{\frac{I}{y}} \tag{267A}$$

make the proper substitutions, and determine the dimensions.

268. **Problem.**—A rectangular cantilever wooden beam, 6″ wide, 8″ deep, and 6′ long, is to carry a weight of 1,000 pounds at its free end. What is its factor of safety?

Solution.—The maximum bending moment is at the fixed end and = 1000 × 6 × 12 = 72,000 inch pounds.
Using equation 260A:

$$s_f = \frac{M_f}{\frac{I}{y}} \tag{268A}$$

Making proper substitutions, we have

$$s_f = \frac{72{,}000}{\frac{6 \times 8^2}{6}} = 1125 \text{ pounds} \tag{268B}$$

$$\text{Factor of safety} = \frac{v_f}{1125} = \frac{6000}{1125} = 5\tfrac{1}{3} \tag{268C}$$

Vertical or Transverse Shear.

269. Another effect of a bending force is to move the consecutive planes of cross-section along each other and thus cause shearing stress. The shearing stress at any cross-section is called the *vertical or transverse shear*, (S_v). As beams are usually horizontal, the term vertical shear is generally used.

270. If in Fig. 270A (same as Fig. 255A) we consider as a "free" body the portion of the beam to the left of the section c, then in order to maintain equilibrium of forces, not considering moments, there must be at c a force equal and opposite to the resultant of F' and F''. As neither of these forces acts at c, it is evident that their intensities must be transmitted from section to section until their resultant reaches c.

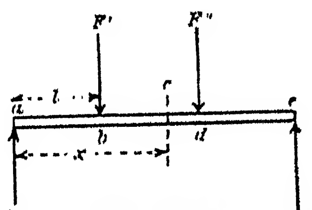


Fig. 270 A

Thus the force F', acting in the end section, will tend to shear off that section, but it will be opposed by the resistance to shearing of the fibers connecting it with the next section, and its effect will be transmitted from section to section, from a to c. In a similar manner the effect of F'' will be transmitted from section to section, from b to c. The resultant shear, or force transmitted to c, will be their difference.

If we take as a "free body" a portion of the beam to the left of F'', we need place at that section a force only equal and opposite to F'. If we worked from the right, the forces necessary to hold the right portions in equilibrium would be the same numerically but contrary in direction.

The shearing at any section is therefore equal to the resultant of all the bending forces acting on either side of the section: or expressed in another way, it is the *algebraic sum of all the bending forces acting on either side of the section.*

271. **Problem.**—A cantilever beam is loaded as shown in Fig. 271A. Find shear at sections b, c, d.

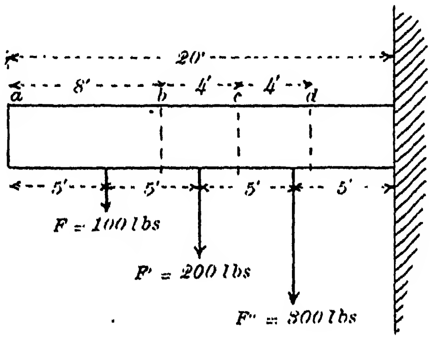


Fig. 271 A

Solution.—Working from the left, and obtaining the algebraic sum of the forces to the left of each section, we have

Left end to F,	$S_v = 0$		(271A)
F' to F'',	$S_v = 100$	(this includes the section at b)	(271C)
F'' to F''',	$S_v = 300$	(" " " " " c)	(271C)
F''' to wall,	$S_v = 600$	" " " " " d)	(271D)

272. For every force that is acting on a beam, shear due to it is transmitted to the right and left from its point of application. If the force acts at the middle of a beam resting on two supports, half of it will be transmitted from the middle through section after section to each point of support. If as represented by Fig. 272A, the load is not in the middle, we have from the general equations of equilibrium

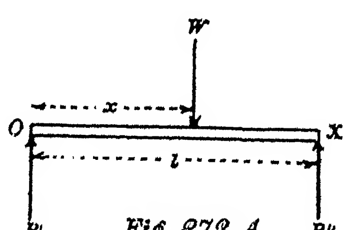


Fig. 272 A

$$(1)\quad R' + R'' = W \tag{272A}$$

And, taking moments around O.

$$(2)\quad R''l = Wx \tag{272B}$$

Solving we obtain,
$$R'' = \frac{Wx}{l} \tag{272C}$$

Similarly, we obtain,
$$R' = \frac{W\,(l - x)}{l} \tag{272D}$$

Hence,
$$R' : R'' :: l - x : x \tag{272E}$$

Therefore, *the weights transmitted to the supports by shear from section to section are inversely proportional to the distances from the point of application to the support.*

273. **Problem.**—A beam 24′ long, supported at both ends, has a load of 8 pounds placed 10′ from its left support. How much of this load is transmitted to each support?

Solution.—The load is transmitted to the supports in portions of $10/24 \times 8$, and $14/24 \times 8$. Of these, $10/24 \times 8$ ($= 3\frac{1}{3}$) pounds goes to the most distant support, that is, to the right. The remainder of the load, $4\frac{2}{3}$ pounds, goes to the left support.

274. **Relation Between Shear and Bending Moment.**—In Fig. 270A, we have for the bending moment of the section c at a distance x from the left end

$$M_f = F'x - F''(x - l) = (F' - F'')\,x + F''l \tag{274A}$$

Since $F' - F'' = S_v$, we have by substitution

$$M_f = S_v x + C \tag{274B}$$

Differentiating
$$\frac{dM_f}{dx} = S_v \tag{274C}$$

Therefore, the vertical shear measures the rate of change of the bending moment along the axis of the beam: that is, the ordinate of the shear curve gives the value of the tangent (slope) of the bending moment curve.

PROBLEMS.

P. 201. A steel bar of circular cross-section, 1″ in diameter, supports a weight of 10,000 pounds in tension. What is the stress per square inch in the bar?

P. 202. A circular wrought-iron rod, $1\frac{1}{4}$″ in diameter, breaks under a tension of 67,500 pounds. Find the breaking stress per square inch.

P. 203. Find the size of a round wrought-iron rod to safely carry a tensile stress of 100,000 pounds.

P. 204. Two strips of steel, whose rectangular cross-sections measure $\frac{1}{2}''$ by 2″, and $\frac{1}{4}''$ by 8″, support a horizontal bar 9′ long. At what point on the bar must a weight of 300 pounds be hung in order that the unit stress may be the same in the two hangers? What is the unit stress in the hangers?

P. 205. A steel tie-rod 30′ long and 4″ square in cross-section is subjected to 40,000 pounds tension. Find the total elongation and the unit stress.

P. 206. A rectangular timber tie is to be 40′ long. If one dimension of the cross-section is 12″, find the other dimension so that the elongation under a pull of 270,000 pounds may not exceed 1.2″.

P. 207. How much will a 100′ steel tape $\frac{1}{2}''$ wide and 1/50″ thick stretch under a pull of 50 pounds?

P. 208. If a bar 1″ in diameter and 8′ long elongates 0.05″ under a stress of 15,000 pounds, how much will a bar $1\frac{1}{4}''$ in diameter and 12′ long, of the same material, elongate under a stress of 30,000 pounds?

P. 209. If the ultimate strength of wrought-iron is 48,000 pounds per square inch, what tensile force will rupture a bar, of uniform cross-section, 6′ long, which weighs 60 pounds? Weight of wrought-iron is 480 pounds per cubic foot.

P. 210. Plot the stress-strain diagram for the tests in P211, using scales as follows:

Stresses.............. 10,000 pounds to the inch.
Strains............... 2,000 times real value.

P. 211. Find the ultimate strength, the modulus of elasticity, and the elastic limit in the following tests:

Diameter, ″.505; sectional area, .20 square inch; gauged length, 3″.

Applied loads		Elongation per inch.	Successive elongation per inch.	Applied loads.		Elongation per inch.	Successive elongation per inch.
Total.	Per sq. in.			Total.	Per sq. in.		
Pounds.	*Pounds.*	*Inch.*	*Inch.*	*Pounds.*	*Pounds.*	*Inch.*	*Inches*
200	1,000	0.	0.	15,000	75,000	0.002767	0.000167
1,000	5,000	.000133	.000133	15,200	76,000	.008200	.005433
2,000	10,000	.000300	.000167	16,000	80,000	.0100	.0018
4,000	20,000	.000633	.000333	17,000	85,000	.0133	.0033
6,000	30,000	.001000	.000367	18,000	90,000	.0167	.0034
8,000	40,000	.001333	.000333	19,000	95,000	.0233	.0066
10,000	50,000	.001700	.000367	20,000	100,000	.0267	.0034
12,000	60,000	.002067	.000367	21,000	105,000	.0333	.0066
14,000	70,000	.002433	.000366	22,000	110,000	.0400	.0067
14,200	71,000	.002500	.000067	23,000	115,000	.0467	.0067
14,400	72,000	.002533	.000033	24,000	120,000	.0633	.0166
14,600	73,000	.002567	.000034	25,000	125,000		
14,800	74,000	.002600	.000033				

P. 212. What is the least compressive force that will crush a stone cube which measures 4″ on an edge?

P. 213. What is the greatest compressive force which may be safely carried by a stone cube which measures 10″ on an edge?

P. 214. A short cylinder of cast-iron, 3″ in diameter, is subjected to a compressive force of 60,000 pounds, applied parallel to the axis of the cylinder. What is the stress per square inch? Is this force sufficient to crush the cylinder?

P. 215. The crushing strength of brick being 2,500 pounds per square inch, find the height of a brick tower of uniform cross-section that will crush under its own weight. Brick weighs 120 pounds per cubic foot.

P. 216. A wooden post, 3′ long, with rectangular cross-section 4″ by 6″, is subjected to a load of 12,000 pounds. Find the distance between two scratches which were 2′ apart before the load was applied.

P. 217. A rod 7 square inches in cross-section is to be subjected to a tensile force of 90 tons. What materials may be used if the elongation is not to exceed 1/10 of one per cent?

P. 218. A wooden post of 8 square inches cross-section and 12′ in length is firmly fastened vertically at both ends. A weight of 1,000 pounds is secured to the post at a height of 9′ from the lower end. How much lower, after the weight is applied, will be the point which, before the weight was applied, was at the height of 4′ from the lower end? Neglect weight of post.

P. 219. A wrought-iron rod, 984′ long, alternately exerts a thrust and pull of 52,910 pounds; its cross-section is 9.3 square inches. Find the loss of stroke.

P. 220. A wrought-iron column, 1′ long, is to sustain safely a load of 6,000 pounds. The dimensions of its section (rectangular) are to be in the ratio of one to four. Find these dimensions, assuming that the safe stress per square inch is 10,000 pounds.

P. 221. A square steel bar, 10′ long, has one end fixed; a sudden pull F is exerted at the other end. Find the sectional area of the bar in order that the maximum elongation is not to exceed 1/750 of the length of the bar. Neglect weight of bar.

P. 222. A steel bar, 10′ long and of rectangular cross-section, 1″ by $\frac{1}{2}$″, is suspended by its upper end and a weight of 500 pounds is suddenly applied at its lower end. Find the extreme elongation, the stress per square inch developed at this position, the final elongation, and the final stress. Neglect weight of bar.

P. 223. How much will a steel rod, 2 miles long, elongate under the action of its own weight, if suspended from its upper end? Steel weighs 500 pounds per cubic foot.

P. 224. A wrought-iron bar is 10′ long, and $\frac{1}{4}$ of a square inch in cross-section; find the weight which, suspended from the lower end of the bar, will, together with the weight of the bar, produce an elongation of $\frac{1}{8}$″. Weight per cubic foot of iron, 480 pounds.

P. 225. If a wrought-iron rail, 30′ long, is prevented from expanding, what will be the stress per square inch from an increase of temperature of 80° F., the coefficient of expansion of wrought-iron being taken at .0000067?

P. 226. A wrought-iron bar 2 square inches in cross-section has its ends immovably fixed between two blocks when the temperature is 60° F. Taking the coefficient of expansion at .0000067 per unit of length for 1°, what pressure will be exerted on the blocks when the temperature is 100° F?

P. 227. What shearing force will a beam of wood 8″ by 12″ in rectangular cross-section safely withstand?

P. 228. Two steel plates are to be held together by a steel bolt; the force tending to slide the plates over each other is 75,000 pounds. What must be the diameter of the bolt for safety against shearing?

P. 229. What force is required to punch a hole 1″ in diameter in a $\frac{1}{2}$″ steel plate?

P. 230. A beam is loaded uniformly with a weight of 200,000 pounds and each end rests on a cast-iron bracket 1″ wide. What should be the depth of the bracket for safety against shearing?

P. 231. The weight of an eight-wheel freight car with load is 100,000 pounds. What should be the diameter of the steel axles for safety against shearing?

P. 232. A wooden capstan shaft has a circular cross-section 12″ in diameter. Four handspikes, 5′ long, measured from the center of the shaft, are used to operate the capstan. What force, gradually applied, can be safely used at the end of each handspike?

P. 233. The screw of a screw-jack is of steel and has a diameter of 2″; it is operated by the direct application of a bar. If a man can gradually apply a force of 300 pounds at the end of the bar, how long may the bar be made and the screw still be safe against failure by torsion?

P. 234. A winding-drum, 20′ in diameter, is used to raise a load of 5 tons. If the driving-shaft is in pure torsion, find the least diameter for a stress not exceeding 3 tons per square inch.

P. 235. What couple applied at the rim of a brake-wheel, 3′ in diameter, will be necessary to cause it to rotate 2 degrees, if it is rigidly attached to a steel shaft 3/2″ in diameter and 5′ long?

P. 236. (Fig. P. 236.) Considering torsion alone, determine:

(a) The force to be applied to the pulley *C*, in order to produce equilibrium.

(b) The safe diameters of the steel shaft, in order that it may be a shaft of uniform strength.

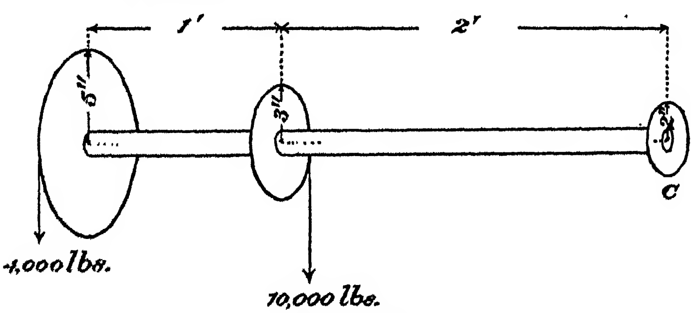


Fig. P. 236.

P. 237. Calculate the horse-power that a round wrought-iron shaft 8″ in diameter, and making 150 revolutions per minute, will transmit with a factor of safety of 6.

P. 238. A solid steel shaft of circular cross-section is to safely transmit 500 horse-power with 200 revolutions per minute. What must be its diameter?

P. 239. (Fig. P. 239.) With the shaft making 200 revolutions per minute, what horse-power can be safely taken off by each machine?

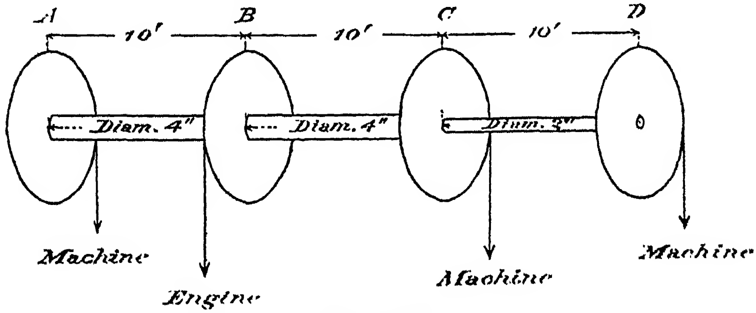


Fig. P. 239.

P. 240. (Fig. P. 240.) A solid steel shaft 6" in diameter is coupled by bolts 1" in diameter on a flange coupling. The centers of the bolts are 5" from the axis. Find the number of bolts in order that their safe strength shall equal that of the shaft.

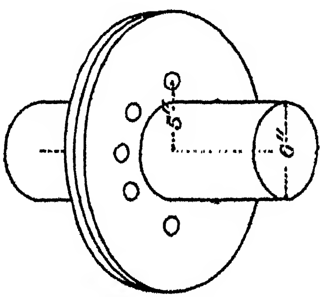


Fig. P. 240.

P. 241. A cantilever beam 20' long has concentrated loads of 100, 200 and 300 pounds at points 5', 10' and 15', respectively, from the free end. Find both the bending moment and the vertical shear at points 8', 12' and 16' from the free end.

P. 242. A beam 30' long, supported at the ends, has concentrated loads of 500 pounds, 400 pounds, 300 pounds, 200 pounds and 100 pounds, at distances from left support of 5', 10', 12', 18' and 20', respectively. Find the bending moment (in foot-pounds) at points 8', 15' and 25' from the left support.

P. 243. (Fig. P. 243). Find shear and bending moment at sections 5' and 10' from left end of beam.

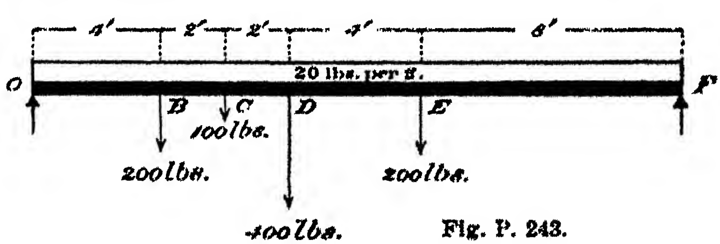


Fig. P. 243.

P. 244. A beam of rectangular cross-section, 8″ wide by 12″ deep, has at its middle point a bending moment of 150,000 foot-pounds. What is the greatest unit fiber stress in the cross-section?

P. 245. A rectangular wooden beam 8″ wide, 14″ deep and 16″ long, rests upon end supports. What concentrated load at the middle point will break the beam?

P. 246. A beam of square cross-section resting on end supports, with a side vertical, is subjected to a certain bending load. The beam is then placed with a diagonal vertical and subjected to the same bending load. Compare the maximum fiber stresses in the two cases.

P. 247. A hollow, circular, cast-iron beam, inside diameter 5″, outside diameter 6″, rests upon end supports 8′ apart. What is the maximum safe load that may be concentrated at its center? Neglect weight of beam.

P. 248. A wooden beam resting on end supports 10′ apart has a cross-section which is an isosceles triangle with a 6″ horizontal base. It carries a uniform load of 120 pounds per linear foot including its own weight. What must be the altitude of its cross-section for a factor of safety of 8?

P. 249. A rectangular cantilever beam of wood, 4″ wide by 8″ deep by 6′ long, is to carry a weight of 1,000 pounds at its free end. What is the factor of safety?

P. 250. A wooden beam of rectangular cross-section has a breadth of 7″. What must be its depth if the greatest bending moment is 75,000 foot-pounds, with a factor of safety of 8?

CHAPTER III.

BEAMS.

301. The term **beam** is generally used to designate a structural piece whose principal function is to sustain a bending stress. The cross-section is usually of considerable size, consisting of several square inches. In this text, beams are taken as of uniform cross-section throughout, unless otherwise stated.

302. **Classes of Beams.**—A beam resting on three or more supports is called a *continuous beam.* If it rests on two points of support only, and the ends are free to move, it is a *simple beam.* If so placed that it has one end fixed and the other free, it is a cantilever. Loads are concentrated or uniformily distributed.

According to the method of loading and supporting we have (Fig. 302A) the following classes of beams: cantilevers, *A* and *B*;

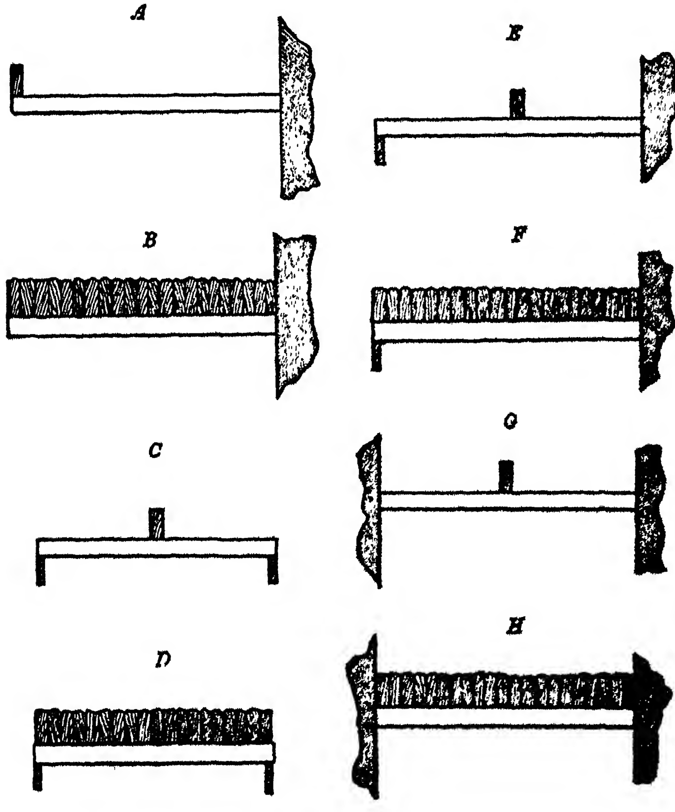


Fig. 302 A

beams resting on two end supports, C and D; beams fixed at one end and supported at the other end, E and F; beams fixed at both ends, G and H.

To prevent confusion, in deducing equation of bending moments, only the axis of the beam will be represented in figures used in the deductions.

303. To find the stresses of bending and shear in a beam, it is necessary to know the forces acting on the beam and the consequent reactions at the points of support. In classes A, B, C, D, above, it is possible to calculate the reactions by the equations of equilibrium. In the remaining classes, it is not possible to determine easily the reactions and thereby to determine the bending moments: it is necessary to determine the equation of the curve of the axis of the beam, called the **curve of mean fiber**, and by use of it to determine the reactions and equations of bending and shear.

CURVE OF MEAN FIBER.

304. To deduce the equation of the curve of mean fiber, it is simpler to deduce it for a cantilever beam and obtain an expression which can be applied to other beams. The expression as obtained will apply to other beams if, in deducing it for a cantilever, no assumptions are made which will not apply equally to other beams.

Let it be required, therefore, to deduce an expression for the curve of mean fiber of a cantilever beam without weight, supporting a load at its extremity.

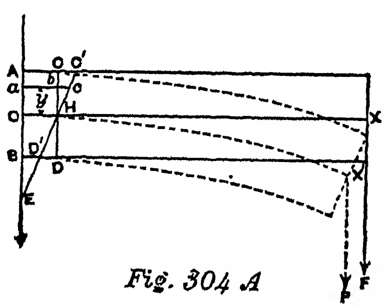


Fig. 304 A

Taking the cantilever in Fig. 304A,

Let CD = section adjacent to fixed section, AB

$C'D'$ = section CD after action of force F

L = length of fiber ab before deflection.

$l = bc$ = elongation of fiber ab

ρ = radius of curvature of mean fiber after deflection $= HE$

Using equation 122A

$$E = \frac{\frac{F}{A}}{\frac{l}{L}} \tag{122A}$$

Substituting for F/A its value in terms of the unit bending stress at a distance y from the axis, we have from paragraph 260,

$$E_f = \frac{(s_f)_1 y}{\frac{l}{L}} \tag{304A}$$

or

$$(s_f)_1 y = \frac{E_f l}{L} \tag{304B}$$

From similar triangles, we have

$$bc : bH :: OH : OE :$$

or

$$l : y :: L : \rho ;$$

Hence

$$l = \frac{yL}{\rho}, \text{ or } \frac{l}{L} = \frac{y}{\rho} \tag{304C}$$

Substituting this value of l/L in equation 304B, we have

$$(s_f)_1 y = \frac{E_f y}{\rho}, \text{ or } (s_f)_1 = \frac{E_f}{\rho} \tag{304D}$$

From equation 259D, in the theory of flexure or bending, we have

$$(s_f)_1 = \frac{M_f}{I} \tag{259D}$$

hence

$$\frac{M_f}{I} = \frac{E_f}{\rho} \tag{304E}$$

or

$$M_f = \frac{E_f I}{\rho} \tag{304F}$$

From calculus we have

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} \tag{304G}$$

in which

dx = the projection of elementary part of any curve on the axis of X;

dy = its projection on the axis of Y.

Since the deflection of the beam is *very small,* as it must be *within the limits of elasticity,* dy is so small as compared with dx that $(dy/dx)^2$ is so very small that it may be taken as zero. Then equation 304G becomes

$$\rho = \frac{1}{\frac{d^2y}{dx^2}} \qquad (304H)$$

Substituting this value of ρ in equation 304F, we have

$$M_f = \frac{E_f I d^2y}{dx^2} \qquad (304I)$$

in which y and x are coordinates of points of the curve.

If, in this expression, the value of M_f can be expressed in terms of x, we can determine, by double integration and proper substitutions for constants, an expression for y in terms of x, which will be the equation of the mean fiber. The expression is general: that is, it applies to curves of mean fiber of beams other than cantilevers.

Application.

305. The curve of mean fiber is of use not only in determining the bending moments and hence the safety of a beam, but also in determining the maximum deflection of a beam which may be safe but may deflect so much that it will be unsuitable, as for example in the ceilings of rooms, where too great deflection will cause the plastering to crack. The curve of mean fiber is also called the deflection curve.

EQUATIONS OF BEAMS.

306. From paragraphs 274 and 304, it is noticed that certain relations exist between the curves of shear, bending moment, and mean fiber. A clear idea of this relation assists materially in determining these curves.

Fig. 306A shows the curves,* drawn in their proper relation to each other, for a beam resting on two supports and supporting a concentrated load at its middle point. Inspection of this figure brings out the following points:

*With colored chalk, shear is shown in red, bending moments in blue, forces in yellow.

a. The shear curve, being the first differential of the bending moment curve, shows at all points the slope of the bending moment curve. In this case, the shear is constant; therefore, the bending moment curve is a straight line with a constant slope.

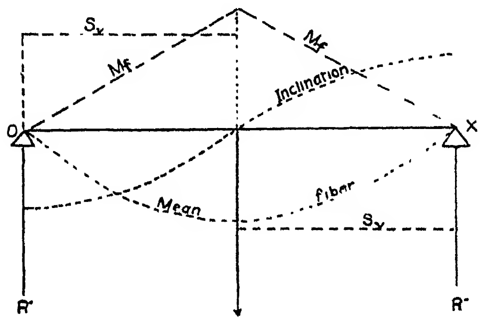


Fig. 306 A

b. The bending moment curve, being the first differential of the inclination curve, shows at all points the slope of the inclination curve. In this case, the slope is greatest at the center as shown by the bending moment.

c. The inclination curve, being the first differential of the mean fiber curve, shows at all points the slope of the mean fiber curve. In this case, it is evident that the inclination curve has its greatest ordinate at the ends and is zero at the center.

In some cases, a reversal of the above order of study is necessary. The value of the bending moment and its equivalent, the second differential of the mean fiber curve, may be all that can be determined. Then, it is necessary to integrate twice, obtaining one of the curves each time. The only difficulty in thus integrating lies in the elimination of the constants of integration. This is done by substituting known values of the curve of integration, and thus obtaining the constant: for example, in the first integration for the curves in Fig. 306A it is known that the slope of the mean fiber curve (and the ordinate of the inclination curve) is zero at the middle point; by this substitution in the equation, the constant of integration for the inclination curve can be obtained.

Cantilever Without Weight, Supporting a Load at its Extremity.

307. Let it be required to find the equations of shear, bending moment, and curve of mean fiber of a cantilever without weight, supporting a load at its extremity.

Represent by Fig. 307A the cantilever, in which c is any section at a distance x from the origin of co-ordinates at the left end.

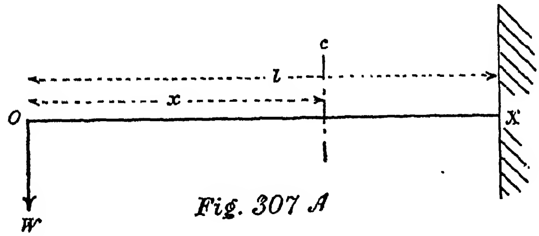


Fig. 307 A

Shear.—By definition, paragraph 270, the vertical shear at any section of the beam, as at c, is equal to W. It is negative because the force is to the left of the section and acts downward. Hence

$$\text{Shear at any section,} \quad S_v = -W \tag{307A}$$

The above equation is true at all sections. Therefore the equation (307A) is the equation of shear for the cantilever.

In plotting the curve of shear, we know that it is a straight line, that it is the same at all points and equal to $-W$. Therefore (Fig. 307B), if we lay off $XB = -W$, the line S_v will be the line whose equation is $S_v = -W$. This line is called the curve of shear.

Bending Moment.—Working from the left of the beam, we have from the definition of the bending moment, paragraph 255,

$$M_f \text{ at } c = -Wx \tag{307B}$$

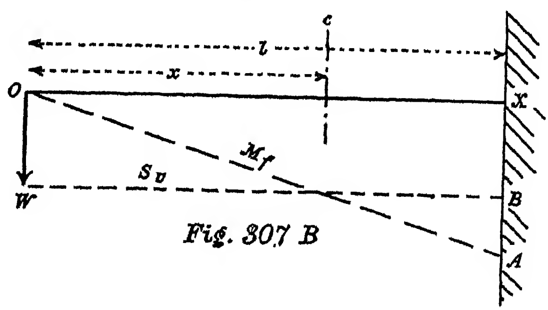


Fig. 307 B

This equation shows the variation of the value of the bending moment from section to section. Since M_f is a numerically increasing function of x, it is greatest numerically at X, where x is greatest and equal to l; it is least numerically at O, where $x = 0$. The maximum bending moment is therefore at X, and

$$M_{fm} = -Wl \tag{307C}$$

In plotting the curve of bending moments, we know that it is a straight line, that at point O it is zero, that at point X it is $-Wl$. Therefore (Fig. 307B), if we lay off $XA = -Wl$, the line M_f will be the line whose equation is $M_f = -Wx$. This line is called the curve of bending moments. From it by inspection we may determine the position of the dangerous section (where the bending moment is a maximum).

Curve of Mean Fiber.—The equation of second differential of any curve of mean fiber is, from paragraph 304,

$$M_f = \frac{E_f I d^2 y}{dx^2} \tag{304I}$$

The equation of M_f for this cantilever is

$$M_f = -Wx \tag{307D}$$

Equating the two, we have

$$\frac{E_f I d^2 y}{dx^2} = -Wx \tag{307E}$$

Integrating with respect to x, we have

$$\frac{E_f I\, dy}{dx} = -\frac{Wx^2}{2} + C \tag{307F}$$

In which dy/dx is the tangent of the angle made by the mean fiber with the axis of X, and C is the constant of integration.

As beam is fixed at X, we know that, at X, $dy/dx = 0$, and $x = l$. *Hence* $C = Wl^2/2$.

Substituting this value in 307F and integrating, we have

$$E_f I y = -\frac{Wx^3}{6} + \frac{Wl^2 x}{2} + C' \tag{307G}$$

Knowing that at X, $y = 0$ and $x = l$, we have

$$C' = \frac{Wl^3}{6} - \frac{Wl^3}{2} = -\frac{Wl^3}{3} \tag{307H}$$

Substituting in 307G, we obtain

$$y = -\frac{W}{6E_f I}(x^3 - 3l^2 x + 2l^3) \tag{307I}$$

which is the equation of the curve of mean fiber. The deflection is a maximum numerically where $x = 0$, hence

$$y_m = -\frac{Wl^3}{3E_f I} \tag{307J}$$

308. **Problem.**—A cantilever projecting to the left, 24′ long, has a load of 220 pounds at its left extremity and 150 pounds 10′ from its left extremity. Write the equations of shear and bending moment and show them graphically, origin at left.

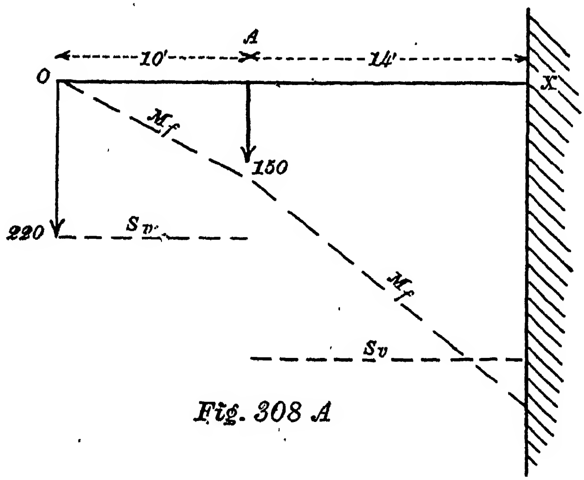


Fig. 308 A

Solution.—*Shear*: The curves are shown in Fig. 308A. Their equations are

From O to A, $S_v = -220$ pounds (308A)

From A to X, $S_v = -370$ pounds (308B)

Bending Moment.—The curves are shown in Fig. 308A. Their equations are

From O to A, $M_f = -220x$ (308C)

From A to X, $M_f = -220x - 150(x - 10)$ (308D)

It is to be noted in the curve of bending moment, even without actual plotting of the curve, that

a. The slopes from O to A and from A to X are constant, this being necessary because these shears are constant.

b. The slope from O to A is less than the slope from A to X, this being necessary because the first shear is less than the second.

A Cantilever Without Weight Uniformly Loaded.

309. By deduction similar to the preceding, we find (Figs. 309A and 309B):

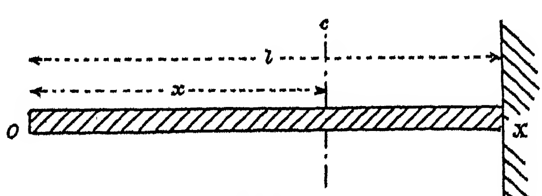


Fig. 309 A

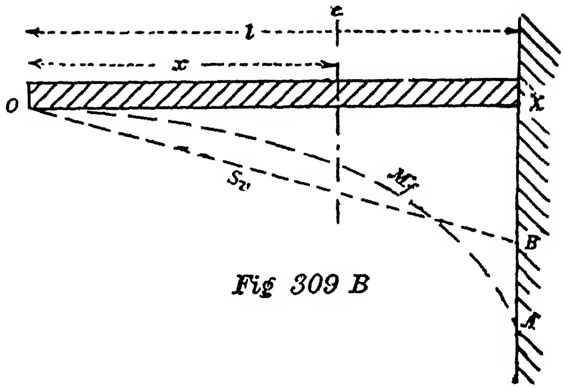


Fig 309 B

$$S_v = -wx \tag{309A}$$

$$S_{vm} = -wl \tag{309B}$$

$$M_f = -wx\left(\frac{x}{2}\right) = -\frac{wx^2}{2} \tag{309C}$$

$$M_{fm} \text{ is at } X \text{ and } = -\frac{wl^2}{2} \tag{309D}$$

$$y = -\frac{w}{24E_fI}(x^4 - 4l^3x + 3l^4) \tag{309E}$$

$$y_m = -\frac{wl^4}{8E_fI} \tag{309F}$$

310. **Problem.**—A cantilever projecting to the left, 14′ long, carries a uniformly distributed load of 10 pounds per linear foot. Write the equations of shear, bending moment, and mean fiber, and show them graphically, origin at the left.

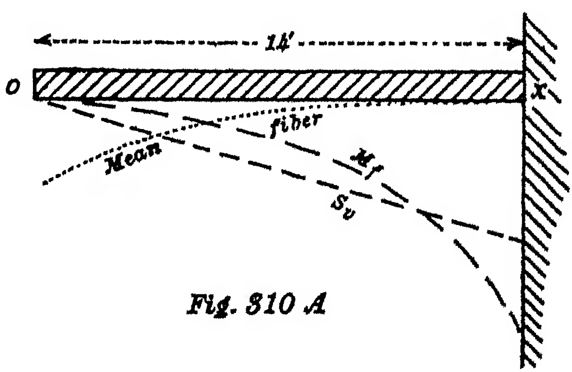


Fig. 310 A

Solution.—The equations of shear and bending moment can be written without reference to the formulas. The equation of the curve of mean fiber can be deduced, or can be obtained by substitution in equation 309E. The curves are shown in Fig. 310A. Their equations are

$$\text{Shear,} \quad S_v = -10x \tag{310A}$$

$$\text{Bending moment,} \quad M_f = -\frac{10x^2}{2} = -5x^2 \tag{310B}$$

$$\text{Mean Fiber,} \quad y = -\frac{10}{24E_fI}[(x^4 - (4)(14)^3x + (3)(14)^4] \tag{310C}$$

A Beam Without Weight, Resting on End Supports and Supporting a Load at its Middle Point.

311. Let it be required to find the equations of bending moment, shear, and curve of mean fiber of the beam stated above.

Represent by Fig. 311A the beam, in which c and d are sections on the left and right of the weight, W, at the middle point.

Reactions.—On starting to work from the left, it is at once apparent that there is an unknown force at the left support. In the general case, this reaction is determined by calculations with use of the

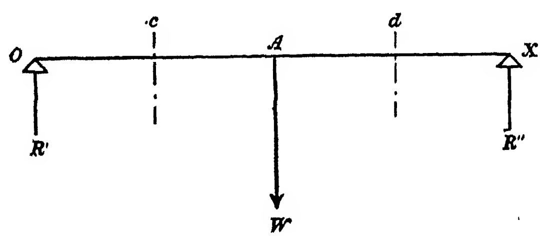


Fig. 311 A

equations of equilibrium; in this case, since the load, W, is at the middle of the beam, we know at once that half of the load is transmitted to each support.

$$\text{Therefore,} \quad R' = R'' = \frac{W}{2} \tag{311A}$$

Shear.—Working from the left support, we have

$$\text{From } O \text{ to } A, \quad S_v = +\frac{W}{2} \tag{311B}$$

$$\text{From } A \text{ to } X, \qquad S_v = +\frac{W}{2} - W = -\frac{W}{2} \tag{311C}$$

These are plotted in Fig. 311B.

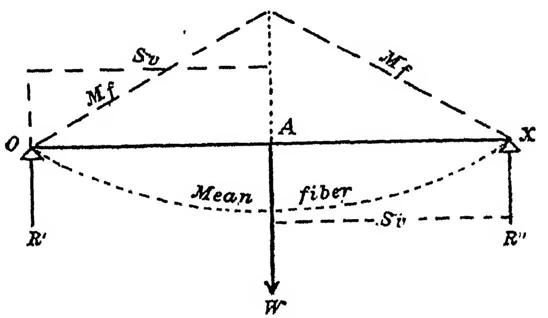


Fig. 311 B

Bending Moment.—Working from the left support, we have

$$\text{From } O \text{ to } A, \quad M_f = \frac{Wx}{2} \tag{311D}$$

$$\text{From } A \text{ to } X, \quad M_f = \frac{Wx}{2} - W\left(x - \frac{l}{2}\right) = -\frac{Wx}{2} + \frac{Wl}{2} \tag{311E}$$

In plotting these two curves of bending moments, from the equations and also from the shear curve, we know that each is a straight line; that at O, where $x = 0$, $M_f = 0$, from equation 311D; that at X, where $x = l$, $M_f = 0$, from equation 311E. The curves as plotted are shown in Fig. 311B. The maximum bending moment is at the middle point and is given by the equation

$$M_{fm} = \frac{Wl}{4} \tag{311F}$$

Curve of Mean Fiber.—The curve of mean fiber is symmetrical to right and left of the middle point, since the beam is symmetrically loaded. We shall consider only the left half.

From paragraph 304, we have

$$M_f = \frac{E_f I d^2 y}{dx^2} \tag{304I}$$

Substituting in equation 311D, we have

$$\frac{E_f I d^2y}{dx^2} = \frac{Wx}{2} \qquad (311G)$$

Integrating, $$\frac{E_f I dy}{dx} = \frac{Wx^2}{4} + C \qquad (311H)$$

Because of symmetrical loading, where $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$; hence

$$C = -\frac{Wl^2}{16}$$

Substituting, $$\frac{E_f I\,dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \qquad (311I)$$

Integrating again, $$E_f Iy = \frac{Wx^3}{12} - \frac{Wl^2x}{16} + C' \qquad (311J)$$

Where $x = 0$, $y = 0$, hence $C' = 0$, and

$$y = \frac{W}{48E_fI}(4x^3 - 3l^2x) \qquad (311K)$$

When plotted, the curve is concave upwards and is of the form shown in Fig. 311B. The maximum value of y is where $x = l/2$.

Hence, $$y_m = -\frac{Wl^3}{48E_fI} \qquad (311L)$$

312. **Problem.**—Write equations of shear and bending moment of beam loaded and supported as shown in Fig. 312A.

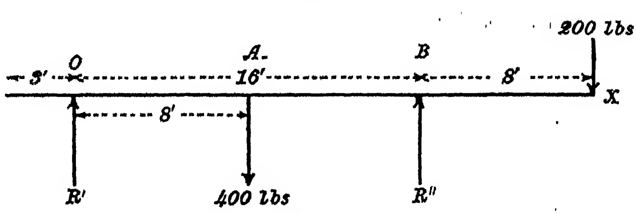


Fig. 312 A

Solution.—Taking the center of moments at the left support, we have

$$+R' \times 0 - 400 \times 8 + R'' \times 16 - 200 \times 24 = 0 \qquad (312A)$$

$$16R'' = 3200 + 4800 = 8000 \qquad (312B)$$

$$R'' = 500 \qquad (312C)$$

Thence, $$R' = 400 + 200 - 500 = 100 \qquad (312D)$$

With these reactions, the equations are

Shear, O to A, $S_v = 100$ (312E)

A to B, $S_v = 100 - 400 = -300$ (312F)

B to X, $S_v = 100 - 400 + 500 = 200$ (312G)

Bending Moment, O to A, $M_f = 100x$ (312H)

A to B, $M_f = 100x - 400\ (x - 8)$ (312I)

B to X, $M_f = 100x - 400\ (x - 8) + 500\ (x - 16)$ (312J)

Beam Resting on End Supports and Uniformly Loaded.

313. By deduction similar to the preceding, we find (Figs. 313A and 313B):

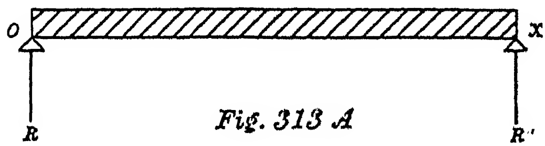


Fig. 313 A

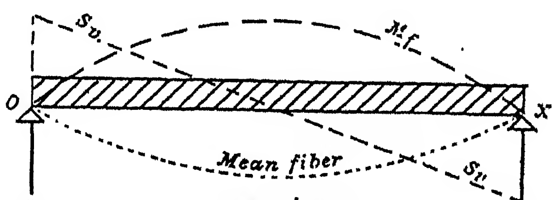


Fig. 313 B

$$S_v = \frac{wl}{2} - wx \qquad (313A)$$

S_{vm} is at the supports and $= \frac{wl}{2}$ and $-\frac{wl}{2}$ (313B)

$$M_f = \frac{wlx}{2} - \frac{wx^2}{2} \qquad (313C)$$

M_{fm} is at the middle point and $= \frac{wl^2}{8}$ (313D)

$$y = \frac{w}{24E_fI}(2lx^3 - x^4 - l^3x) \qquad (313E)$$

y_m is at the middle point and $= -\frac{5wl^4}{384E_fI}$ (313F)

314. **Problem.**—A wooden beam of uniform strength, supported at the ends, is 12′ long, 6″ wide and supports a uniformly distributed load of 1000 pounds per lineal foot. Find the depth at the end to safely resist shear, and the depth at the middle point to safely resist bending moment.

Solution.—The reactions are equal and each $= \dfrac{1000 \times 12}{2} = 6000$ pounds (314A)

Shear, $S_{vm} = 6000$ pounds (314B)

Using the equation, $a_s = \dfrac{F}{A}$

Remembering that a_s for wood $= 1000$ pounds, that $A = bD$, and substituting, we have

$$1000 = \frac{6000}{6 \times D} \tag{314C}$$

$$D = \frac{6000}{1000 \times 6} = 1'' \tag{314D}$$

Bending Moment, M_f at middle point $= 6000 \times (6 \times 12) - (1000 \times 6) \times (3 \times 12)$
$= 216{,}000$ inch pounds (314E)

Using the equation, $a_f = \dfrac{M_f}{\dfrac{I}{y}}$

Remembering that a_f for wood $= 1000$ pounds, that $I/y = bD^2/6$, and substituting, we have

$$1000 = \frac{216{,}000 \times 6}{6D^2} \tag{314F}$$

$$D^2 = 216 \tag{314G}$$

$$D = 14.7'' \tag{314H}$$

A Beam Without Weight, Fixed Horizontally at Both Ends and Supporting a Load at its Middle Point.

315. Let it be required to find the equation of shear, bending moment, and mean fiber of the beam described above.

Represent the beam by Fig. 315A, in which c and d are sections on the left and right of the weight, W, at its middle point.

Reactions.—It is not possible to determine the reactions by the general equations of equilibrium; therefore we obtain expressions

for them by certain substitutions for constants of integration in which we make use of certain known properties of the curve of mean fiber and determine in the equation certain values derived from the values of the reactions.

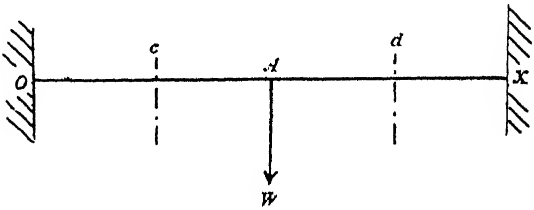


Fig. 315 A

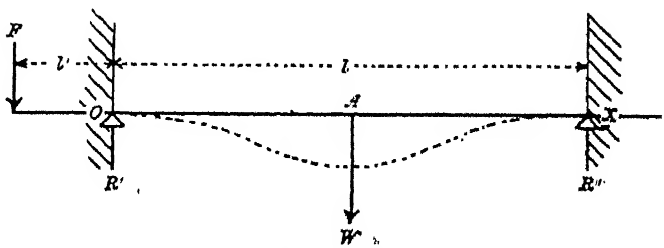


Fig. 315 B

We know, as shown in Fig. 315B, that since the beam is fixed at the ends and symmetrically loaded the curve of mean fiber at *O*, *A*, and *X* is horizontal and that its tangent (dy dx) at these points is equal to zero.

We know, also, that in order to make the beam fixed at the ends, when the beam is loaded, some force must be exerted to hold the ends in position and prevent their movement.

Assume R' as the vertical reaction at the left support, and F as the other force to the left of it with a lever arm, l', necessary to make the beam horizontal at O.

Then, since the beam is symmetrically loaded,

$$R' = \frac{W}{2} + F \qquad (315A)$$

The equation of the second differential of the curve of mean fiber (equation of curve of bending moments) is

$$\frac{E_f I d^2 y}{dx^2} = M_f \qquad (315B)$$

From O to A,

$$M_f = -F(l' + x) + \left(\frac{W}{2} + F\right)x = \frac{Wx}{2} - Fl' \qquad (315C)$$

Substituting in 315B, we have

$$\frac{E_f I d^2y}{dx^2} = \frac{Wx}{2} - Fl' \qquad (315D)$$

Integrating, $$\frac{E_f I dy}{dx} = \frac{Wx^2}{4} - Fl'x + C \qquad (315E)$$

As stated above, the deflection of the beam is such that $dy/dx = 0$ where $x = 0$.

Substituting in equation 315E, we have

$$C = 0 \qquad (315F)$$

Substituting this in equation 315E, we have,

$$\frac{E_f I dy}{dx} = \frac{Wx^2}{4} - Fl'x \qquad (315G)$$

We also know that $dy/dx = 0$ where $x = l/2$. Substituting this in equation 315G, we obtain

$$0 = \frac{Wl^2}{16} - \frac{Fl'l}{2} \qquad (315H)$$

Hence, $$F = \frac{Wl}{8l'} \qquad (315I)$$

and $$R' = \frac{W}{2} + \frac{Wl}{8l'} \qquad (315J)$$

Shear.—From the definition of shear, we have

From O to A, $$S_v = R' - F = \frac{W}{2} \qquad (315K)$$

From A to X, $$S_v = \frac{W}{2} - W = -\frac{W}{2} \qquad (315L)$$

Bending Moment.—Working from the left, we have

From O to A,

$$M_f = -F(l' + x) + R'x = -\frac{Wl}{8l'}(l' + x) + \left(\frac{W}{2} + \frac{Wl}{8l'}\right)x = \frac{Wx}{2} - \frac{Wl}{8} \qquad (315M)$$

$$\text{From } A \text{ to } X,\ M_f = \frac{Wx}{2} - \frac{Wl}{8} - W\left(x - \frac{l}{2}\right) \qquad (315N)$$

Curve of Mean Fiber.—Substituting for M_f, in equation 315B, its value in equation 315M, we have

$$\frac{E_f I d^2 y}{dx^2} = \frac{Wx}{2} - \frac{Wl}{8} \qquad (315O)$$

Integrating, and making $C = 0$ as in equation 315F, we have

$$\frac{E_f I dy}{dx} = \frac{Wx^2}{4} - \frac{Wlx}{8} \qquad (315P)$$

Integrating again,

$$E_f I y = \frac{Wx^3}{12} - \frac{Wlx^2}{16} + C' \qquad (315Q)$$

Where $x = 0$, $y = 0$; hence $C' = 0$; and equation 315Q becomes

$$y = \frac{W}{48 E_f I}(4x^3 - 3x^2 l) \qquad (315R)$$

The above deduction applies only to the left half of the curve of mean fiber. The curve is symmetrical for the right half, but deduction of its equation will require use of equation 315N.

The maximum deflection is at A, where the value of y is

$$y_m = -\frac{Wl^3}{192 E_f I} \qquad (315S)$$

Beam Fixed Horizontally at Both Ends and Uniformly Loaded.

316. By deduction similar to the preceding, we find (Fig. 316A)

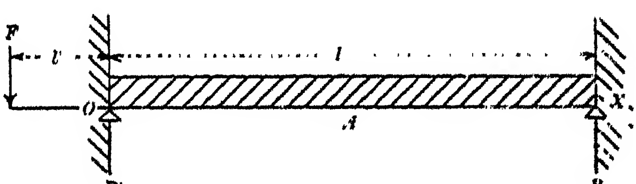


Fig. 316 A

$$S_v = \frac{wl}{2} - wx \qquad (316A)$$

$$M_f = \frac{E_f I d^2 y}{dx^2} = \left(\frac{wl}{2} + F\right) x - \frac{wx^2}{2} - F(l' + x)$$

$$= \frac{wlx}{2} - \frac{wx^2}{2} - Fl'$$

$$= \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wl^2}{12} \tag{316B}$$

$$M_{fm} \text{ is at the ends and } = -\frac{wl^2}{12} \tag{316C}$$

$$y = \frac{w}{24E_fI}(2lx^3 - x^4 - l^2x^2) \tag{316D}$$

$$y_m \text{ is at the center and } = -\frac{wl^4}{384E_fI} \tag{316E}$$

Beam Without Weight Fixed Horizontally at One End, Resting on a Support at the Other, and Supporting a Load at the Middle Point.

317. The equation of shear, bending moment, and curve of mean

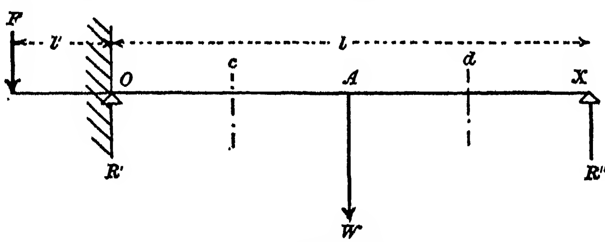


Fig. 317 A

fiber, Fig. 317A, we find by deduction similar to preceding deductions to be

$$O \text{ to } A, \quad S_v = \frac{11}{16}W \tag{317A}$$

$$A \text{ to } X, \quad S_v = -\frac{5}{16}W \tag{317B}$$

$$O \text{ to } A, \quad M_f = -\frac{3}{16}Wl + \frac{11}{16}Wx \tag{317C}$$

$$A \text{ to } X, \quad M_f = -\frac{5}{16}Wx + \frac{5}{16}Wl \tag{317D}$$

$$M_f \text{ is greatest at } O \text{ and } = -\frac{3}{16}Wl \tag{317E}$$

$$O \text{ to } A,\ E\,Iy = -\frac{3}{32}Wlx^2 + \frac{11}{96}Wx^3 \tag{317F}$$

$$A \text{ to } X,\quad E_fIy = -\frac{5}{96}Wx^3 + \frac{15}{96}Wlx^2 - \frac{12}{96}Wl^2x + \frac{2}{96}Wl^3 \tag{317G}$$

y_m is greatest at some point between A and X (where $\delta y/\delta x = 0$)

$$\text{and} = \frac{Wl^3}{108E_fI} \tag{317H}$$

Beam Without Weight, Uniformly Loaded and Fixed Horizontally at One End, and Resting on a Support at the Other.

318. By deduction (Fig. 318A) similar to the preceding, we obtain

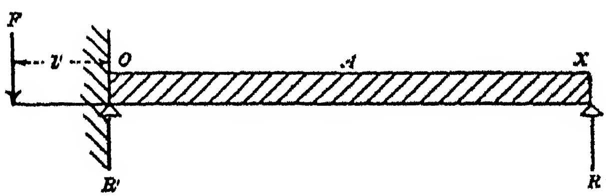


Fig. 318 A

$$S_v = -wx + \frac{5}{8}wl \tag{318A}$$

$$M_f = -\frac{wx^2}{2} + \frac{5}{8}wlx - \frac{wl^2}{8} \tag{318B}$$

$$E_fIy = -\frac{wx^4}{24} + \frac{5}{48}wlx^3 - \frac{wl^2x^2}{16} \tag{318C}$$

As in paragraph 317, the maximum deflection is between the center and right support, where $dy/dx = 0$, or where $x = (9/16)l$ approximately.

Hence
$$y_m = \frac{wl^4}{180E_fI} \tag{318D}$$

319. From preceding equations, we construct the following table:

Table 319A: Maximum Values of Bending Moments, Fiber Stress, and Deflections in Beams of Uniform Cross-section, whose length is l and whose load is W or wl.

Method of Loading and Supporting.	Maximum Moments M_{fm}	Maximum Fiber Stress $s_{fm} = \frac{M_{fm}}{\frac{I}{y}}$	Maximum Deflections y_m
Cantilever with load at end	Wl	$\frac{Wly}{I}$	$\frac{1}{3}\frac{Wl^3}{E_fI}$
Cantilever uniformly loaded	$\frac{wl^2}{2}$	$\frac{wl^2y}{2I}$	$\frac{1}{8}\frac{wl^4}{E_fI}$
Beam resting on end supports with load at middle point	$\frac{Wl}{4}$	$\frac{Wly}{4I}$	$\frac{1}{48}\frac{Wl^3}{E_fI}$
Beam resting on end supports, uniformly loaded	$\frac{wl^2}{8}$	$\frac{wl^2y}{8I}$	$\frac{5}{384}\frac{wl^4}{E_fI}$
Beam fixed horizontally at both ends, with load at middle	$\frac{Wl}{8}$	$\frac{Wly}{8I}$	$\frac{1}{192}\frac{Wl^3}{E_fI}$
Beam fixed horizontally at both ends, uniformly loaded	$\frac{wl^2}{12}$	$\frac{wl^2y}{12I}$	$\frac{1}{384}\frac{wl^4}{E_fI}$
Beam fixed horizontally at one end and resting on end support at the other, with load at middle	$\frac{3Wl}{16}$	$\frac{3Wly}{16I}$	$\frac{1}{108}\frac{Wl^3}{E_fI}$
Beam fixed horizontally at one end and resting on end support at the other, uniformly loaded	$\frac{wl^2}{8}$	$\frac{wl^2y}{8I}$	$\frac{1}{180}\frac{wl^4}{E_fI}$

320. **Maximum Deflections.**—In the construction of the floors of buildings, to prevent injury to plastered ceilings of rooms below, it is engineering practice to limit the deflection in each beam to about 1/360 of its span. Under this limitation, the proper load of the beam may be determined from the calculation for maximum deflections above, and not from that giving the safe fiber stress, since the allowable deflection may limit the load before the surface fibers have reached their safe fiber stress.

321. **Problem.**—A rectangular wooden beam 6″ wide and 30′ long is supported at its ends and weighs 50 pounds per cubic foot. Find its depth, if its maximum deflection is 1″ under its own weight.

Solution.—From table, paragraph 319, we have

$$y_m = \frac{5wl^4}{384E_fI}$$

From statement of the problem, we have

$$y_m = 1''$$

$$W = \frac{6}{12} \times \frac{D}{12} \times 30 \times 50$$

$$l = 30' = 30 \times 12''$$

$$E_f = 1{,}000{,}000 \text{ pounds}$$

$$I = bD^3/12 = D^3/2$$

$$D = ?$$

Substituting in above, we have

$$1 = \frac{5 \times \frac{6}{12} \times \frac{D}{12} \times 30 \times 50 \times (12 \times 30)^3}{384 \times 1{,}000{,}000 \times d^3/2}$$

Solving, we obtain

$$D = 8.7''$$

General Equation of Bending Moments.

322. For convenience in obtaining bending moments, it is often desirable to obtain the bending moment at one section, and then by use of this bending moment and the remaining forces, to find the bending moment at other sections. This is practicable, as will be shown in the deduction below.

Taking Figs. 315B and 316A, we have for the bending moment at any section between O and A:

Fig. 315B, $$M_f = -F(l' + x) + \left(\frac{W}{2} + F\right)x \qquad (322\text{A})$$

$$M_f = -Fl' + \frac{Wx}{2} \qquad (322\text{B})$$

Substituting S_v for its value $W/2$, we have

$$M_f = -Fl' + S_v x \qquad (322\text{C})$$

Fig. 316A, $$M_f = -F(l' + x) + \left(\frac{wl}{2} + F\right)x - \frac{wx^2}{2} \qquad (322\text{D})$$

$$= -Fl' + \frac{wlx}{2} - \frac{wx^2}{2} \qquad (322\text{E})$$

Substituting S_v for its value $wl/2$, at the left support, we have

$$M_f = -Fl' + S_v x - \frac{wx^2}{2} \qquad (322\text{F})$$

Analyzing equations 322C and 322F and remembering that Fl' is the bending moment of F with respect to the left support, we note the principle that *the bending moment at any stated section is equal to the bending moment at any preceding section, plus the bending moment due to the shear at this preceding section, plus the bending moment of the forces between this preceding section and the stated section.*

323. **Problem.**—A beam 20′ long and fixed at both ends, is loaded uniformly with 60 pounds per foot. It has a bending moment up to the left support of 2000 foot pounds. What is its bending moment at a point 12′ from left support?

Solution.—Using equation 322F

$$M_f = -Fl' + S_v x - \frac{wx^2}{2} \qquad (322F)$$

Substituting, we have for bending moment 12′ from left support

$$M_f = -2000 + 600\ (12) - 4320$$
$$M_f = 880 \text{ foot pounds}$$

CONTINUOUS BEAMS.

324. A continuous beam is one resting on more than two supports, or having its ends so fixed that they will not move. Since for parallel forces there are only two equations of equilibrium (paragraph 113) and it is possible to determine only two unknown quantities, it is evident that other equations are necessary where a beam is continuous, that is, has three or more supports, and consequently three or more unknown reactions. These equations are provided by the *theorem of three moments.**

325. Let it be required to find the equations of bending moments for a continuous beam.

Represent by Fig. 325A a continuous beam in which each section

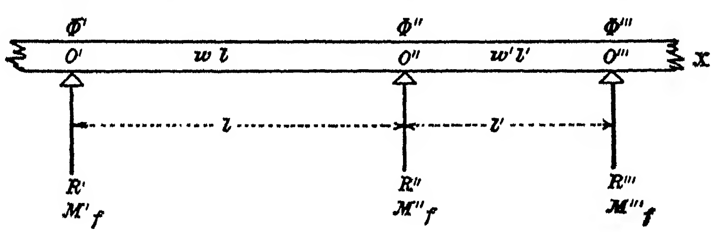


Fig. 325 A

*The theorem of three moments applies only to beams not fixed at the ends. For beams fixed at the ends, it is necessary to use a very complicated deduction (omitted) involving the principles of the Theorem of Least Work.

is uniformly loaded as shown, and the angles φ', φ'', etc., are the angles made by the axis of the loaded beam with the axis of X at the various supports. The loads are represented as uniform, for simplicity in deduction; but the method of deduction will apply to similar beams not uniformly loaded.

The deduction consists in applying the General Equation of Bending Moments to each support and sector to its right, and eliminating unknown quantities and constants of integration thereby deriving an equation containing known quantities and the bending moments at three points of support.

1st.—Beginning at the point O', and using the General Equation of Bending Moments given in paragraph 322, we have

From O' to O'', $$M_f = \frac{E_f I d^2 y}{dx^2} = M'_f + S'_r x - \frac{wx^2}{2} \qquad (325A)$$

Integrating, $$\frac{E_f I\, dy}{dx} = M'_f x + \frac{S'_r x^2}{2} - \frac{wx^3}{6} + C \qquad (325B)$$

At O'', where $x = l$, we know that $dy/dx = \tan\varphi''$. Substituting, we have

$$C = E_f I \tan\varphi'' - M'_f l - \frac{S'_r l^2}{2} + \frac{wl^3}{6} \qquad (325C)$$

Substituting this value of C in equation 325B, we have

$$E_f I\left(\frac{dy}{dx} - \tan\varphi''\right) = M'_f x + \frac{S'_r x^2}{2} - \frac{wx^3}{6} - M'_f l - \frac{S'_r l^2}{2} + \frac{wl^3}{6} \qquad (325D)$$

Integrating, we have

$$E_f I(y - \tan\varphi'' x) = \frac{M'_f x^2}{2} + \frac{S'_r x^3}{6} - \frac{wx^4}{24} - M'_f l x - \frac{S'_r l^2 x}{2} + \frac{wl^3 x}{6} + C' \qquad (325E)$$

At O', where $x = 0$, we know that $y = 0$, hence $C' = 0$. At O'', where $x = l$, we know that $y = 0$.

Substituting, we have

$$-E_f I \tan\varphi'' = \frac{M'_f l}{2} + \frac{S'_r l^2}{6} - \frac{wl^3}{24} - M'_f l - \frac{S'_r l^2}{2} + \frac{wl^3}{6} \qquad (325F)$$

$$= -\frac{M'_f l}{2} - \frac{S'_r l^2}{3} + \frac{wl^3}{8} \qquad (325G)$$

In the above equation, S'_v may be eliminated by obtaining its value from the General Equation of Bending Moments at O'', where $x = l$.

This equation is,
$$M''_f = M'_f + S'_v l - \frac{wl^2}{2} \tag{325H}$$

or
$$S'_v = \frac{M''_f}{l} - \frac{M_f'}{l} + \frac{wl}{2} \tag{325I}$$

Substituting this value in equation 325G, we have

$$-E_f I \tan \varphi'' = -\frac{M'_f l}{2} - \frac{M''_f l}{3} + \frac{M'_f l}{3} - \frac{wl^3}{6} + \frac{wl^3}{8} \tag{325J}$$

$$= -\frac{M''_f l}{3} - \frac{M'_f l}{6} - \frac{wl^3}{24} \tag{325K}$$

2nd.—Starting anew with the origin at O'', and working in a similar manner, we have for the bending moment at any section between O'' and O'''

$$M_f = E_f I \frac{d^2y}{dx^2} = M''_f + S''_v x - \frac{w'x^2}{2} \tag{325L}$$

Integrating,
$$E_f I \frac{dy}{dx} = M''_f x + \frac{S''_v x^2}{2} - \frac{w'x^3}{6} + C \tag{325M}$$

At O'', where $x = 0$, we know that $dy/dx = \tan \varphi''$. Substituting, we have

$$E_f I \left(\frac{dy}{dx} - \tan \varphi''\right) = M''_f x + \frac{S''_v x^2}{2} - \frac{w'x^3}{6} \tag{325N}$$

Integrating,
$$E_f I \,(y - \tan \varphi'' x) = \frac{M''_f x^2}{2} + \frac{S''_v x^3}{6} - \frac{w'x^4}{24} + C' \tag{325O}$$

At O'', where $x = 0$, we know that $y = 0$, hence $C' = 0$. At O''', where $x = l'$, we know that $y = 0$. Substituting, we have

$$-E_f I \tan \varphi'' = \frac{M''_f l'}{2} + \frac{S''_v l'^2}{6} - \frac{w'l'^3}{24} \tag{325P}$$

In the above equation, S''_v may be eliminated by obtaining its

value from the General Equation of Bending Moments at O''', where $x = l'$.

This equation is
$$M'''_l = M''_l + S''_r l' - \frac{w'l'^2}{2} \qquad (325Q)$$

or
$$S''_r = \frac{M'''_l}{l'} - \frac{M''_l}{l'} + \frac{w'l'}{2} \qquad (325R)$$

Substituting this value of S''_r in equation 325P, we have

$$-E_lI\tan\varphi'' = \frac{M''_l l'}{2} + \frac{M'''_l l'}{6} - \frac{M''_l l'}{6} + \frac{w'l'^3}{12} - \frac{w'l'^3}{24} \qquad (325S)$$

$$= \frac{M''_l l'}{3} + \frac{M'''_l l'}{6} + \frac{w'l'^3}{24} \qquad (325T)$$

We now have two independent equations, each of which contains the unknown, $-E_lI\tan\varphi''$. Equating these two values of $-E_lI\tan\varphi''$ in equations 325K and 325T, and transposing, we have

$$M'_l l + 2M''_l(l + l') + M'''_l l' = -\frac{wl^3}{4} - \frac{w'l'^3}{4} \qquad (325U)$$

This is the equation of the *theorem of three moments* deduced for continuous beams whose segments are unequal but each uniformly loaded.

It is evident that we can obtain as many such equations as there are supports to the beam less two. Thus we can determine $(n - 2)$ independent equations for a beam resting on n supports. If, therefore, we know the bending moment at two of the supports, we can determine all of the others.

If the ends of a continuous beam are not fixed, the bending moments at these end supports can always be determined when we know the forces acting on the beam outside of these supports. This gives us values for two bending moments. Therefore, in such cases, we can always determine the bending moments by use of the theorem of three moments.

326. **Problem.**—A beam 30 feet long rests on four supports, spaced 10' apart, and is uniformly loaded with a weight of 12 pounds per foot. Find the equation of bending moments.

Solution.—Using equation 325U

$$M'_f l + 2M''_f(l + l') + M'''_f l' = -\frac{wl^3}{4} - \frac{w'l'^3}{4}$$

$$M''_f l' + 2M'''_f(l' + l'') + M^{iv}_f l'' = -\frac{w'l'^3}{4} - \frac{w''l''^3}{4}$$

Making $l = l' = l''$, and $w = w' = w''$, we have

$$M'_f l + 4M''_f l + M'''_f l = -\frac{wl^3}{2}$$

$$M''_f l + 4M'''_f l + M^{iv}_f l = -\frac{wl^3}{2}$$

Since there is no force outside of the end supports, we know that $M'_f = M^{iv}_f = 0$. Substituting, we have

$$4M''_f l + M'''_f l = -\frac{wl^3}{2}$$

$$M''_f l + 4M'''_f l = -\frac{wl^3}{2}$$

Combining and solving, we have

$$M''_f = M'''_f = -\frac{wl^2}{10}$$

Representing the reactions by R', R'', etc., and using the General Equation of Bending Moments (paragraph 322), we have

$$M''_f = R'l - \frac{wl^2}{2}$$

But
$$M''_f = -\frac{wl^2}{10}$$

Hence
$$R'l - \frac{wl^2}{2} = -\frac{wl^2}{10}$$

Solving, we obtain
$$R' = \frac{4wl}{10}$$

With this known value of R', we can obtain the equation of bending moments at any section between the first two supports. In a similar manner, R'', etc., can be calculated and the bending moments obtained at any section.

HORIZONTAL SHEAR.

327. If two planks are placed one above the other and supported at the ends, the effect of a vertical force at the middle will be to cause the two planks to slide upon each other. This is an evidence of horizontal shear.

If the two planks are nailed together, the effect of the vertical force will be to cause a shearing stress on the nails; but the two planks will not slide on each other if the nails are strong enough to overcome this shearing stress.

If a beam is supported at the ends, and a vertical force is applied at the middle, it will cause a shearing stress similar to that of the two planks; but the internal stresses of the beam will be developed to overcome this horizontal shear, and they may be sufficient to prevent any horizontal movement of one part of the beam on another.

328. Let it be required to deduce an expression for the horizontal shear in beams acted on by vertical forces.

Represent by Fig. 328A a cantilever supported at the left end, with a load, W, at its right end, with two vertical planes $A'\ B'\ C'\ D'\ E'\ F'\ G'\ H'$ and $A''\ B''\ C''\ D''\ E''\ F''\ G''\ H''$ separated by the distance δx, and with the horizontal plane, $C''\ C'\ H'\ H''$, parallel to the neutral surface, $G''\ G'\ D'\ D''$.

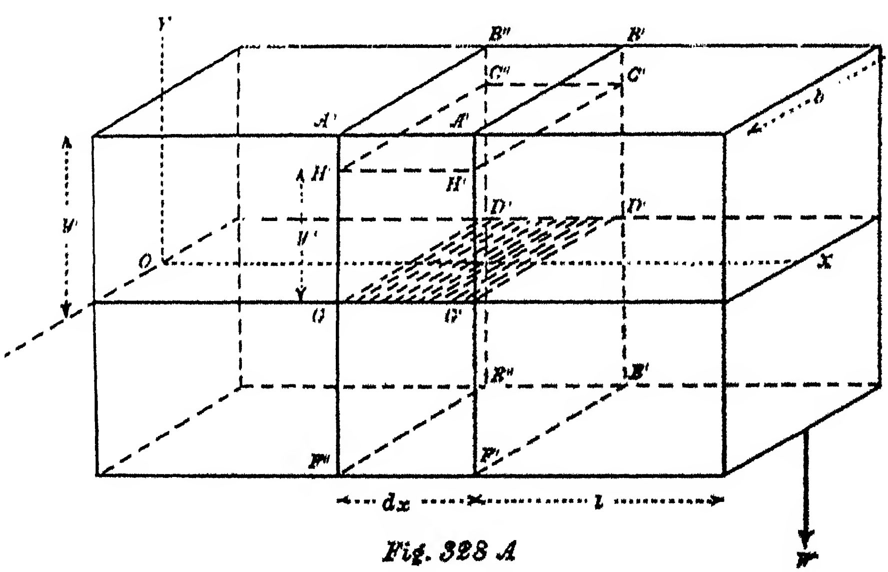


Fig. 328 A

If we can find a general expression for the horizontal shearing stress in the rectangle $C'' C' H' H''$ which prevents movement by the prism $B'' B' A' A'' C'' C' H' H''$, we shall have the desired expression for horizontal shear in beams acted on by vertical forces.

Considering the prism $A' B' C' H' A'' B'' C'' H''$ with its plane $C'' C' H' H''$ at a distance y'' from the neutral surface, assume that

$dy\,dz$ = elementary area of cross-section.

$(s_f)_1$ = unit bending stress in an elementary area at unit distance from the neutral surface.

Then, as in paragraph 259,

$(s_f)_1 y\,dy\,dz$ = bending stress in an elementary area at distance y from the neutral axis.

Applying this to planes $A' B' E' F'$ and $A'' B'' E'' F''$

$(s'_f)_1 y\,dy\,dz$ = bending stress in an elementary area of the plane $A' B' C' H'$ at distance y from neutral surface.

$(s''_f)_1 y\,dy\,dz$ = bending stress in an elementary area of the plane $A'' B'' C'' H''$ at distance y from neutral surface.

$$(s'_f)_1 \int_{z=-\frac{b}{2}}^{z=+\frac{b}{2}} \int_{y=y''}^{y=\frac{D}{2}} y\,dy\,dz = \text{bending stress in area } A'\,B'\,C'\,H' \text{ of plane } A'\,B'\,E'\,F'. \qquad (328A)$$

$$(s''_f)_1 \int_{z=-\frac{b}{2}}^{z=+\frac{b}{2}} \int_{y=y''}^{y=\frac{D}{2}} y\,dy\,dz = \text{bending stress in area } A''B''C''H'' \text{ of plane } A''\,B''\,E''\,F''. \qquad (328B)$$

In the above equations, each double integral is the sum of the products of all the elementary areas by their distances from the neutral axis. This is an expression for the *static moment of an area about its axis of rotation,* and from Mechanics is equal to the total area multiplied by the distance of its center of gravity from the axis.

Taking A_v = area of $A' B' C' H'$ or $A'' B'' C'' H''$.

D' = distance of the center of gravity of either area from neutral axis.

Then $A_v D'$ = static moment of either area with respect to neutral axis.

Substituting in equations 328A and 328B, we have

$(s'_f)_1 A_v D' =$ bending stress in area $A'\ B'\ C'\ H'$

$(s''_f)_1 A_v D' =$ bending stress in area $A''\ B''\ C''\ H''$

Considering the small prism $A'\ B'\ C'\ H'\ A''\ B''\ C''\ H''$ as a "free body", the horizontal shearing stress on the base $C'\ H'\ H''\ C''$ must be equal to the difference between the bending stresses on $A'\ B'\ C'\ H'$ and $A''\ B''\ C''\ H''$.

That is $$s_s b dx = [(s''_f)_1 - (s'_f)_1] A_v D' \qquad (328C)$$

It is necessary to obtain this expression in terms of the bending moment and vertical shear at any section.

From equation 259D, we have

$$(s_f)_1 = \frac{M_f}{I} \qquad (259D)$$

Applying this to the values of $(s'_f)_1$ and $(s''_f)_1$, and taking

$M_f =$ bending moment at section $A'\ B'\ E'\ F'$,

$M_f + dM_f =$ bending moment at section $A''\ B''\ E''\ F''$,

we have

$$(s'_f)_1 = \frac{M_f}{I} \qquad (328D)$$

$$(s''_f)_1 = \frac{M_f + dM_f}{I} \qquad (328E)$$

Substituting in equation 328C, we have

$$s_s b dx = \left[\frac{M_f + dM_f}{I} - \frac{M_f}{I}\right] A_v D' = \frac{dM_f A_v D'}{I} \qquad (328F)$$

$$s_s = \left(\frac{dM_f}{dx}\right)\frac{A_v D'}{bI} \qquad (328G)$$

But $\dfrac{dM_f}{dx} = S_v =$ vertical shear in section $A'\ B'\ E'\ F'$.

Substituting in equation 328G, we have

$$s_s = \frac{S_v A_v D'}{bI} \qquad (328H)$$

That is, *the unit horizontal shearing stress at any point of a surface parallel to the neutral surface, is equal to the result obtained by multiplying the vertical shear at the section by the static moment of the area of section between this surface and the surface of the*

beam, and dividing this product by the moment of inertia of the section of the beam multiplied by the breadth of the beam at the surface considered.

329. Let A_h = total area of a horizontal surface between sections with the same vertical shear.

Then $s_s A_h$ = total shear in the horizontal surface.

$$= \frac{S_v A_v D' A_h}{bI} \tag{329A}$$

330. This equation is true only if the beam is of uniform cross-section so that I is constant. Hence in the same horizontal surface, since $A_v D'$ and b are also constant, the value of s_s in the equation 328H will vary with S_v; or, *in a beam of uniform cross-section, the unit shearing stress in any surface which is parallel to the neutral surface, will vary with the vertical shear.*

The unit horizontal shear will therefore be uniform at every point of a horizontal surface of a beam whose vertical shear is constant, and the total horizontal shear will be equal to $s_s bl$. The unit horizontal shear will be variable in the horizontal surfaces of a beam whose vertical shear is not constant. The total shear in the plane will be equal to the mean value of s_s multiplied by bl.

331. In the same plane of cross-section S_v will be constant, and the value of s_s in equation 328H will vary with $A_v D'/b$; or, *the unit horizontal shear in any plane of cross-section will vary directly with the static moment of the area included between the surface of the beam and the surface of horizontal shear, and inversely with the breadth.*

332. In a rectangular beam of uniform cross-section whose depth is D and whose breadth is b, we have

$$A_v = b\left(\frac{D}{2} - y\right), \text{ and } D' = \frac{\frac{D}{2} + y}{2}. \text{ Hence } A_v D' = \frac{b}{2}\left[\left(\frac{D}{2}\right)^2 - y^2\right] \tag{332A}$$

$$\text{Therefore (Eq. 328H)} \quad s_s = \frac{S_v A_v D'}{bI} = \frac{S_v}{bI}\frac{b}{2}\left[\left(\frac{D}{2}\right)^2 - y^2\right] \tag{332B}$$

The unit horizontal shear in any plane of cross-section will vary in a direction perpendicular to the neutral axis with the expression $(D/2)^2 - y^2$; it will be zero at the surface where $y = D/2$, and will be a maximum at the neutral surface where y = zero.

333. **Vertical Shear.**—In paragraph 238 we have shown that the

unit vertical and horizontal shears at any point in a beam must be equal to each other, hence the vertical shear must vary in the plane of cross-section, and must be a maximum at the neutral axis.

If, in equation 332B, we substitute for I its value $(1/12)bD^3$, and for y its value zero at the neutral axis, we have

$$s_s = \frac{3}{2}\left(\frac{S_v}{bD}\right) \tag{333A}$$

or the unit horizontal shear at the neutral fiber is 3/2 the unit vertical shear assumed to be uniformly distributed over the area of cross-section.

In a rectangular beam, therefore, the true maximum unit vertical shear at the axis is 3/2 its assumed value on the basis of uniform distribution. However, it is usual to omit consideration of this variation in vertical shear and to assume it constant throughout the cross-section; and this is generally satisfactory in work of designing, because beams are designed ordinarily to resist bending forces and have a large excess strength to resist vertical shear.

COMBINED STRESSES.

334. **Tension and Flexure.**—If a beam is subjected at the same time to both tensile and bending forces, the maximum unit stress will be in the fiber in which the sum of the tensile stresses due to the two forces is a maximum. This fiber will usually be the surface fiber on the tension side of the danger section. In Fig. 334A, taking s_m = maximum longitudinal unit fiber stress in beam,

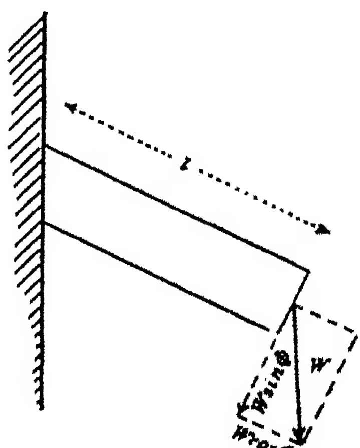


Fig. 334 A

then,
$$s_m = s_t + s_f = \frac{F}{A} + \frac{M_f y}{I} \tag{334A}$$

335. **Compression and Flexure.**—Similarly, for a force producing compressive and bending forces, we have

$$s_m = s_c + s_f = \frac{F}{A} + \frac{M_f y}{I} \tag{335A}$$

336. **Flexure and Shear.**—A force of flexure produces longitudinal stresses in the fibers of the cross-section of a beam, and shearing stresses in each horizontal plane and in each vertical plane of cross-section.

The joint effect of the stresses will be to bring a combined stress on some oblique plane.

By deduction (omitted) we obtain the equations

$$s_{fm} = \frac{s_f}{2} \pm \sqrt{s_s^2 + \frac{s^2_f}{4}} \tag{336A}$$

$$s_{sm} = \pm \sqrt{s^2_s + \frac{s^2_f}{4}} \tag{336B}$$

337. **Longitudinal Stress and Torsion.**—As torsion is a shearing stress, the formulas above given are also applied to stresses produced by a combined longitudinal and torsional force.

By similar deduction (omitted) we obtain the equations

$$s_{tm} = \frac{s_t}{2} + \sqrt{s_{tr}^2 + \frac{s_t^2}{4}} \tag{337A}$$

or

$$s_{cm} = \frac{s_c}{2} + \sqrt{s_{tr}^2 + \frac{s_c^2}{4}} \tag{337B}$$

$$s_{trm} = \sqrt{s^2_{tr} + \frac{s^2_t \text{ (or } s_c^2)}{4}} \tag{337C}$$

PROBLEMS.

P. 301. A cantilever, projecting to the left, 20′ long, carries a concentrated weight of 200 pounds at its free end.

(a) Write equations and construct curves of shear and bending moment with the origin at the free end; (1) the units being in inches, (2) the units being in feet.

(b) Select I-beam, (1) allowable stress=16,000 pounds per square inch. (2) allowable stress=12,500 pounds per square inch.

P. 302. A cantilever projecting to the left, 15′ long, carries a concentrated weight of 1,000 pounds at the free end.

(a) Write equations and draw curves of bending moment and shear.

(b) Find fiber stress if the beam is of wood, rectangular, 8″ wide by 9″ deep.

(c) Select the proper I-beam for a maximum fiber stress of 16,000 pounds per square inch, I vertical.

(d) Select the lightest channel to be placed with the web vertical, maximum fiber stress being 16,000 pounds per square inch.

P. 303. (Fig. P. 303.) Write the equations of bending moments and shear, with origin at free end, and show them graphically. Find moments and shear at weights. Neglect weight of beam.

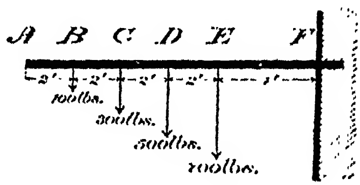


Fig. P. 303.

P. 304. A cantilever is firmly fixed horizontally into a wall and acted on by an upward force of 200 pounds, 15′ from the support, and by a downward force of 100 pounds, at the end, 10′ further out. Neglecting the weight of the beam, determine the equations of bending moment and shear, and draw the curves for same.

P. 305. A cantilever projecting to the left, 20′ long, carries a uniformly distributed load of 2 pounds per linear inch.

(a) Write equations and draw curves of bending moment and shear with the origin at the left end.

(b) Assume the beam to be made of wood. In order to be safe, how wide should it be, if its depth is 6″?

P. 306. (Fig. P. 306.) (a) Write equations and draw curves of bending moments and shear.

(b) Find values of M and S_n at O, A, B, and C.

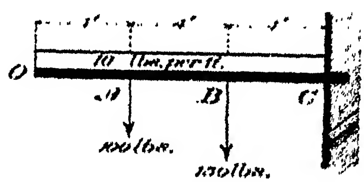


Fig. P. 306.

P. 307. (Fig. P. 307.) (a) Write equations and draw curves of bending moments and shear.

(b) Find values of bending moment and shear at $x = 4$, 10, 20 and 30.

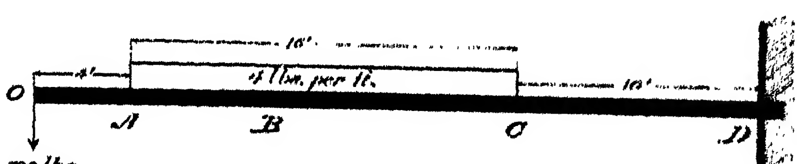


Fig. P. 307.

P. 308. (Fig. P. 308.) Write equations and draw curves of bending moment and shear, and write values at $x=0, 8$ and 16.

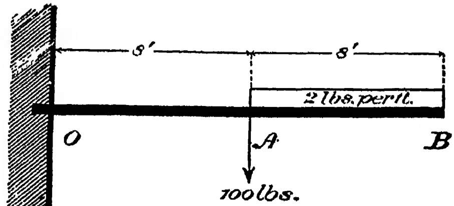


Fig. P. 308.

P. 309. (Fig. 309.) Write the equations of bending moment and shear; find values for bending moment and shear at the position of the loads and draw lines of shear and bending moment. Neglect weight of beam.

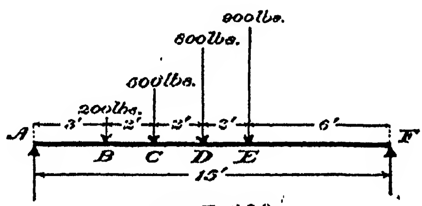


Fig. P 309.

P. 310. (Fig. P. 310.) (a) Write equations and draw curves of bending moment and shear.

(b) Find values of bending moment and shear at $x=0$, 40, 60, 80, and 100.

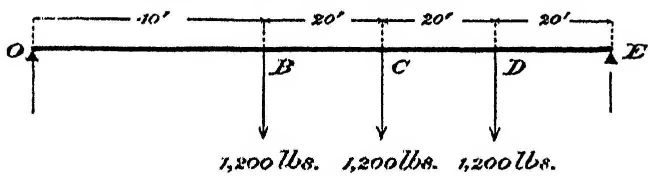


Fig. P. 310.

P. 311. (Fig. P. 311.) (a) Write equations and plot curves of bending moment and shear.

(b) Determine maximum shear, maximum bending moment, and dangerous sections for shear and bending moment.

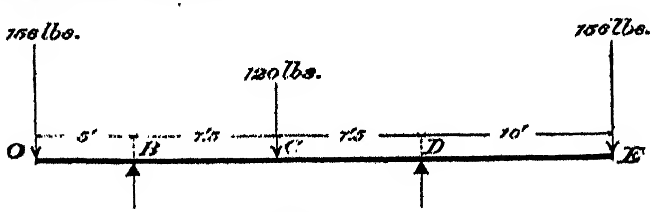


Fig. P. 311.

P. 312. (Fig. P. 312.) Write equations and draw curves of shear and bending moment.

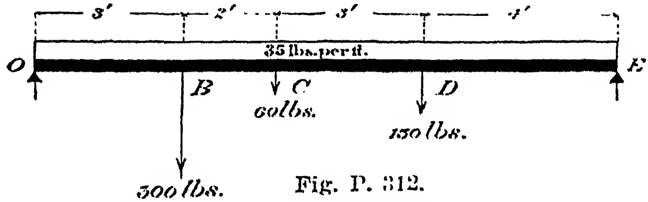


Fig. P. 312.

P. 313. (Fig. P. 313.) Draw lines of shear and write values of ordinates at points where values change abruptly.

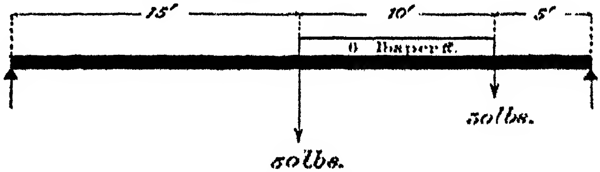


Fig. P. 313.

P. 314. (Fig. P. 314.) (a) What concentrated load placed at O will make $R'' = 0$?

(b) With this load in position, write equations and draw curves of bending moment and shear.

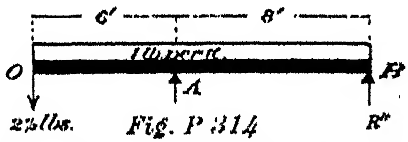


Fig. P 314

P. 315. (Fig. P. 315.) Write equations and draw curve of shear and bending moment. Hinges at A and B.

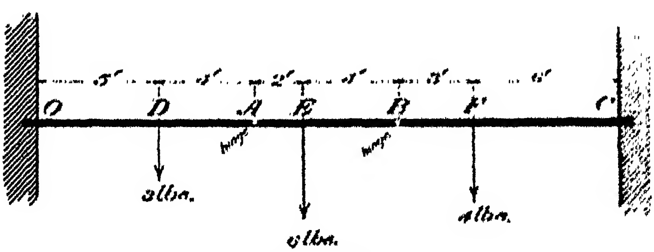


Fig. P. 315.

P. 316. (Fig. P. 316.) Write equations and draw curve of shear and bending moment. Supports at A, B, C, and D.

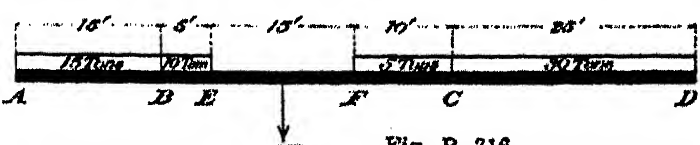


Fig. P. 316.

P. 317. A beam 20′ long, weighing 20 pounds per lineal foot, is placed upon a support, dividing it into segments of 16′ and 4′ and is kept horizontal by a downward force W at the middle point of the smaller segment.

(a) Find the value of W, and the reaction of the point of support.

(b) Draw lines of shear and bending moment.

P. 318. A man and 8 boys carry a stick of timber, the man at one end and the boys at a common point.

(a) Find the position of this point, if the man is to carry twice as much as each boy.

(b) Plot curves of shear and bending moment.

P. 319. (Fig. P. 319.) Determine the value of W and construct to scale the curves of shear and bending moment. Hinge at C.

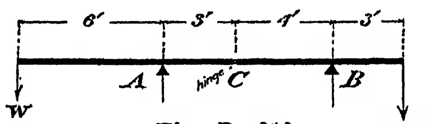


Fig. P. 319.

P. 320. A beam 10′ long, resting on end supports, has a dead load varying uniformly from 0 at the left to 100 pounds per lineal foot at the right end.

(a) Find the reactions.

(b) Write equation of shear, origin left.

(c) Write equation of bending moment, origin left.

(d) Determine maximum value of bending moment in foot-pounds and position of dangerous section.

(e) Determine maximum shear.

(f) Draw curves of shear and bending moment.

(g) Find bending moment and shear at $x=3$, 6 and 9.

P. 321. A wooden cantilever beam, 12′ long, 6″ wide and 8″ deep, supports a weight of 300 pounds at the free end. Find the equation of the mean fiber deflection in inches, origin being at free end. Also find greatest deflection in inches.

P. 322. A wooden cantilever beam, 6″ wide, 8″ deep and 10′ long, supports a weight of 1,000 pounds at the free end. Find the maximum deflection due to this load.

P. 323. (Fig. P. 323.) Deduce the equation of the mean fiber, the unit of x being in inches.

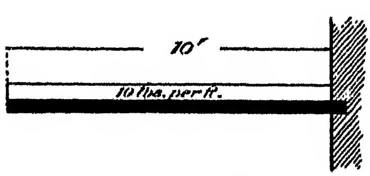


Fig. P. 323.

P. 324. What is the deflection at the end of a cantilever of wrought-iron 20′ long, 2″ wide and 4″ deep, under the action of its own weight and a load of 100 pounds concentrated at the free end? Construct curve of bending moment to scale. $E = 24{,}000{,}000$. Weight of iron equals 480 pounds per cubic foot.

P. 325. A wooden beam, 15′ long supported at the ends, of rectangular cross-section 2″ by 8″, bears a load of 500 pounds concentrated at its middle point. Find the equation of the mean fiber and the value of the greatest deflection; 8″ dimension is vertical.

P. 326. A 9″ steel I-beam, 21 pounds per foot, supported at its ends and 10′ long, bears a concentrated load of 25 tons at its center. Determine the maximum deflection due to this load.

P. 327. A floor is to support a total uniform load of 100 pounds per square foot. The 10″ steel I-beams, 25 pounds per foot, have a span of 20′ and are spaced 6.5′ between centers. Does the maximum deflection of the beam exceed 1/360 of the span?

P. 328. A wooden beam, 10′ long and 3″ in square cross-section, is fixed at the ends and supports at its middle point a concentrated load of 100 pounds.

(a) Deduce and plot the equation of the curve of bending moment.

(b) Deduce the equation of the curve of mean fiber.

(c) Draw the curve of mean fiber, after determining the deflection at the following points: (1) points of support; (2) middle point of beam; (3) points of inflection.

(d) Is the beam safe, if so what is its factor of safety?

P. 329. (Fig. P. 329.) A steel I-beam is fixed at both ends and loaded as shown. The moment of inertia of cross-section of the beam about neutral axis is 2,087.2 (inches^4). Find the greatest deflection.

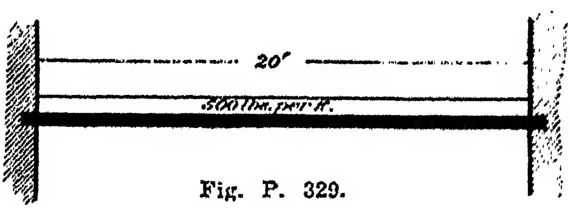


Fig. P. 329.

P. 330. A beam 20′ long and fixed at the ends carries a uniform load of 12 pounds per linear foot. Determine where the curve of mean fiber of the beam has its points of inflection.

P. 331. A wooden floor beam, 18′ long, of rectangular cross-section 2″ wide, is fixed at one end and supported at the other, and bears a uniformly distributed load of 50 pounds per lineal foot. What must be the depth of the beam in order that the greatest deflection shall not exceed 1/360 of the span?

P. 332. A steel I-beam, 20′ long, is fixed at one end and supported at the other, and bears a uniformly distributed load of 200 pounds per lineal foot. Select the beam that will safely bear the load.

P. 333. A wooden beam, 8″ by 14″ cross-section, 16′ long and supported at each end, bears a total uniform load of 100 pounds per lineal foot. How much

greater will be the maximum deflection when the short side is vertical than when the long side is vertical?

P. 334. A beam 16′ long, 2″ wide and 6″ deep, has a maximum deflection of 0.3″. Determine the maximum deflection of a beam of the same material 12′ long, 3″ wide and 8″ deep, with the same loading and manner of support.

P. 335. A continuous beam, 36′ long, rests on three supports spaced at intervals of 18′, and is loaded with 500 pounds per lineal foot; find the bending moments in inch-pounds at points 10′ to the right of the left hand support, at 2′ to the right of the middle support, and at that point where the bending moment is numerically greatest.

P. 336. A continuous beam of two equal spans is uniformly loaded. Will the greatest fiber stress be greater or less than if the beam be sawn in two over the middle point of support, and in what ratio?

P. 337. A continuous uniformly loaded beam 30′ long is supported at both ends and at a point 10′ from left end. Find reactions and plot curves of shears and bending moments, origin at left end of beam.

P. 338. A continuous beam, 60′ long, is uniformly loaded with 200 pounds per lineal foot and rests upon four supports equally spaced. Find the reactions at the points of support; the bending moments at the same points and at the middle points of the three spans, all in inch-pounds, and draw the curve of bending moments.

P. 339. A continuous uniformly loaded beam, 50′ long, is supported at the ends and at two points each distant 5′ from the center. Find reactions and draw curves of shears and bending moments.

P. 340. A continuous beam of three equal spans, has the second span uniformly loaded and no loads on the outer spans. Determine the reactions. Draw curves of bending moment and shear, origin at left end of beam. Neglect weight of beam.

P. 341. Find the maximum horizontal shearing unit stress in a cantilever beam 6″ wide, 8″ deep, and 10′ long, if it supports a weight of 1,000 pounds at its free end.

P. 342. (Fig. P. 342.) The cantilever is made up of two pieces bolted one on top of the other, each 1.5″ deep by 3″ wide. How many 1/2″ wrought-iron bolts are necessary to safely resist the horizontal shear in AB? How many in BC?

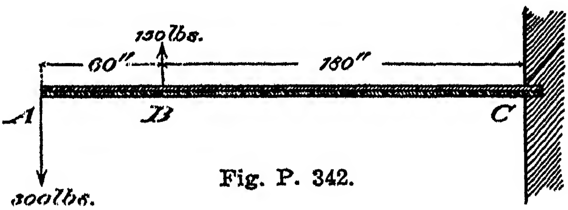


Fig. P. 342.

P. 343. A wooden beam, 18′ long, 8″ deep, and 6″ wide, supported at the ends, is made up of two pieces 4″ thick and 6″ wide, placed one on the other and held together by wooden pins 1″ in diameter. A weight of 600 pounds rests on the beam at its middle point. How many wooden pins must be used to insure a factor of safety of 5? Neglect weight of beam.

P. 344. A wooden beam, 18′ long, is made of two pieces 12″ by 12″, placed one upon the other, and rests upon end supports. The beam carries a uniformly distributed load of 100 pounds per lineal foot. If the two pieces are bolted together, what is the total shear on the bolts? Neglect weight of beam.

P. 345. A beam 10′ long and 6″ by 6″ in cross-section, resting on end supports, is made up of three 2″ by 6″ wooden planks superimposed. Three bolts, one at each end and one in the middle of the beam, hold the planks together.

(a) If the beam is subjected to a uniformly distributed load of 100 pounds per lineal foot, what is the shearing stress of each bolt? Neglect weight of beam.

(b) If the bolt at one end should fail, what would be the shear on each of the other two bolts?

P. 346. A wooden beam is composed of two pieces, placed one on the other and is 16′ long, 12″ deep, and 8″ wide, and rests on end supports. The top piece is 8″ wide and 8″ deep; the bottom piece, 8″ wide and 4″ deep. The two pieces act as a solid beam because of seven pieces of wood 2″ by 2″ by 8″ with grain horizontal being notched into both pieces at intervals of 2′. Find the safe load upon the middle of the beam. Factor of safety equals 5.

P. 347. A 10″ steel I-beam, 30 pounds per foot, resting on supports 20′ apart, carries a load of 6,000 pounds at its middle point. Determine the maximum horizontal unit shear if the center of gravity of each half-section is 4.5″ from the neutral axis.

P. 348. What is the size of a square wooden beam of 12′ span, which sustains a load of 300 pounds at the center and has at the same time to sustain a constant tension (longitudinally) of 2,000 pounds, the maximum allowable stress per square inch being 1,000 pounds? Neglect weight of beam.

P.349. A 6″ 12.25 pound steel I-beam projects downward from a wall at an angle of 45° with the horizontal, and is 10′ long. What is the safe load that can be suspended from the outer end?

(a) Neglecting weight of beam.

(b) Considering weight of beam.

P. 350. A beam, 10′ long, 4″ by 4″ in cross-section, projects upward from a wall at an angle of 60° with the vertical. A weight of 1,000 pounds is suspended from the free end. What is the maximum fiber stress, and where is it? (cos. 60° = ½).

P. 351. A steel bar, 8″ by 8″ in rectangular cross-section is firmly fixed in a wall and is pointing downward at an angle of 66° 25′ with the horizontal, so that 4′ 2″ project from the wall. It carries a weight of 2,000 pounds at the lower end. Find the maximum fiber stress, and locate the fiber that has no stress. Below what section of the bar will all fibers be in tension? (sin. 66° 25′ = .9165; cos. = .4000).

P. 352. A 12″ 35 pound I-beam, 8′ long, rests against a vertical wall at one end and on the floor at the other, and makes an angle with the floor of 60°. A weight of 5,000 pounds is supported at the middle of the beam. What is the maximum fiber stress in the beam? Neglect weight of beam.

P. 353. A bar of iron is under a direct tensile stress of 5,000 pounds per square inch, and a shearing stress of 3,500 pounds per square inch. Find the maximum unit tensile and shearing stresses.

P. 354. A wooden beam, 8″ wide, 12″ deep and 16′ long, is loaded with 2,000 pounds at the middle point and rests on end supports. Find the maximum tensile stress on a fiber 4″ below the neutral fibre at the middle section; also the maximum shearing stress at the same point. Neglect weight of beam.

P. 355. A wooden beam, 4″ wide by 12″ deep, rests on end supports 12′ apart, and carries a total uniformly distributed load of 6,400 pounds. Find the maximum longitudinal stress of a fiber 2″ below the upper surface of the beam at its middle section, and the maximum shear at the same fiber one foot from the left end.

P. 356. A circular steel shaft, 4″ in diameter, is 8′ long and transmits 25 horse-power through a pulley at its middle point, making 220 revolutions per minute. The belt on the pulley has a tension of 100 pounds. Find the maximum unit torsional and tensile stresses in the extreme upper and lower fibers of the shaft due to combined bending and torsion.

P. 357. A capstan post is 18″ square and 4′ long from base to arm, the arm is 12′ long to side of post, and has a force of 200 pounds applied to it at the end. What is the maximum unit tensile stress developed in the post, due to torsion and bending? Capstan bar is perpendicular to face of post. $I_p = b^4/6$.

P. 358. A circular wrought-iron shaft is subjected simultaneously to a bending moment of 8,000 (inch-pounds) and a twisting moment of 15,000 (inch-pounds). Determine the least diameter of the shaft if the maximum tensile strength is not to exceed 10,000 pounds per square inch, and the shearing stress 8,000 pounds per square inch.

P. 359. A wrought-iron shaft, 3″ in diameter, and making 140 revolutions per minute is supported in bearings 16′ apart. If a load of 210 pounds is brought by a belt and pulley at the middle, what horse-power can be transmitted with a maximum shearing stress of 8,000 pounds per square inch?

P. 360. Determine the factor of safety for a wrought-iron shaft, 3″ in diameter, resting on bearings 12′ apart, when transmitting 25 horse-power at 100 revolutions per minute and bearing a load of 200 pounds at the middle.

P. 361. Deduce the formulas in paragraph 309.
P. 362. " " " " " 313.
P. 363. " " " " " 316.
P. 364. " " " " " 317.
P. 365. " " " " " 318.

P. 366. Draw, without calculations, the curves of shear, bending moment, inclination, and mean fiber, showing their relationship to each other, of the beams shown in:

a. Fig. P. 303.	*e.* Fig. P. 309.	*i.* Fig. P. 313.
b. Fig. P. 306.	*f.* Fig. P. 310.	*j.* Fig. P. 315.
c. Fig. P. 307.	*g.* Fig. P. 311.	*k.* Fig. P. 316.
d. Fig. P. 308.	*h.* Fig. P. 312.	*l.* Fig. P. 323.

CHAPTER IV.

MOVING LOADS.

401. If a load does not remain fixed in position in or on a structure, but moves over it, the load is called a moving or live load. A road roller or a railway train moving over a bridge is a *moving or live load*. In the case of the road roller, since the bridge is usually a highway bridge, it is generally desirable to consider its weight as represented by one moving load concentrated under the center of the road roller. In the case of the railway train, since the bridge is usually proportionately long as compared with each car, it is generally desirable to consider its cars as represented by a uniformly distributed load of the length of the railway train: its locomotive is represented by a number of concentrated loads representing the weights on the locomotive wheels.

402. In either case, the problem of finding the stress due to the live load in any one position is the same as the problem already considered for concentrated loads or uniformly distributed loads fixed in position. However, as the structure must be designed to support the maximum stress, it is first necessary to determine the particular positions of the live load which will give the maximum stress on the whole structure or on certain parts of the structure. It is not possible to place the maximum load in any one position so that it will produce maximum stresses at all sections. *It is necessary to consider the live load in all possible positions on the structure; and it is generally necessary to draw curves of maximum stresses for every section of the structure under the most unfavorable position of the live load; and then design each portion of the structure to support the maximum stress.*

403. For complete consideration of the stresses, it is also necessary to consider the stresses due to the fixed loads. These are called the *dead loads*. Taking, for example, a railway bridge; it has the dead load due to the weight of the structure and the live load due to the railway trains which cross it; the dead and the live loads together develop the maximum stresses in the members of the bridge.

404. Let it be required to determine shears and bending moments in a beam without weight resting on end supports, under the action of a single concentrated live load. Determine also the maximum stresses at every section.

Represent the beam by Figs. 404A and 404B, with the live load W, with C as any section under consideration, and with x as the distance from the origin to any position of the live load.

Shear.—(Fig. 404A). The equation is obtained for shear at any section as the load moves over the beam. From this the maximum shear is determined at each section for any position of the load.

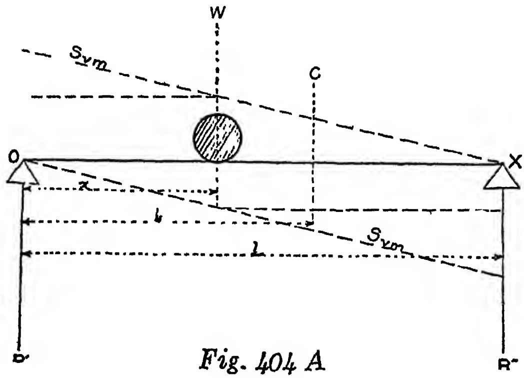


Fig. 404 A

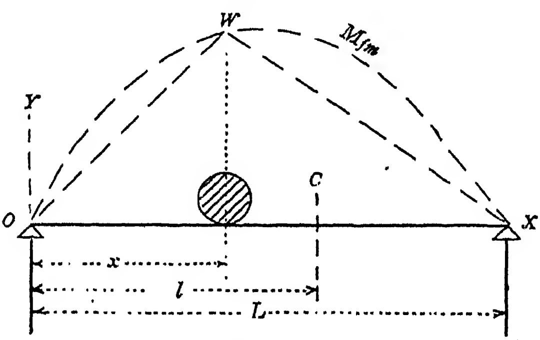


Fig. 404 B

a. While the load moves from O to C, we have

$$S_v \text{ at } C = -W + \frac{W(l-x)}{l} = -\frac{Wx}{l}$$

Since $x < L$, S_v will be negative and will increase with x until x reaches its maximum value, l, when we shall have

$$S_v \text{ at } C = -\frac{Wl}{L} \qquad (404\text{A})$$

b. While the load moves from C to X, we have at any section, C,

$$S_v \text{ at } C = \frac{W(L-x)}{L} \qquad (404\text{B})$$

Since $x < l$, S_v will be positive and will decrease as x increases. The maximum value is when x is at its minimum or equal to d.

We then have

$$S_v \text{ at } C = \frac{W(L-l)}{L} \qquad (404\text{C})$$

Therefore, *if a concentrated live load moves over a beam without weight, the live load shear at any section will be numerically greatest when the live load is at the section.* This gives the two curves of S_{rm} as shown in Fig. 404A for maximum shear at each section.

To find the section having the greatest maximum shear, we take equations 404A and 404C and substitute such values as will give the maximum for S_r. We find that the maximum is when $l = L$ for equation 404A, and when $l = 0$ for equation 404C. These values of l are when the live load is at the supports, and are equal. Therefore, *if a concentrated live load moves over a beam without weight, the live load shear is a maximum at the supports; this occurs when the load is at the supports.**

Substituting the value of $l = L$ in equation 404A, we find this maximum live load shear to be

$$S_{rm} = W \qquad (404D)$$

Bending Moment.—(Fig. 404B). The equation is obtained for bending moment at any section as the load moves over the beam. From this the maximum bending moment is determined at each section for any position of the load.

a. While the load moves from O to C, we have at any section, C,

$$M_f \text{ at } C = R'd - W(l - x) = \frac{Wl(L - x)}{L} - W(l - x) = Wx\left(1 - \frac{l}{L}\right) \qquad (404E)$$

Since $l < L$, M_f will be positive and will increase with x, until x reaches its maximum value, l, when we shall have

$$M_f \text{ at } C = Wl - \frac{Wl^2}{L} \qquad (404F)$$

b. While the load moves from C to X, we have at any section, C,

$$M_f \text{ at } C = R'l = Wl - \frac{Wxl}{L} = Wl\left(1 - \frac{x}{L}\right) \qquad (404G)$$

Since $x < L$, M_f will be positive and will decrease while x increases.

The maximum value is when $x = l$, when we shall have the same value of M_f as in equation 404F.

Therefore, *if a concentrated live load moves over a beam without weight, the live load bending moment at any section of the beam will be numerically greatest when the live load is at the section.* This gives the curve M_{fm} as shown in Fig. 404B for maximum bending moment at each section.

* The maximum shear is at a support when the live load is actually a very small distance from the support; for all practical purposes it is taken as at the support.

To find the section having the maximum bending moment, we take equation 404F for the maximum bending moment at the section at a distance l from the origin of co-ordinates, and substitute x for l to obtain the equation for the maximum moment at any section at a distance x from the origin, thus

$$M_f = Wx - \frac{Wx^2}{L} \qquad (404\text{H})$$

If we differentiate this equation with respect to x and make this differential equal to zero, the resulting value of x locates the point where the tangent of the curve is zero, that is, where the curve is highest which is where the bending moment is a maximum.

Thus

$$\frac{dM_f}{dx} = W - \frac{2Wx}{l} = 0 \qquad (404\text{I})$$

Whence

$$x = \frac{L}{2} \qquad (404\text{J})$$

Therefore, *if a concentrated live load moves over a beam without weight, the live load bending moment is a maximum at the middle point of the beam; this occurs when the load is at the middle point.*

Substituting in equation 404H, we find the maximum bending moment to be

$$M_{fm} = \frac{WL}{4} \qquad (404\text{K})$$

405. **Problem.**—A concentrated load of 100 pounds rolls over a beam 80′ long, without weight, resting on end supports. Find the maximum shear and bending moment at a point 20′ from the left support.

Solution.—From paragraph 404, we know that both the shear and bending moment at the point will be maximum when the live load is at the point. With this loading, we have the reaction at the left support equal to 75 pounds. The stresses at the point 20′ from left support are:

Shear = 75 pounds.
Bending Moment = 75 × 20 foot pounds.

406. Let it be required to determine maximum shears and bending moments in a **beam with uniform dead load resting on end supports under the action of a single concentrated live load.**

Represent by Figs. 406A and 406B, the beam with the uniformly distributed load as shown.

Shear.— (Fig. 406A).

Since the uniformly distributed dead load does not change position as the live load moves over the beam, the shear due to dead load is fixed for the section and is represented by the line "S_r *for dead load*". It is thus evident that the maximum shear at any section will be dependent upon the position of the live load which will

give the maximum shear at the section. From paragraph 404, the maximum shear at a section occurs when the live load is at the section. Hence, since the sign is the same as that of the shear for the dead load, it is necessary only to add the two in order to obtain the maximum total shear for the section.

Therefore, *if a concentrated live load moves over a uniformly loaded beam, the shear at any section of the beam due to live and dead loads will be numerically greatest when the live load is at the section.*

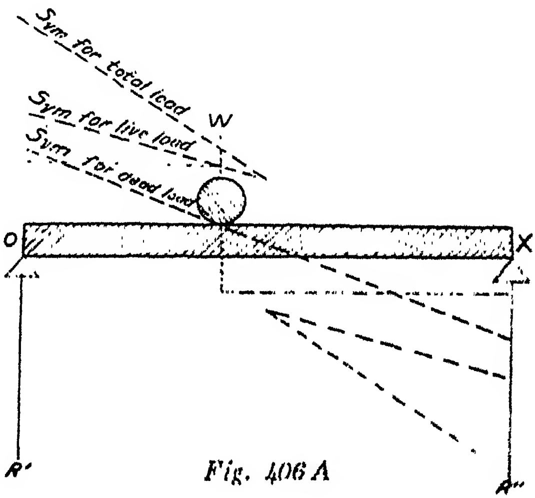


Fig. 406 A

Similarly, since the maximum shear is at the supports for both live and dead loads separately, it will be there for their sum.

Therefore, *if a concentrated live load moves over a uniformly loaded beam, the shear due to live and dead loads is a maximum at the supports; this occurs when the live load is at the supports.*

Bending Moment.—(Fig. 406B).

Since this uniformly distributed dead load does not change position in any way as the live load moves over the beam, its bending moment is represented by the curve "*M_f for dead load*" in the figure. From this it is evident that, due to dead load alone, the bending moment at any section is a fixed quantity regardless of the live load. Hence, the greatest bending moment at a section due to both live and dead loads will be when the bending moment due to live load is greatest; and from paragraph 404 this is when the live load is at the section. (See Fig. 406B.)

Therefore, *if a concentrated live load moves over a uniformly loaded beam, the bending moment at any section of the beam due*

to live and dead loads will be numerically greatest when the load is at the section.

Similarly, since the maximum bending moment is at the middle point for both live and dead loads separately, it will be there for their sum. (See Fig. 406B.)

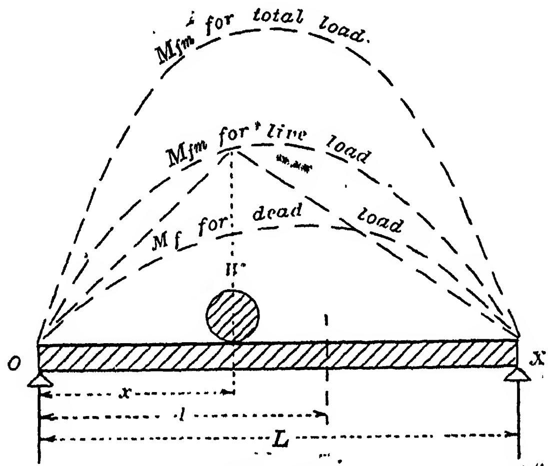


Fig. 406 B

Therefore, *if a concentrated live load moves over a uniformly loaded beam, the bending moment due to live and dead loads is a maximum at the middle of the beam; this occurs when the live load is at the middle of the beam.*

407. **Problem.**—A concentrated load of 1200 pounds rolls over a beam 90′ long, resting on end supports and carrying a dead load of 40 pounds per linear foot. Find the maximum shear and bending moment.

Solution.—From paragraph 406, we know that the maximum shear is at the supports when the live load is at the supports. Taking the left support,

$$R' \text{ under this condition} = \frac{90 \times 40}{2} + 1200 = 3000 \qquad (407A)$$

$$S_{vm} = 3000 \text{ pounds} \qquad (407B)$$

From paragraph 406, we know that the maximum bending moment is at the middle of the beam when the live load is at the middle,

$$R' \text{ under this condition} = \frac{90 \times 40}{2} + \frac{1200}{2} = 2400 \qquad (407C)$$

Working from left,

$$M_{fm} \text{ at middle point} = 2400 \times 45 - 40 \times 45 \times \frac{45}{2} \qquad (407D)$$

$$= 67{,}500 \text{ foot pounds}$$

408. From the preceding discussions, it is known (1) that the sign of the shearing stress due to a concentrated load changes at the load from positive to negative; (2) that the sign of the shearing stress due to a uniformly distributed load changes at the middle point. From this, it is evident that between the concentrated load and the middle point, the shearing stress is reduced and may even have its sign changed. This condition exists in the case of a road roller, moving over a highway bridge, in which we consider the road roller as a live load which may be concentrated at any section, and consider the weight of the bridge as a uniformly distributed dead load.

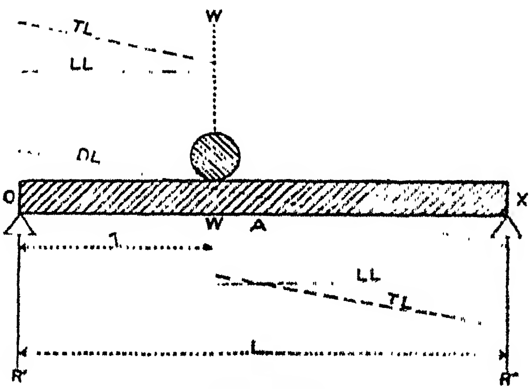


Fig. 408 A

The change of stress in shear is shown in Fig. 408A, in which

Curve *DL* represents the shear due to dead load.

Curve *LL* represents the shear due to live load in the position shown.

Curve *TL* represents the shear due to total load in the position shown.

It is seen that in this case the total shear changes from positive to negative between the live load and the center.

The same result is obtained by use of the equations of shear.

Thus, *for any section between O and W, with live load at W,* we have

$$S_x = w\left(\frac{L}{2} - d\right) + W\left(\frac{L - x}{L}\right) \qquad (408A)$$

in which d equals the distance to the section and x equals the distance to the load.

In the expression, $w\left(\frac{L}{2} - d\right)$ is the shear due to the dead load. If we consider only the part of the beam between O and W, $\frac{L}{2} > d$,

and $L > x$; hence both their signs are positive and their sum is numerically greater than either. This is shown graphically in the figure.

For any section between W and A, with live load at W, we have

$$S_v = w\left(\frac{L}{2} - l\right) - \frac{Wx}{L} \tag{408B}$$

Since $\frac{L}{2} > l$, the shear due to the dead load is positive. The resultant shear at any section will be positive if $w\left(\frac{L}{2} - l\right) > \frac{Wx}{L}$; it will be zero when $w\left(\frac{L}{2} - l\right) = \frac{Wx}{L}$, or when at any section the shears due to the dead and live loads are equal and opposite; it will be negative when $w\left(\frac{L}{2} - l\right) < \frac{Wx}{L}$, or the shear due to the dead load is numerically less than the shear due to the live load. This is shown graphically in the figure.

For any section between A and X, we have

$$S_v = w\left(\frac{L}{2} - l\right) - \frac{Wx}{L} \tag{408C}$$

Since $\frac{L}{2} < l$, both terms of the second member are negative, hence the resultant shear at the section is negative and is numerically equal to the sum of the shears of the dead and live loads.

This is shown graphically in the figure.

Therefore, as seen in the figure and deduced from the equations we have the principle that *if a concentrated live load moves over a uniformly loaded beam, the dead and live loads shears will have like signs between the load and nearer support, as well as between the middle point of the beam and farther support; but unlike signs between the load and the middle point.*

409. **Problem.**—A concentrated load of 160 pounds rolls over a beam 80′ long, resting on end supports and carrying a dead load of 15 pounds per linear foot. Find the point where the shear due to both loads is zero; that is, changes its sign from positive to negative, when the live load is 30′ from the left support.

Solution.—Using equation 408B,

$$S_v = w\left(\frac{L}{2} - l\right) - \frac{Wx}{L}$$

Working from the left and substituting proper values, we have

$$0 = 15\left(\frac{80}{2} - l\right) - \frac{160 \times 30}{80}$$

$$l = 36'$$

Application.

410. This principle is especially applicable in designing trusses for highway bridges. As shown in the Chapter on Bridges, it is necessary to introduce counterbraces to take up the reversals of stress due to concentrated live loads.

411. In the case of a vehicle with light load on one pair of wheels and very heavy load on the other pair of wheels, as with large guns, it is desirable to know the position which will give the maximum bending moment.

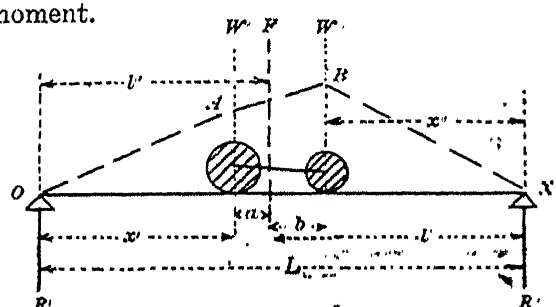


Fig. 411 A

In Fig. 411A represent the live loads on the two wheels by W' and W'', their resultant by F, and their bending moments by M'_f and M''_f.

In the general curve $OABX$ of bending moments due to the two live loads, the line AB may be horizontal or may slope towards A or B.

Hence, the maximum bending moment will be at one of the loads. Therefore, it is necessary only to find the positions of the loads when the bending moment at each is a maximum, and then to determine which of the resulting maximum bending moments is the greater.

(1) For the section at W' we have

$$M'_f = R'x' = \frac{Fx'l'}{L} \qquad \text{(411A)}$$

But
$$l' = L - x' - a \qquad \text{(411B)}$$

Hence
$$M'_f = \frac{F}{L}(Lx' - x'^2 - ax') \qquad \text{(411C)}$$

Differentiating with respect to x', we have

$$\frac{dM'_f}{dx'} = \frac{F}{L}(L - 2x' - a) \qquad \text{(411D)}$$

Making the first differential coefficient equal to zero to obtain the maximum M'_f, we have

$$\frac{F}{L}(L-2x'-a)=0, \text{ or } x'=\frac{L-a}{2} \tag{411E}$$

Therefore, when W' is at a distance of $(L-a)/2$ from O, M'_f is a maximum.

Substituting this value of x' in equation 411B, we have

$$l'=(L-a)-\frac{L-a}{2}=\frac{L-a}{2} \tag{411F}$$

which is the same as the value x'. Hence, when M'_f is a maximum, $x'=l'$, or W' (the greater load) is as far from O as F (the resultant) is from X.

(2) Working from the right support, we have for the bending moment at W''

$$M''_f=R''x''=\frac{Fx''l''}{L} \tag{411G}$$

But
$$l''=L-x''-b, \tag{411H}$$

Hence,
$$M_f''=\frac{F}{L}(Lx''-x''^2-bx'') \tag{411I}$$

$$\frac{dM''_f}{dx''}=\frac{F}{L}(L-2x''-b) \tag{411J}$$

Making $\frac{dM''_f}{dx''}=0$, we have

$$x''=\frac{L-b}{2} \tag{411K}$$

Therefore, when W'' *is at a distance* $(L-b)/2$ from X, M''_f is a maximum.

Substituting this value of x'' in the value of d'' above, we have $l''=(L-b)/2$. Hence, when M''_f is a maximum, $x''=l''$, or W'' is as far from X as F is from O.

That is, the smaller load is as far from one support as the resultant is from the other.

(3) To determine at which load the bending moment is a maximum, we substitute for x' in equation 411A, its value, d', which corresponds to M'_{fm}

$$M'_{fm}=\frac{Fx'l'}{L}=\frac{Fl'^2}{L}. \tag{411L}$$

Similarly, substituting l'' *for* x'' in equation 411G, we have

$$M''_{fm} = \frac{Fl''^2}{L} \tag{411M}$$

To determine which is the greater, l' *or* l'', we know that at their points of maximum bending moments

$$l' = x' = \frac{L-a}{2} \text{ and } l'' = x'' = \frac{L-b}{2} \tag{411N}$$

Since the resultant is nearer the greater weight, W'', we have $a < b$, hence $\frac{L-a}{2} > \frac{L-b}{2}$.

Hence, $$M'_{fm} > M''_{fm} \tag{411O}$$

That is, the maximum bending moment is at the greater weight.

Combining the two results deduced above, we have the principle that, *if two unequal concentrated live loads move over a beam without weight, resting on two supports, the danger section will be at the greater load when that load is as far from one support as the resultant of the loads is from the other:* that is, when the middle of beam is half way between greater load and resultant.

412. Let it be required to determine shears and bending moments **in a beam without weight resting on end supports, under the action of a uniformly distributed live load as long as the beam.** Determine also the maximum stresses at every section.

Represent the beam by Figs. 412A and 412B, with C as any section under consideration, and with x as the distance from the origin to the head of the live load.

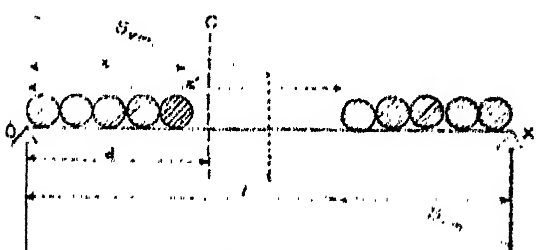


Fig. 412 A

Shear.—(See Fig. 412A).

There are four conditions of shear at C:

a. As head of load moves from O to C, $S_c = -\frac{wx^2}{2L}$.

b. As head of load moves from C to X, $S_c = +w\left(x - l - \frac{x^2}{2L}\right)$.

c. As tail of load moves from O to C, $S_v = w\left(\frac{L}{2} - l + \frac{x'^2}{2L}\right)$.

d. As tail of load moves from C to X, $S_v = w\left(\frac{L}{2} - x' + \frac{x'^2}{2L}\right)$.

Since in *a* and *c* above, x and x' cannot be greater than l, and in *b* and *d* above, x and x' cannot be greater than L and since $L/2 > l$, we find the maximum shear to be in *c* and *d* and to be equal to $w(L/2 - l + l^2/2L)$ when the tail of the load is at the section and the load covers the greater segment of the beam.

Therefore, *if a uniformly distributed load as long as the beam moves over a beam without weight, the live load shear at any section of the beam will be greatest when the tail of the load is at the section and the load covers the greater segment of the beam.*

To find the section having the greatest maximum shear, we take the expression for maximum shear at any section $w(L/2 - l + l^2/2L)$ and substitute values to find the maximum shear at any section. The expression is a maximum when $l = 0$, or when the tail of the load is at the left support. Therefore, *if a uniformly distributed load as long as the beam moves over a beam without weight, the live load shear is a maximum at the supports; this occurs when the load covers the beam.*

Bending Moment.—(Fig. 412B).

It is evident that similar expressions will be obtained for the

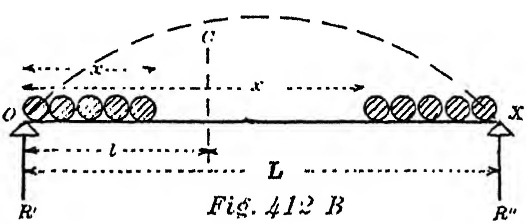


Fig. 412 B

bending moments while the head moves on the beam as will be obtained while the tail moves off the beam; therefore, only the first will be considered.

a. While the head of the live load moves from O to C, we have for bending moment at C

$$M_l \text{ at } C = R'l - wx\left(l - \frac{x}{2}\right) = \frac{wxl\left(L - \frac{x}{2}\right)}{L} - wx\left(l - \frac{x}{2}\right)$$

$$= wxl - \frac{wx^2l}{2L} - wxl + \frac{wx^2}{2} = \frac{wx^2}{2}\left(1 - \frac{l}{L}\right) \qquad \textbf{(412A)}$$

Since $l < L$, M_f will be positive and will increase as x increases, until x reaches its maximum value d when we shall have

$$M_f = \frac{wl^2}{2}\left(1 - \frac{l}{L}\right) = \frac{wl}{2} \times \frac{l}{L}(L - l) \tag{412B}$$

b. While the head of the live load moves from C to X, we have for bending moment at C

$$M_f \text{ at } C = R'l - \frac{wl^2}{2} = \frac{wxl\left(L - \frac{x}{2}\right)}{L} - \frac{wl^2}{2} = wxl - \frac{wx^2 l}{2L} - \frac{wl^2}{2} \tag{412C}$$

This is a maximum when $x = L$, when we shall have

$$M_f = \frac{wl}{2}(L - l) \tag{412D}$$

The value of M_f at C is greater in equation 412D than in equation 412B.

Therefore, *if a uniformly distributed live load as long as the beam moves over a beam without weight, the live load bending moment at any section of the beam will be greatest when the live load entirely covers the beam.*

To find the section having the greatest maximum bending moment, we take equation 412D for the maximum bending moment at the section at a distance l from the origin of co-ordinates, and substitute x for l to obtain the equation for the maximum bending moment at any section at a distance x from the origin.

Thus $$M_f = \frac{wx}{2}(L - x) \tag{412E}$$

Differentiating this, and placing the first differential equal to 0, we have

$$\frac{dM_f}{dx} = \frac{wL}{2} - wx \tag{412F}$$

$$x = \frac{L}{2} \tag{412G}$$

$$M_{fm} = \frac{wL^2}{8} \tag{412H}$$

Therefore, *if a uniformly distributed live load as long as the beam moves over a beam without weight, the live load bending moment is a maximum at the middle point of the beam; this occurs when the live load covers the entire beam.*

413. **Problem.**—Find the maximum bending moment at a point 30′ from the left support of a 70′ beam without weight, if a uniformly distributed live load 80′ long, weighing 22 pounds per foot, rolls across it.

Solution.—From paragraph 412, we know that the maximum bending moment at any section occurs when the live load covers the entire beam.

$$R' \text{ under this condition} = \frac{22 \times 70}{2} = 770 \qquad (413\text{A})$$

$$M_f \text{ at } 30' \text{ point} = 770 \times 30 - 22 \times 30 \times \frac{30}{2} \qquad (413\text{B})$$

$$= 23{,}100 - 9{,}900 = 13{,}200 \text{ foot pounds} \qquad (413\text{C})$$

414. Let it be required to determine maximum shears and bending moments in a **beam with uniform dead load resting on end supports, under the action of a uniformly distributed live load as long as the beam.**

Let w be dead load and w' be live load per unit of length.

Shear.—(Fig. 414A).

There are the same four conditions of shear in this case as were considered in paragraph 412 where the beam was without weight.

a. As head of live load moves from O to C, we have

$$S_v = w\left(\frac{L}{2} - l\right) - \frac{w'x^2}{2L} \qquad (414\text{A})$$

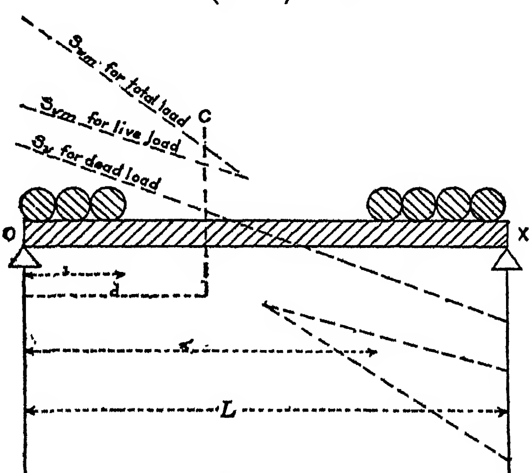


Fig. 414A

The first term of the second member is the dead load shear at C which is constant for all values of x, and is positive since $L/2 > l$. The second term is the live load shear at C which increases numerically as x increases. The resultant shear at C will

therefore decrease algebraically as x increases, and will have its greatest value when $x = 0$, and its least value when $x = l$.

b. While the head of the load moves from C to X, we have

$$S_c = w\left(\frac{L}{2} - l\right) + w'\left(x - l - \frac{x^2}{2L}\right) \tag{414B}$$

The first term of the second member is the dead load shear at C and is identical with the first term of the equation 414A.

The second term is the live load shear at C and increases algebraically as x increases, since its first differential coefficient is positive. The resultant shear will therefore increase algebraically as x increases, and will have its least value when $x = l$, and its greatest value when $x = L$.

c. While the tail of the load moves from O to C, we have

$$S_c = w\left(\frac{L}{2} - l\right) + w'\left(\frac{L}{2} - l + \frac{x'^2}{2L}\right) \tag{414C}$$

The first term of the second member is the dead load shear at C and is identical with its values in the preceding equations. The second term is the live load shear at C, and since $L > 2l$, it is positive and increases algebraically with x'. The resultant shear at C therefore increases algebraically with x', and is least when $x' = 0$, and greatest when $x' = l$.

d. While the tail of the load moves from C to X, we have

$$S_c = w\left(\frac{L}{2} - l\right) + w'\left(\frac{L}{2} - x' + \frac{x'^2}{2L}\right) \tag{414D}$$

The first term of the second member is the dead load shear at C and is identical with its values in the preceding equations. The second term of the second member is the live load shear at C and decreases algebraically as x' increases, since its first differential co-efficient is negative. The resultant shear at C will therefore decrease algebraically as x' increases, and will be greatest when $x' = l$, and least when $x' = L$.

Hence, we see that the resultant shear at C decreases algebraically while the head of the live load moves from O to C; it increases algebraically while the head of the load moves from C to X and while the tail of the live load moves from O to C, and finally decreases algebraically while the tail of the live load moves from C to X.

Therefore, *if a uniformly distributed live load moves over a uniformly loaded beam resting on end supports, the shear at any section decreases while the head of the live load moves from the left support to the section, increases while the head moves beyond the*

section, increases while the tail moves from left support to the section, decreases while the tail moves beyond the section.

This result is clearly seen by consideration of the method of transmitting shear by internal stresses from section to section of the beam until it reaches the points of support. Taking each of the four cases separately:

a. As head of load moves from left support to the section: part of every unit weight w is transmitted through the section to the right support with a value equal to $wx/L-w$, which is negative; therefore, the shear already due to the dead load decreases because of the live load.

b. As head of load moves from section to right support: part of every unit weight to the right of the section is transmitted through the section to the left support and its value is positive; therefore, the shear already due to the live and dead loads is increased.

c. As tail of load moves from left support to the section: part of fewer unit weights is transmitted through the section to the right support with a negative value as shown in *a* above; therefore, the shear already due to the live and dead loads is increased.

d. As tail of load moves from section to right support: part of fewer unit weights is transmitted through the section to the left support with the positive value as shown in *b* above; therefore, the shear already due to the live and dead loads is decreased.

The maximum shear at any section due to live and dead loads is greatest when the maximum shear due to the live load has the same sign as the shear due to the dead load. This is obtained when the tail of the live load is at the section and the load covers the greater segment of the beam.

Therefore, *if a uniformly distributed live load moves over a uniformly loaded beam resting on end supports, the maximum shear at any section due to live and dead loads will be numerically greatest when the tail of the live load is at the section and the load covers the greater segment of the beam.*

Similarly, since the maximum shear is at the supports for both live and dead loads separately, it will be there for their sum.

Therefore, *If a uniformly distributed live load moves over a uniformly loaded beam resting on end supports, the shear due to live and dead loads is a maximum at the supports; this occurs when the live load covers the entire beam.*

Bending Moment.—(Fig. 414B).

Since the bending moment is a maximum at any section when the live load covers the entire beam, and also when the dead load

covers the entire beam, it is evident that it is a maximum when the live and dead loads entirely cover the beam. (Fig. 414B).

Therefore, *if a uniformly distributed live load moves over a uniformly loaded beam resting on end supports, the bending moment at any section of the beam will be greatest when the live load covers the entire beam.*

Similarly, the maximum bending moment is at the middle section for both the live and dead loads; hence it is at the middle for both of them.

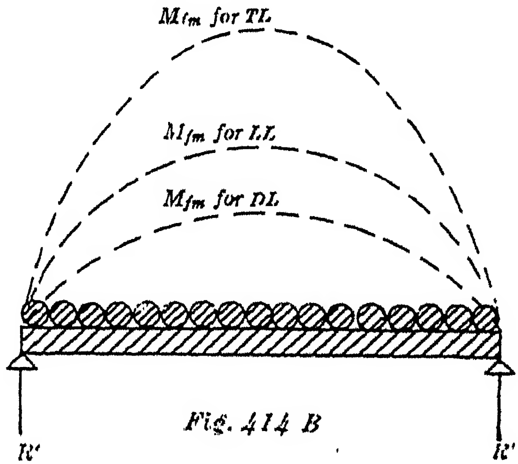


Fig. 414 B

Therefore, *if a uniformly distributed live load moves over a uniformly loaded beam resting on end supports, the bending moment is a maximum at the center of the beam; this occurs when the live load covers the entire beam.*

415. **Section of Zero Shear.**—From the above discussions, it is seen that as the uniformly distributed live load moves over the beam, the signs of the live and dead load shears are different between the middle of the beam and the head (or tail) of the live load. In each of the four cases above, there is a section where the shear is zero, called the "section of zero shear".

By placing $S_n = 0$ in equations 414A, 414B, 414C, 414D, the section may be determined (deduction omitted) where the shear due to live load equals that due to dead load, producing zero shear.

As head of the live load moves from O to X, the section of zero shear moves from the middle and meets it at some point between O and the middle, whose distance from O depends upon the ratio w/w'; it then moves back to the middle point of the beam.

In a similar manner it is shown that while the tail of the load

moves from O to X, the section of zero shear moves to a corresponding point between the middle and X, which it reaches simultaneously with the tail of the live load; it then moves back to the middle point.

Application.

416. The section of zero shear and the consequent change of sign of shear are specially considered in designing trusses for bridges. As shown in the Chapter on Bridges, it is necessary to introduce many counterbraces near the center of these bridges to take up the reversals in stress in the members which transmit the shear due to a uniformly distributed live load such as a railway train.

PROBLEMS.

P. 401. A wheel, supporting 10 tons, rolls over a beam of 20′ span. Place the wheel in such a position as to give the maximum bending moment. Find its value.

P. 402. A beam, 20′ long, without weight, rests on end supports. Over it moves a single concentrated load of 50 pounds. Section C is 4′ from the left hand support. (Units in pounds and foot-pounds).

(a) Find the maximum bending moment at C.

(b) Find the value and location of the maximum bending moment of the entire beam.

(c) Same, for the minimum bending moment.

(d) Find maximum and minimum shear at C.

(e) Find the value and location of the maximum shear of the entire beam: 1st, numerical; 2d, algebraic. Same for minimum shear.

(f) Write equation and draw curve showing bending moment at section under load as load moves across beam.

(g) Write equations and draw curves showing shear; 1st, at first section at elementary distance to left of load as load moves across beam; 2d, at first section at elementary distance to right of load as load moves across beam.

P. 403. A single concentrated load of 1,000 pounds moves over a steel beam, 20′ long, whose section modulus is 173.9 (inches^3). What is the maximum fiber stress in the section 5′ from the end? What is the maximum fiber stress in the beam? Neglect weight of beam.

P. 404. A concentrated live load of 100 pounds has rolled over a beam 100′ long, resting on end supports and carrying a dead load of 5 pounds per lineal foot.

(a) What was the greatest bending moment which occurred at the section 25′ from the left hand support?

(b) What was the greatest bending moment which occurred in the beam?

P. 405. A concentrated live load of 100 pounds has rolled over a beam 100′ long, resting on end supports and weighing 5 pounds per lineal foot. Find the equation of and plot the curve whose ordinate at any point shall be the maximum bending moment which occurred at that point.

P. 406. A concentrated live load of 4,000 pounds moves over a beam 80′ long,

resting on end supports and carrying a uniformly distributed dead load of 40 pounds per lineal foot.

(a) Find the numerical maximum bending moment at the section 16′ from the right hand support.

(b) Find the numerical maximum shear at the section 16′ from the right hand support.

(c) Find the numerical maximum bending moment in the beam.

(d) Find the numerical maximum shear in the beam.

P. 407. A beam is 40′ long between supports and carries a weight of 500 pounds per lineal foot. A concentrated live load of 10,000 pounds moves from left to right. Draw curves of shear and bending moment due to dead load, and curves which will enable shear and bending moment due to total loads to be determined at all points of the beam. What and where are maximum shear and bending moment?

P. 408. How heavy a man can safely cross a 2″ by 12″ plank laid flat across a span of 18′? Neglect weight of plank.

P. 409. A concentrated load of 5,000 pounds moves across a 10″ 25 pound steel I-beam resting on end supports 30′ apart. What is the factor of safety in flexure? Treat weight of beam as dead load. Section modulus is 24.4″.

P. 410. A road-roller, 8′ between front and rear axles, passes over a bridge whose floor is supported by 4 steel I-beams 24′ long. The weight on front wheels is 4 tons, and on rear wheels is 12 tons. Select the I-beams, from the Handbook, in order that the load may be safely borne. Assume that one I-beam takes half the weight of the roller. Neglect weight of floor and I-beams.

P. 411. Two wheels, 4′ apart, each supporting 7 tons, roll over a beam of 7-1/2′ span. Find the maximum bending moment for the whole span, and also the curve of the maximum bending moment at each point.

P. 412. Two wheels, 6′ apart, supporting the one 4 tons and the other 8 tons, travel over a beam of 24′ span.

(a) Find the maximum bending moment for the whole span.

(b) Write equation and draw the single curve of maximum numerical shearing force, positive or negative, at any point.

(c) Write equation and draw curve of the maximum bending moment at any point.

P. 413. Two wheels, 4′ apart and each supporting 7 tons, roll over a beam of 8′ span. Find the maximum bending moment for the whole span, and the position of the load which gives it. Find also the maximum shear in the beam, and the position of the load which gives it.

P. 414. In what positions will a loaded wagon of 8′ between axles produce the maximum shear and bending moment in the joists of a bridge 15′ long? What will be the value of these stresses (shear and bending moment), assuming each wheel to carry 1/4 of the load of the wagon?

P. 415. A car carrying a load of 14 tons (1 ton = 2,000 pounds), rolls over a simple bridge made of two beams, each 8′ long, with planks laid across them. Each wheel of the car is assumed to carry one-fourth of the load. The axles are 4′ apart. In order to carry the load safely, considering bending alone, how wide must the beams be if they are 12″ deep? Neglect weight of bridge.

P. 416. A beam, 100′ long, resting on end supports, weighs 10 pounds per lineal foot and carries a uniformly distributed live load of 20 pounds per lineal foot whose length is greater than that of the beam.

(a) Show position of live load to produce maximum bending moment at section 10′ from left hand support.

(b) Find bending moment at this section.

(c) Show position of live load to produce maximum bending moment in beam.

(d) Find maximum bending moment in beam.

P. 417. A beam, 60′ long, resting on end supports, carries a dead load of 30 pounds per lineal foot. A live load, longer than the beam and weighing 120 pounds per lineal foot, crosses the bridge.

(a) Find the numerical maximum bending moment at section 10′ from right support.

(b) Find numerical maximum shear at section 10′ from right support.

(c) Find numerical maximum bending moment in beam.

(d) Find numerical maximum shear in beam.

P. 418. A beam 100′ long, resting on end supports, carries a uniform dead load of 10 pounds per lineal foot. A uniform live load of 20 pounds per lineal foot, whose length exceeds that of the beam, passes over the beam.

(a) Find the greatest bending moment produced at the section 10′ from the left support. Show by figure the positions of the loads which produce it.

(b) Find the maximum bending moment which occurs in the beam. Show by a figure the positions of the loads which produce it, and the section of the beam at which it occurs.

(c) Find the numerical maximum value of the shear at a point 10′ from the left hand support. Show by a figure the positions of the loads which produce it, and draw the lines of dead load shear, live load shear, and total shear with the loads in this position.

P. 419. A beam 20′ long, without weight, rests on end supports. Over this beam moves a uniformly distributed load of 15 pounds per lineal foot and 20′ long. Section C is 4′ from the left hand support; section C' is 16′ from the left hand support. (Units in pounds and foot-pounds).

(a) Find the maximum bending moment at C.

(b) Find the maximum and minimum bending moments of the entire beam.

(c) Find the numerical and algebraic maximum and minimum shear at C.

(d) Find the numerical and algebraic maximum and minimum shear of the entire beam.

(e) When the head of the load is at C'; write equation and draw curve of bending moment.

(f) Same, for shear.

(g) When the tail of the load is at C'; write equation and draw curve of bending moment.

(h) Same, for shear.

P. 420. A uniformly distributed live load of 180 pounds per square foot moves over a bridge whose roadway is 46′ wide. The floor is supported by 4 I-beam stringers spaced 15′ 4″ center to center each 18′ long. Select the proper stringers from the Handbook. Assume that the floor and stringers weigh 25 pounds per square foot.

P. 421. If a uniform live load, shorter than the beam upon which it moves, passes over a given section, prove that the bending moment at that section is a maximum when that section divides the load into segments whose lengths are proportional to the lengths of the segments of the beam on either side of the section.

P. 422. A beam is 40′ long between supports on which its ends rest. A live load, 10′ in length, rolls over it. Find the position of this load which will make the bending moment a maximum at the point 15′ from one end of the beam. Determine the value of this maximum bending moment. Neglect the moment due to dead load.

P. 423. A beam 100′ long, resting on end supports, carries a uniform dead load of 10 pounds per lineal foot. A concentrated live load of 200 pounds passes over the beam. Find the numerical maximum shear produced in the beam. Show by a figure the position of the loads which produce this shear, and draw the lines of dead load shear, and total shear with the loads in this position.

P. 424. Fig. P. 424 represents a beam without weight carrying a concentrated live load. $PQRS$ is the line of shear when the live load is at section T.

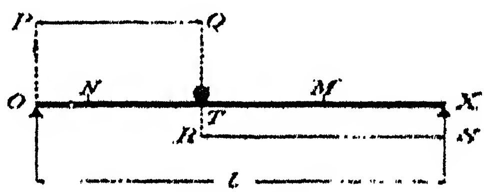


Fig. P. 424.

(a) Draw the locus of the point Q as the live load passes over the beam. Draw the locus of the point R as the live load passes over the beam.

(b) What is the vertical distance between these lines (loci)?

(c) Assuming these loci, draw the line of shear when the live load is at M.

(d) What position of the live load will give the maximum shear at N? Is this the algebraic or numerical maximum?

(e) What position of the live load will give the maximum shear at M? Is this the algebraic or numerical maximum?

(f) What position of the live load will give the least algebraic shear at N?

P. 425. A concentrated live load of 2,000 pounds rolls over a beam which is 24′ long, supported at the ends, and carrying a uniformly distributed dead load of 200 pounds per lineal foot? What is the greatest vertical shear in the beam and where is it? What is the greatest vertical shear at a section 5′ from the end?

In the above problem, find the farthest point from the middle of the beam where the vertical shear, due to dead and live load, is zero.

P. 426. A rolled joist, weighing 150 pounds per lineal foot and 20′ long, carries a uniformly distributed dead load of 6,000 pounds. Two wheels, 5′ apart, the one bearing 5,000 pounds and the other 3,000 pounds, roll over the joist. Find the maximum shear at the supports, at the center, and at 5′ from end.

P. 427. A railway girder, 50′ in the clear, carries a uniformly distributed dead load of 50 tons. Find the maximum shear at 20′ from the end when a train weighing 1-1/4 tons per lineal foot crosses the girder. Find the maximum shear at 10′ from the end. The live load is long enough to cover the entire bridge.

P. 428. A railway girder, 40′ in the clear and 5′ deep, carries a uniformly distributed load of 40 tons. Find the maximum shearing stress at 15′ from one end when a train weighing 1-1/4 tons per lineal foot crosses the girder. Also find the minimum theoretical thickness of the web at a support, 4 tons being the safe shearing stress per square inch of the metal.

P. 429. A beam 100′ long, resting on end supports, weighs 5 pounds per lineal foot and carries a concentrated live load of 100 pounds. When the live load is 25′ from the left hand support, find graphically the point of zero shear in the beam.

P. 430. A uniformly distributed live load of 200 pounds per lineal foot moves over a beam 20′ long, supported at the ends, which bears 75 pounds dead load per lineal foot. Find the greatest shear at the section 5′ from the end.

In the above problem find the point in the beam where the section of zero vertical shear is farthest from the middle of the beam.

P. 431. Find the position of maximum bending moment on a beam without weight resting on two supports, if two equal concentrated live loads move over it, the distance apart of the live loads being less than the length of the beam. Use the nomenclature of Fig. 411A.

CHAPTER V.

PRESSURE AND FLOW OF WATER.

501. In designing systems of water supply and sewerage, and in calculations for cross-sections for dams, it is necessary to know not only the laws governing the stresses and actions of solids but also the laws governing the pressure and flow of water.

502. **Physical Properties.**—Water is classified as a *fluid*, because it does not resist a change of form when under the action of a distorting force; it is usually classed as a *perfect fluid* because it changes form immediately upon coming under the action of the distorting force, whereas a viscous fluid changes form slowly.

503. The weight of water at its maximum density is 62.428 pounds per cubic foot. Above 4° C, it expands with increase of temperature, but this expansion is so little, being about 1% for each 50 degrees, that it is not generally considered; and for all practical purposes, the weight of water is taken as 62.5 pounds per cubic foot. Below 4° C, water expands until it freezes at 0° C, at which point it increases in volume about 1/12 and can exert a pressure of about 30,000 pounds per square inch in a water pipe one inch in diameter.

504. The quantity of water is expressed either in units of volume or in units of capacity. In the United States the unit of volume is the cubic foot; where the metric system is used, the unit is the cubic meter. In the United States the unit of capacity is the standard gallon, which contains 231 cubic inches; where the metric system is used it is the liter, containing 0.264 standard gallons, or the dekaliter, containing 2.64 gallons.

1 cubic foot = 7.4805 standard gallons $\doteq$ 62.5 pounds.
1 standard gallon = 0.1337 cubic feet = 8.356 pounds.

505. Water is considered in civil engineering as incompressible. Actually, under great pressure, as in gun construction, it is reduced in volume by 30% to 40%. In civil engineering work the change of volume is so small that it is neglected in all practical problems of design.

STATIC PRINCIPLES.

506. Certain principles have been determined concerning the action of water. These principles are stated below; they are deter-

mined by experiments, and by deductions arising from results obtained from these experiments. They are the basis for the theory and practice of engineering construction requiring consideration of the pressure and flow of water.

507. *Every molecule of still water is subjected to equal pressure from all directions.* It is assumed that the molecule has no weight and there is no cohesion between the molecules, and that the molecules move without friction. This assumption is not strictly correct, since the molecule has some weight and there is some cohesion between the molecules but these are so little that they are neglected. Accepting the above assumptions, it is evident that the pressure on each molecule must be the same in all directions, as otherwise it would move. Considering the weight of the molecule the pressure on its lower surface equals the pressure on its upper surface, plus its weight.

508. *The upper or free surface of still water is horizontal; that is, normal to the action line of the force of gravity.*

Taking each molecule as a "free body", it is evident that the separate water pressures on its opposite sides must be equal and balance each other. Also, since the pressure is equal from all directions, it is evident that the areas on the sides where adjacent molecules are in contact with the molecule under consideration must all be equal. If this were not so the molecule would move. Therefore, the separate molecules form a surface which is horizontal, or normal to the action lines of gravity.

509. If there is motion on the surface, it is due to the action of a force whose action line is not vertical, or is due to the fact that the surface is not horizontal. The motion in the first case may be caused by the action of an extraneous force, such as wind, which upsets the equilibrium of the molecules and causes them to move. The motion in the second case is caused by the differences in surfaces of contact on the sides of each molecule, there being more pressure coming from the higher side where there is more surface in contact, causing the molecule to move in the direction of the lower side.

510. *The unit pressure on each molecule of a body of still water varies directly with its depth below the free surface.*

Since it is assumed that there is neither cohesion nor friction, the water pressure on the upper surface of a molecule is due to the weight of the molecules above it. As these are incompressible and of equal weight per unit of depth, the pressure on the upper

surface of the molecule must be proportional to its depth. The pressure on its lower surface must be the same, plus the weight of the molecule itself which is disregarded; and the pressure from the sides must also be the same, as shown in paragraph 507.

511. Expressing this principle in an equation, we have

$$p = 62.5\ h \text{ and } p' = \frac{62.5}{144} h \tag{511A}$$

in which p = pressure in pounds per square foot, p' = pressure in pounds per square inch, and h = depth in feet, called the *head, pressure head,* or *pressure.*

If the weight or pressure of the atmosphere which rests on the water be added, the total unit pressure will be $p+p_a$ or $p'+p'_a$, in which p_a and p'_a are the pressures of the atmosphere in pounds per square foot and square inch. At sea level, the pressure of the atmosphere equals 2112 pounds per square foot or 14.7 pounds per square inch.

512. This principle is used in determining the height to which water will rise in a vacuum. Thus, since the weight of the column of water in the vacuum must equal the atmospheric pressure, we have

$$14.7 = \frac{62.5}{144} h \tag{512A}$$

Thence, $$h = 34 \text{ feet, approx.} \tag{512B}$$

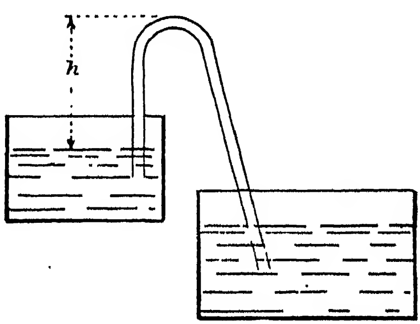


Fig. 513 A

513. The siphon (Fig. 513A) is based on this principle. Disregarding friction in the pipe, it is necessary only that h be less than 34 feet and that the pipe be emptied of air, or rather filled with water. The pressure of the atmosphere will then force the

water to the top of the curve in the pipe, and the difference of weights of the two columns of water in the pipe will cause flow from the higher surface to the lower until the two reach the same level.

514. **Problem.**—A vessel 20′ long by 10′ wide by 12′ high is full of water and is to be emptied by a siphon into a vessel 8′ long by 5′ wide by 65′ high. How much below the bottom of the first vessel must be the bottom of the second vessel? Disregard the small quantity of water in the siphon.

Solution.—When all of the water is in the second vessel, it will be 60′ deep. Therefore, the bottom of the second vessel must be 60′ below the bottom of the first; otherwise the water will cease to run in the siphon before the first vessel is emptied.

515. *The pressure of still water is, at every point, normal to the surface pressed.*

This principle follows from the hypothesis that there is no friction or cohesion between the molecules. Assuming that the force of pressure acts obliquely to the surface pressed, then because of lack of friction, its component not normal to the surface would move the molecule. But it does not move the molecule, as shown by the fact that the water is still. Therefore, the pressure is normal to the surface pressed, as stated in the principle above.

516. The pressures on the elementary areas of a plane surface will form a system of parallel forces whose resultant can be represented by a single force. The pressure on the elementary areas of a curved surface will in general form a system of non-parallel forces which cannot be represented by a single force; such pressures can, however, be so represented in pipes and spheres.

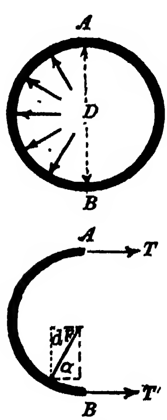


Fig. 517 A

517. To find the force tending to tear apart a section of pipe under equal pressure throughout, assume a section of the pipe as shown in Fig. 517A. Taking the left half of the pipe as a "free body" and considering the forces necessary to maintain equilibrium, we know that the pressure on this half of the pipe must equal the two tensile forces, T and T'. Assuming p as the force on a unit of interior surface, and resolving it into components normal and parallel to T and T', we have

$$\begin{aligned} &\text{Pressure on an elementary area} = p\,dA \\ &\text{Pressure normal to } T \text{ and } T' = p \sin \alpha \, dA \\ &\text{Pressure parallel to } T \text{ and } T' = p \cos \alpha \, dA \end{aligned} \qquad (517A)$$

The normal components of the pressures on all the areas balance each other, and need not be considered. The total force parallel to T and T' must be equal to their sum, therefore

$$T + T' = p \, \Sigma \cos \alpha \, dA \qquad (517B)$$

As, $p \, \Sigma \cos \alpha \, dA = pD$, for unit length of pipe with diameter = D,

we obtain $T + T' = pD$; ditto (517C)

For a pipe of length l, we have

$$T + T' = pD \, l \qquad (517D)$$

518. *The total water pressure on any immersed plane surface is equal to the weight of a prism of water, whose base is the area under pressure, and whose height is the depth of the center of gravity of the surface.*

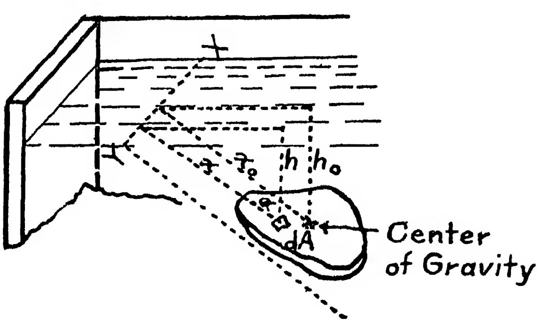


Fig. 518 A

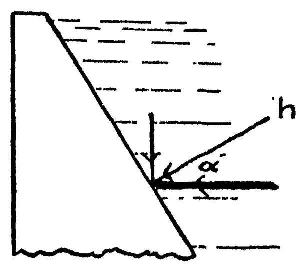


Fig. 519 A

This is evidently true for a plane surface immersed in a horizontal position. To prove it for an inclined surface, represent by Fig. 518A the immersed plane surface, making the angle α with the vertical.

Considering the elementary area, dA, and remembering that the unit pressure varies directly with its depth below the surface, we have

$$dp = 62.5 \, hdA \qquad (518A)$$

For the total pressure, p, we have

$$\begin{aligned} p &= \int 62.5 \, x \cos \alpha dA \\ &= 62.5 \cos \alpha \int x dA \qquad (518B) \end{aligned}$$

The integral of xdA is the moment of the whole area around

XY as an axis and equals A multiplied by the distance from the axis to the center of gravity of the area. Substituting, we have

$$p = 62.5 \cos \alpha \, x_o A \tag{518C}$$

But $$x_o \cos \alpha = h_o \tag{518D}$$

Hence, $$p = 62.5 \, A h_o \tag{518E}$$

519. In *design of dams,* calculations are simpler if the normal pressure is resolved into its horizontal and vertical components.

If, instead of the immersed plane, we take the side of a dam (Fig. 519A) holding h feet of water ($=2h_o$ feet in this case) we have, from equation 518 E,

$$\text{Horizontal component of } p = 62.5 \, A \, \frac{h}{2} \cos \alpha \tag{519A}$$

$$\text{Vertical component of } p = 62.5 \, A \, \frac{h}{2} \sin \alpha \tag{519B}$$

For one linear foot of the dam, we know that

$$A = 1 \times \frac{h}{\cos \alpha} \tag{519C}$$

Hence, for one linear foot of the dam, we have

$$\text{Horizontal component of } p = 62.5 \, \frac{h^2}{2} \tag{519D}$$

Vertical component of $p = 62.5 \, \frac{h^2}{2} \tan \alpha =$ weight of one linear foot of the triangular prism of water between the dam and the surface. (519E)

520. *The resultant water pressure on a plane area acts at the center of percussion of the area with respect to the axis formed by the line of intersection of the plane area and the surface of the water.*

The elementary pressures form a system of parallel forces, perpendicular to the plane area. Their resultant is parallel to the elementary pressures and acts on the plane area at a point called the *center of pressure.*

As the pressures increase uniformly from zero at the surface, it is seen that if there were no reactions opposite to the water pressures the plane area would rotate about its line of intersection with the surface. This line is the *spontaneous axis,* and the center of pressure becomes *the center of percussion.*

The distance of the center of percussion from its spontaneous

axis (Fig. 520A) is obtained from the following equation derived from Mechanics:

$$L = \frac{I}{Al'} = \frac{r^2 + l'^2}{l'} \qquad (520A)$$

in which L = distance of center of percussion from the spontaneous axis;
l' = distance of center of gravity of area from the same line;
I = moment of inertia of area about the same line;
Al' = moment of area about the same line;
r = radius of gyration of the plane area about an axis through its center of gravity parallel to its spontaneous axis.

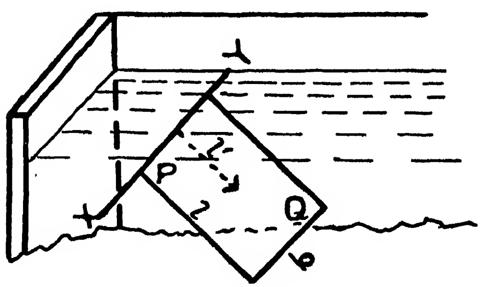


Fig. 520 A

The value of l' and A for any plane area can easily be determined from the conditions, and the values of r and I are given in engineering manuals. By substituting, the value of L can be determined.

521. *The center of pressure of a rectangular plane area whose upper edge lies in the surface is two-thirds the depth of its lower edge below the surface.*

Representing the plane area by PQ in Fig. 520A, we have

$$A = bl \qquad (521A)$$

$$r^2 = \frac{l^2}{12} \qquad (521B)$$

$$l' = \frac{l}{2} \qquad (521C)$$

Substituting in equation 520A, we obtain

$$L = \frac{\frac{l^2}{12} + l'^2}{l'} = \frac{\frac{l^2}{12} + \frac{l^2}{4}}{\frac{l}{2}} = \frac{2}{3} l \qquad (521D)$$

As the depth at a point two-thirds of l from upper edge is the depth at a point two-thirds of the depth of the lower edge, the principle is proven.

This is the principle which is used in calculating the overturning effect of the water pressure on dams. See Fig. 519A.

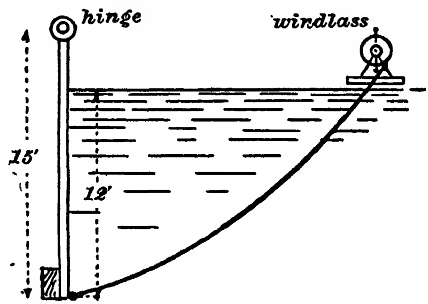


Fig. 522 A.

522. **Problem.**—A water-way is closed by a gate as shown in Fig. 522A. It is opened by a windlass as shown. The gate is 8′ wide. Find the horizontal component of the tension on the cable necessary to open the gate with the water 12′ high as shown.

Solution.—From paragraph 519, we have, by considering 1′ width of gate,

Horizontal component of pressure on 1′ width $= 62.5h^2/2 \times 12 \times 12/2 = 4500$ pounds (522A)

Horizontal component of pressure on 8′ width $= 4500 \times 8 = 36{,}000$ pounds (522B)

The center of this pressure is 2/3 of the depth of water, which is 8′ below the surface and 11′ from the hinges. The lever arm of the cable is 15′. Equating the two moments, we have

Horizontal component of tension on cable $\times\ 15 = 36{,}000 \times 11$ (522C)

Hence, horizontal component of tension on cable $= 26{,}400$ pounds (522D)

523. *If a pressure is applied on any unit area of surface of water filling a closed vessel, this pressure is transmitted equally to all other unit areas within or at the surface of the water.* This principle results from the incompressibility of water and the lack of

friction between its particles. It is the principle used in the hydraulic jack, where strong unit pressure on a small area is transmitted as equal unit pressure to a large area and becomes a very large pressure on this large area. In Fig. 523A, the unit pressure downward on p is transmitted with equal unit pressure to lift the weight W. Thus if p has an area one-fourth of W, it can lift the weight W with the same unit pressure but requires only one-fourth the total pressure. It is to be noted, however, that the work performed by p in moving over its path must equal that performed by W in moving over its path. That is, since p has an area one-fourth of w, it must move over a path four times the length of the path of W.

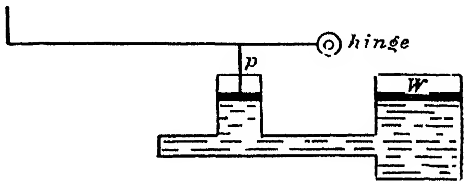


Fig. 523 A

524. **Problem.**—In Fig. 523A, the total pressure on p is 300 pounds, and the relative diameters are 6″ for p and 18″ for W. What weight can be lifted?

Solution.—

$$\text{Area of } p : \text{Area of } W :: \pi 3^2 : \pi 9^2 \qquad (524A)$$

$$\text{Area of } W = \frac{\pi 81 \times \text{area of } p}{\pi 9} = 9 \times \text{area of } p \qquad (524B)$$

$$\text{Pressure} = 9 \times 300 = 2700 \text{ pounds} \qquad (524C)$$

525. *Every body in water is subjected to a vertical upward pressure equal to the weight of water displaced.* This is the principle of Archimedes. The upward pressure is called the *buoyancy of water.*

This buoyancy is evident from the principle that the unit pressure varies with the depth below the surface, and (paragraph 511) equals the weight of the water above it. Take any elementary vertical prism of the body with its upper surface h feet below the surface and its lower surface h' feet below it:

Then, pressure on upper surface $= 62.5\, h$
pressure on lower surface $= 62.5\, h'$

Resultant upward pressure $= 62.5\ (h' - h)$, which is the weight of the column of water displaced by the elementary vertical prism.

A solid lighter than water will therefore displace its weight of

water because when body floats (in equilibrium) the downward pressure (weight of solid) equals upward pressure (weight of water displaced). A solid heavier than water will lose in weight an amount equal to the weight of an equal volume of water.

526. **Problem.**—In air, one cubic foot of concrete weighs 150 pounds, and cast iron weighs 3 times as much. What are their relative weights in water?

Solution.—In water, each is buoyed up by the water with a force equal to 62.5 pounds, the weight of a cubic foot of water. Then,

Weight of concrete in water : Weight of cast iron in water : : 150 — 62.5 : 450 — 62.5 : : 87.5 : 387.5.

Pressure in Pipes and Spheres.

527. **Pipes.**—If we consider a pipe filled with water to be divided into two equal parts by a plane through its axis, it is evident that the water pressure on one half will be equal and opposite to the water pressure on the other half. These equal pressures tend to tear the pipe into two equal parts along two rectilinear elements in which the axial plane intersects the pipe. (See Fig. 517A.) From equation 517D, the total force tending to tear the pipe apart equals the unit pressure multiplied by the area of the plane of cross-section through the axis $= pDl$. This force is prevented from tearing the pipe apart by the tensile strength of the two thicknesses of metal on opposite sides of the pipe.

Hence, if l = length of pipe;

D = interior diameter of the pipe;

t = thickness of pipe;

We have, Area of plane of cross-section of water $= Dl$,

and Metal holding pipe together $= 2tl$.

Hence, $$pDl = 2s_t tl \tag{527A}$$

Solving, we obtain for the thickness of pipe

$$t = \frac{pD}{2s_t} \tag{527B}$$

528. **Hollow Spheres.**—In a sphere filled with water under pressure, the ring of metal considered as taking the tensile stress is one between two concentric circumferences cut from the sphere by a plane of a great circle.

If r = radius of interior of sphere,

t = thickness of metal,

We have, Area of plane of cross-section of water $= \pi r^2$

and Metal holding sphere together $= \pi(r+t)^2 - \pi r^2$

Hence, $p\pi r^2 = s_t[\pi(r+t)^2 - \pi r^2] = s_t\pi(2rt+t^2)$ (528A)

If t is very small in comparison with r, the value t^2 may be omitted, and we have

$$2s_t t = pr \tag{528B}$$

Solving, we obtain for the thickness of sphere

$$t = \frac{pr}{2s_t} \tag{528C}$$

529. **Problem.**—What should be the thickness of metal of an 8″ cast iron water main to resist safely a water pressure of 300 pounds per square inch?

Solution.—The allowable tensile strength of cast iron is 3,000 pounds. Substituting proper values in equation 527B, we have

$$t = \frac{pD}{2a_t} = \frac{300 \times 8}{2 \times 3000} = \frac{8}{20} = .4''$$

FLOW OF WATER IN CLOSED CHANNELS.

Discharge Through Orifices.

530. An **orifice** is an opening in the side of a vessel containing water. The stream of water which issues from an orifice is called a *jet*.

531. *The theoretical velocity of water flowing through an orifice is equal to the velocity acquired by a body falling freely in vacuo through a distance equal to the head on the orifice.* This is Torricelli's Theorem, and is expressed by the equation

$$V = \sqrt{2gH} \tag{531A}$$

in which H is the head to the center of the orifice.

The theoretical discharge, Q, is equal to the area of the orifice, A, multiplied by the velocity. That is,

$$Q = AV = A\sqrt{2gH} \tag{531B}$$

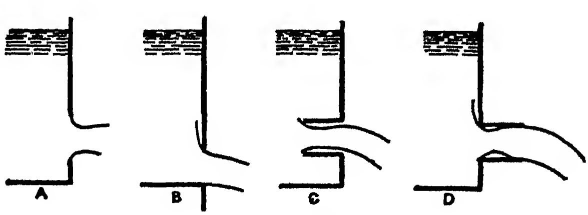


Fig. 531

If the side of the vessel is thin and the orifice is not near the bottom or a side (Fig. 531A) the actual form of the jet is as shown. By experiments, it is found that the actual V is about 0.97 the theoretical V, and the actual A of the jet is about 0.64 the A of the orifice. Therefore, the actual discharge is expressed by the equation

$$Q = 0.97 \times 0.64\, A \sqrt{2gH} = 0.62\, A \sqrt{2gH} \qquad (531C)$$

532. **Short Tubes.**—Short tubes inserted into orifices are of two kinds: (1) re-entrant, where the tube projects into the vessel and has its outer end flush with the outer face of the vessel; (2) projecting, where the inner surface is flush with the inner face of the vessel.

The Borda tube (Fig. 531C) is a cylindrical re-entrant tube so short that the jet passes through the tube without coming into contact with its sides. Its discharge is less than that of any other form, due to the crowding of the molecules seeking entrance. The formula of discharge for a Borda tube is

$$Q = 0.5\, AV = 0.5\, A \sqrt{2gH} \qquad (532A)$$

The standard tube (Fig. 531D) is a projecting cylindrical tube, of length such that the jet expands before it leaves the tube so as to completely fill the cross-section at the outlet. The actual contraction is therefore nothing, A of the jet $= A$ of the tube, and the coefficient of discharge must be equal to the coefficient of velocity. By actual measurement this coefficient has been found to be 0.82. The increase in the discharge over that through a simple orifice is explained by the reduction of back pressure due to the formation of a vacuum in the tube at the point where the jet is contracted. This vacuum is caused by the rushing water carrying with it the confined air.

The formula for the standard tube is, therefore

$$Q = 0.82\, AV = 0.82\, A \sqrt{2gH} \qquad (532B)$$

533. **Miner's Inch.**—The miner's inch (Fig. 533A) is a unit of measurement of discharge of water, largely used in the western mining states where more accurate devices are not available. It is the discharge in a unit of time (usually taken as one second) through an orifice, 1″ square, under a head usually of 6″ above center of orifice. Substituting these values in equation 531C, reducing all units to feet, we have

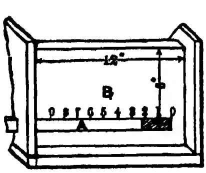


Fig. 533 A

$$Q = 0.62 \times \frac{1}{144}\sqrt{2g\frac{6}{12}} = \frac{0.62}{144}\sqrt{2 \times \frac{32.2}{12} \times 6} = 0.0244 \text{ cubic feet sec.} \tag{533A}$$

The above formula is derived from the formula (531C) based on the hypothesis that the orifice is in a thin plate. In actual practice, the sides of this miner's inch orifice are boards about an inch thick; therefore, the formula is based on the wrong hypothesis, and a coefficient of 0.72 is more nearly correct than 0.62.

534. The practical method of measuring the miner's inch is shown in the figure. The shutter, A, is moved backward and forward to maintain the discharge through the opening such that the level of the water, the head of the water, is maintained at 6″ above the center of the orifice. The width of the opening is then read off on the scale, showing the number of miner's inches being discharged.

Discharge Through Pipes.

535. Represent by Fig. 535A a reservoir of water, discharging through a standard tube and a pipe. The total head, H_t, is expended in forcing the water to enter the pipe, to flow through the

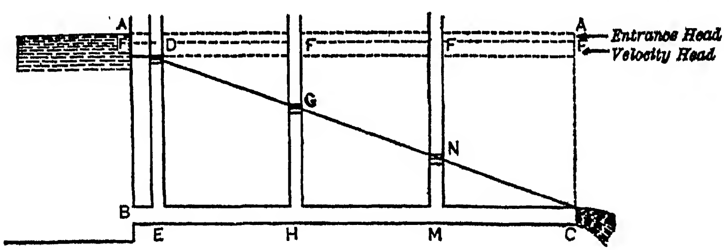


Fig. 535 A

pipe against friction, and out at its end with a certain velocity. Expressing this by an equation, we have

$$H_t = H_e + H_f + H_v \tag{535A}$$

These are mutually dependent. For example, if the pipe is lengthened, a greater total head will be required to maintain the same velocity, because the loss due to friction head is increased. If, however, the pipe is lengthened and the total head is not increased, there is a readjustment of H_f, and H_v; the loss due to friction is still increased but it is not so much increased

because the velocity is decreased and this lessens slightly the loss due to friction.

536. **Entrance Head.**—Theoretically, from equation 531A,

$$V_t = \sqrt{2gH_t}, \quad \text{or} \quad H_t = \frac{V_t^2}{2g} \qquad \text{(536A)}$$

That is, the total velocity bears this relation to the total head. But actually, because of losses due to friction and losses at entrance, the velocity is reduced to V_a, and we have, by introducing the constant C so as to allow for this difference,

$$V_a = C\sqrt{2gH_t}, \quad \text{or} \quad C^2H_t = \frac{V_a^2}{2g} = H_v \qquad \text{(536B)}$$

The amount of head lost at entrance, which changes the velocity from V_t to V_a must be

$$\text{Lost head} = H_t - H_v = H_t - C^2H_t \qquad \text{(536C)}$$

Substituting for H_t its value in terms of actual velocity of the water as given in equation 536B, we have

$$\text{Lost head} = \frac{V_a^2}{2gC^2} - \frac{C^2V_a^2}{2gC^2} = \left(\frac{1}{C^2} - 1\right)\frac{V_a^2}{2g} \qquad \text{(536D)}$$

This is the equation for the loss of head at entrance in any discharging device. Applying this to the standard tube entrance of the pipe shown in Fig. 535A, and remembering that C above is for a standard tube, we have, from equation 532B,

$$C = 0.82 \qquad \text{(536E)}$$

Substituting in equation 536D, and remembering that the actual velocity is due to velocity head, we obtain

$$\text{(536F)}$$

$$\text{Head lost at entrance} = \left[\frac{1}{(0.82)^2} - 1\right]\frac{V_a^2}{2g} = 0.5\frac{V_a^2}{2g} = 0.5H_v$$

Therefore, in the ordinary pipe, *the entrance head is one-half the velocity head.*

537. **Friction Head.**—The roughness of the interior surface of the pipe checks the flow and produces eddies and cross-currents which interfere with the free flow of the water in the pipe. The frictional resistance thus offered has been found by experiment to:

may be reduced by many causes other than those named above. Such causes are: sharp bends, eddies, changes in diameter, etc. These are generally of so small effect that they are not considered; if important in a particular system, special equations may be found in text books or engineering handbooks.

542. **Long Pipes.**—The preceding discussions apply to both short and long pipes. However, when the pipe is long, the equations can be much simplified by eliminating values which are proportionally so small as to render it unnecessary to consider them.

From Fanning's table, paragraph 540, it is seen that as the length of the pipe is increased, more and more of the total head, H_t, is expended in overcoming the frictional resistance in the pipe and less and less is expended at the entrance and in producing velocity. In practice, it is customary to consider the total head, H_t, as entirely expended in overcoming the frictional resistance in pipes with a length not less than one thousand diameters and in which water has a velocity not more than 10 feet per second; such pipes are called *long pipes*.

Under this assumption, H_e and H_v are disregarded, and H_f becomes H_t. The equations for long pipes are, therefore, (Eq. 537D),

$$V = \sqrt{\frac{2gH_tD}{4\mu l}} \tag{542A}$$

$$Q = 0.7854D^2\sqrt{\frac{2gH_tD}{4\mu l}} = 3.152\sqrt{\frac{H_tD^5}{\mu l}} \tag{542B}$$

543. **Problem.**—A pipe 1′ in diameter has a length of 1000′. The total head is 100′. What are the entrance, friction, and velocity heads? $\mu = .006$.

Solution.—Substituting proper values in equation 539C, we have

$$V = \sqrt{\frac{2gH_t}{1.5 + \frac{4 \times .006 \times 1000}{1}}} = \sqrt{\frac{2 \times 32.2 \times 100}{1.5 + 24}} = \sqrt{\frac{6440}{25.5}} = 15.89 \text{ feet per sec.}$$

$$\text{Velocity Head} = H_v = \frac{V^2}{2g} = \frac{6440}{25.5 \times 2 \times 32.2} = 3.92'.$$

$$\text{Entrance Head} = \frac{3.92}{2} = 1.96'.$$

$$\text{Friction Head} = 100 - 3.92 - 1.96 = 94.12'.$$

544. **Problem.**—Assuming the pipe in Problem 543, what will be the velocity if it is considered as a long pipe?

Solution.—

$$V = \sqrt{\frac{2gH_tD}{4\mu l}} = \sqrt{\frac{2 \times 32.2 \times 100 \times 1}{4 \times .006 \times 1000}} = \sqrt{\frac{6440}{24}} = 16.4 \text{ ft. per sec.}$$

545. **Problem.**—Using Fanning's Table, find the entrance, friction, and velocity heads. Compare the velocity in the table with the answer to problem 544.

Solution.—Taken directly from the table, the velocity heads are 2.37′, 92.94′, and 4.69′. The velocity in the table is 17.38 feet per sec., which is 0.98 feet per sec. more than the velocity calculated in problem 544.

546. **Hydraulic Gradient.**—In Fig. 535A, if a cap is placed on the outlet at C, the water will cease to flow and will rise in each of the piezometer tubes to the level of the water in the reservoir. If the cap is removed, the water in each will fall. At E, where the water has started flowing in the pipe, the loss in head will be entrance head plus velocity head, giving a total loss of one and one-half times the velocity head. This head will be small if the pipe is fairly long, since the velocity of flow in the pipe will be small. The fall in the remaining tubes will be due to the loss of head due to friction between E and H, and between E and M. If the pipe is of uniform size, the friction loss is equal per unit of length, that is, proportional to the length.

Hence, the loss E to H : loss E to M : : EH : EM (546A)

When the pipe is of the same size throughout, the water level in the three tubes at E, H, and M will be in a straight line. This is called the *hydraulic gradient*, which is the line joining the points to which water will rise in piezometer tubes.

547. As the height of the water in each of the tubes represents the pressure on the inner surface of the pipe at the base of the tube, its height is called the pressure head at that point.

At any point of the pipe, therefore, the pressure head is the vertical distance from the hydraulic gradient to the axis of the pipe; it is equal to the friction head from that point to the end of the pipe.

Hence, at any point of the pipe, the *sum of the entrance head, the velocity head, the friction head, and the remaining pressure head is equal to the total head.*

548. In a very long pipe, such as the water main connecting the inlet of a gravity system of water supply with its reservoir, it is seen from the table in paragraph 540, that the entrance and velocity heads are so small that they need not be considered, and the hydraulic gradient is drawn as in Fig. 548A, as practically the whole head is employed in overcoming the frictional resistance. As

previously stated, this line, *AC*, is straight only when the pipe is of uniform size. When there are different sizes of pipe in the

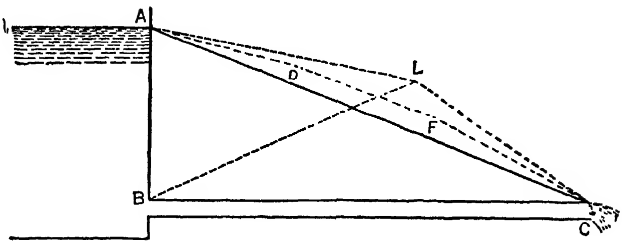


Fig. 548 A

pipe line, the hydraulic gradient is not straight, but takes such forms as *ALC* and *ADFC*.

The slope of the hydraulic gradient is found from the equation

$$H_f = \frac{4}{D} \mu l \frac{V^2}{2g} \tag{537D}$$

Transposing the length, l, to the left member, we obtain

$$\text{Slope} = \frac{H_f}{l} = \frac{4\mu V^2}{2gD} \tag{548A}$$

Knowing the difference of level between the outlet and the reservoir, and the length of the pipe connecting them, this slope can easily be computed, if the pipe is of the same diameter throughout, by dividing the difference of level by the total length of pipe.

549. From equation 548A, we have

$$V^2 = \frac{2gD\,H_t}{4\mu l} \text{ (for long pipes).} \tag{549A}$$

From this it is seen that the velocity of flow in a long pipe and consequently its discharge is not affected by any change of location that does not change H_t/l, its slope.

In laying long pipe it is usually necessary to cross valleys and and ridges. From equation 549A, it is seen that as long as the pipe is kept free from obstructions and below the grade line, the vertical irregularities in the line have no effect on the velocity of flow other than that due to increase in length. In the valleys, however, the pipe must be thicker, because of the greater pressure head.

Should the pipe line be laid above the hydraulic grade line, as the line *BLC*, Fig. 548A, the part above the hydraulic grade line

will form a siphon and be subject to the laws governing the siphon. As air collects in the top of the siphon it will interrupt the flow, and the discharge will be less than it would be if the entire line was below the hydraulic gradient.

550. **Problem.**—The elevation of the surface of the water in a reservoir is 459′, and its outlet 3 miles away has an elevation of 228′. The top of a ridge over which the pipe line must run is 1 mile from the reservoir, and its elevation is 398′. Is it necessary to cut into the ridge in order to keep the flow of water free from air bubbles due to siphon effect; and if so, how much?

Solution.—The hydraulic gradient has a slope of

$$\frac{459-228}{3} = 77' \text{ per mile.}$$

Therefore, at the top of the ridge the elevation of the pipe line should be $459-77=382'$. This will necessitate a cut of $398-382=16'$.

Weirs.

551. An orifice of regular form cut in the upper edge of a vertical wall is called a *weir*. It serves the purpose of measuring the discharge. The edge over which water flows is called the crest. Most weirs are rectangular; and the crest is the width of the bottom of the rectangular notch.

552. Weirs are of two forms: with end contractions (Fig.552A), and without end contractions (Fig. 552B).

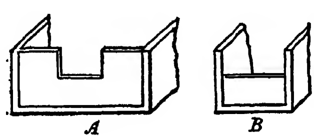


Fig. 552

Many formulas have been deduced from experiment for the discharge over various forms of weirs. These may be found in works on hydraulics which explain their derivation and limitations.

Francis' formula for a weir with end contractions is

$$Q = 3.33\left(b - \frac{1}{5}H\right)H^{\frac{3}{2}} \tag{552A}$$

His formula commonly applied to a weir without end contractions is

$$Q = 3.33bH^{\frac{3}{2}} \tag{552B}$$

in which Q = discharge in cubic feet per second;
b = breadth of weir in feet;
H = height of surface (upstream from the weir) above the crest of the weir in feet.

553. For rough measurement, H may be read on a rod as shown in Fig. 553A, placed far enough above the crest so that the surface

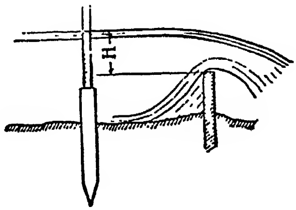


Fig. 553 A

has not begun to slope quickly to the weir but still has its normal slope.

For accurate measurement, a hook-gauge is used. The hook points upwards and is lifted up from below until a slight pimple is observed on the surface. By means of a vernier, readings may be made to one-thousandth of a foot.

In engineering handbooks the values of Q are tabulated corresponding to $b = 1'$ and values of H differing by $0.01'$.

The above formulas of Francis are for weirs measuring discharge from reservoirs in which the water is still; thus the velocity over the weir is that due to head above the crest of the weir alone. If the weir is in a running stream, there will also be the velocity head of the running stream; and the total velocity at the weir for the same head over the crest will be greater than in the case of still water, and the discharge will be greater than that given by equations 552A and 552B. To obtain this discharge, it is necessary to measure the velocity of the stream, calculate the head, H_v, which will produce that velocity and substitute H_v+H for H alone in equations 552A and 552B.

FLOW OF WATER IN OPEN CHANNELS.

554. An open channel is a conduit in which water flows without any pressure above atmospheric pressure, other than that due to the actual weight of the water carried.

Open channels are divided into:

a. Artificial Channels with uniform flow;
b. Artificial Channels with non-uniform flow;
c. Natural Channels.

Theoretical Determination of Velocity.

555. To obtain a formula for the flow of water in open channels, let Fig. 555A represent a channel with uniform cross-section and

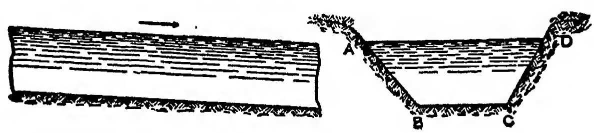


Fig. 555 A

uniform slope; then, unless acted upon by forces other than atmospheric pressure and gravity, the slope of the water will also be uniform.

Pass two parallel planes perpendicular to the line of flow, and consider the prism of water included between these two planes as a "free body".

Let A = area of cross-section of channel;
l = length of prism = distance between the two planes;
h = fall in length l.

Then h/l = sine of angle of slope of the surface.

The terminal sections are equal in all respects; and the pressure transmitted from the upper section, through the prism of water included between the two sections, to the lower section, is equal to that carried forward by the lower section. Therefore, the only force which tends to move the "free body" is the component of the weight of the water itself which acts parallel to the surface.

Representing this force by F_p, we have

$$F_p = 62.5Al\frac{h}{l} \qquad (555A)$$

As the velocity with which the water enters the prism is equal to that with which it leaves it, there is no increase in force in the prism itself; therefore, the force given by the weight of the water must be equal to the resistance which opposes its movement. This is the resistance of the bed and sides of the channel. This

resistance, as in paragraph 537 for pipes, varies directly with the roughness of interior surface of the channel, directly with its wetted area, and directly with some function of the velocity, usually taken as $V^2/2g$. The resistance of the free surface of the water is neglected, as it is found by experiment to be negligible.

Let L = length of wetted perimeter; then Ll = wetted area;
μ' = friction constant.

Then, Frictional Resistance $= \dfrac{Ll\mu' V^2}{2g}$ (555B)

Equating the two equations, 555A and 555B, we have

$$62.5Al\frac{h}{l} = \frac{Ll\mu' V^2}{2g} \tag{555C}$$

$$V^2 = \frac{2g\,62.5}{\mu'}\frac{h}{l}\frac{A}{L} \tag{555D}$$

Substituting μ for $\mu'/62.5$, we have

$$V = \sqrt{\frac{2g}{\mu}\frac{h}{l}\frac{A}{L}} \tag{555E}$$

556. **Chezy's Formula.**—Substituting

R = hydraulic mean radius, for A/d,
S = sine of slope of the surface, for h/l,

we obtain $$V = \sqrt{\frac{2g}{\mu}}\,\sqrt{RS} = C\,\sqrt{RS} \tag{556A}$$

This equation is called the Chezy formula and is the basis of those since devised. As originally employed, a constant value was given to C.

557. **Bazin's Formula.**—By an elaborate system of experiments, Bazin proved that C was not constant, but that its value was greatly affected by the character of the walls of the channel. Also that in a channel in which $\sqrt{S}$ was constant, C did not vary exactly with the ratio $V/\sqrt{R}$.

The formula deduced by Bazin from his experiments was

$$V = \frac{87}{0.552 + \dfrac{m}{\sqrt{R}}}\sqrt{RS} \tag{557A}$$

in which m is a factor for roughness whose values depend on the character of the walls of the channel. These values are:

Walls of the Channel	m
1. Cement and carefully planed wood.......	0.06
2. Smooth ashlar, brick, unplaned wood.....	0.16
3. Rubble masonry........................	0.46
4. Earth.................................	0.85
5. Channels carrying detritus and coarse gravel, mountain torrents.............	1.75

These experiments were made principally in an artificial canal 2 meters wide, 1 meter deep, and about 600 meters long.

558. **Abbot's Formula.**—About the same time an elaborate study of the Mississippi River was made by Captain Andrew A. Humphreys and Lieutenant Henry L. Abbot of the Corps of Engineers, U. S. Army.

The formula deduced by these engineer officers is, in its simplest form,

$$V = \frac{C}{\sqrt[4]{S}} \sqrt{RS} \qquad (558A)$$

When the results of the American investigations were compared with the French, it was seen at once that the two formulas were each applicable only to the character of the channel in which the experiments were conducted. Bazin's formula could not be applied to a channel of the dimensions of the Mississippi, nor could the formulas deduced by Humphreys and Abbot be applied to the small channels experimented on by Bazin.

559. **Kutter's Formula.**—The theory of the flow of water in open channels was next made the subject of careful study by a Swiss engineer, Kutter, who sought an empirical formula which would conform to the data obtained by the American and French engineers, as well as that furnished by other experimenters.

He found that the value of C varies with the following general laws:

a. It increases with R, and most rapidly when R is small.

b. It increases with the decrease in the roughness of the bottom and the walls of the channel. This effect is greatest when the velocity is small.

c. It increases with S if R is less than one meter and if the bottom and sides of the channel are smooth.

d. It decreases as S increases if R exceeds one meter, and also in small channels if the surface of the bottom and sides is very rough.

The formula below, deduced as the result of the experiments and known as Kutter's formula, is the one now generally employed.

$$V = \left\{ \frac{41.66 + \frac{1.811}{n} + \frac{.00281}{S}}{1 + \left(41.66 + \frac{.00281}{S}\right)\frac{n}{\sqrt{R}}} \right\} \sqrt{RS} = C\sqrt{RS} \qquad (559A)$$

in which V = mean velocity in feet per second;
R = hydraulic mean radius or hydraulic mean depth in feet;
S = sine of slope.

560. The value of the coefficient, n, as determined from the results of experiments, varies with the character of the bottom and sides of the channel as follows:

Planed timber, glazed and enameled surfaces	0.009
Smooth clean cement	0.010
Same, with one-third sand	0.011
Unplaned timber	0.012
Ashlar and rough brick	0.013
Smooth brick masonry	0.015
Rubble	0.017
Firm gravel	0.020
Rivers and canals free from stones and weeds	0.025
Rivers and canals in fair condition	0.030
Rivers and canals in bad condition	0.035

561. Kutter's formula is applicable to pipes as well as to artificial and natural channels. As the formula is a complicated one, tables and graphic diagrams have been computed and placed in engineering manuals giving the value of C corresponding to different channels. In all ordinary computations, these tables are used.

562. The discharge is determined, as in all cases, by multiplying the area, A, by the velocity, V. Thus, we have

$$Q = AV \qquad (562A)$$

Direct Measurement of Velocity.

563. In natural channels, and in some artificial channels, it is often unsatisfactory to rely on the formulas deduced above. It is more satisfactory to make direct measurements of the velocity of the stream and multiply this velocity by the area of the stream.

564. The section of the channel selected for measurement must be of sufficient length so that the water is flowing steadily through it; there must be no eddies and back currents. The velocity is measured in one of the following ways, depending upon the accuracy desired:

a. A single surface measurement of the channel;
b. A surface measurement of each of certain vertical sections;
c. A surface and one or more sub-surface measurements of each of certain vertical sections.
d. Many surface and sub-surface measurements, sufficient to determine curves of velocity.

565. **Single Surface Measurement.**—A surface float is placed in the center of the stream and the time is accurately taken in which it goes a known distance. As a rough approximation, the average velocity for the whole area of cross-section may be taken as three-fourths of the maximum surface velocity.

566. **Surface Measurement of Vertical Sections.**—The channel is divided into a number of vertical sections (see Fig. 566A), each of the same width, and the surface velocity is taken in each. Many measurements indicate that the mean velocity in the vertical section is nine-tenths of the surface velocity. To determine the dis-

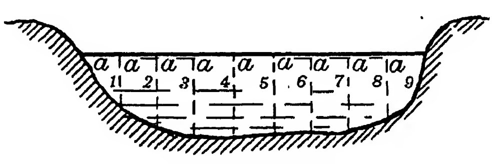


Fig. 566 A

charge of the water in the whole cross-section, the area of each vertical section is determined, then multiplied by the average velocity of the water in it, and the products added.

567. **Surface and Sub-surface Measurements of Vertical Sections.**—In very irregular channels, and when great accuracy is required in any channel, it is necessary to determine the velocity at many

points in each section, as the average velocity may not be nine-tenths of the surface velocity. The points are taken preferably at regular depths, and the proper area for each point is multiplied by the velocity as shown by the measuring instrument. Many experiments indicate that the average mean velocity is at about six-tenths of the depth, and the maximum velocity is somewhere between the surface and one-third of the depth.

568. **Velocity Contours.**—If sufficient measurements are made of the velocity and all points of equal velocity are joined by curves, there will be obtained (Fig. 568A) a series of curves which have

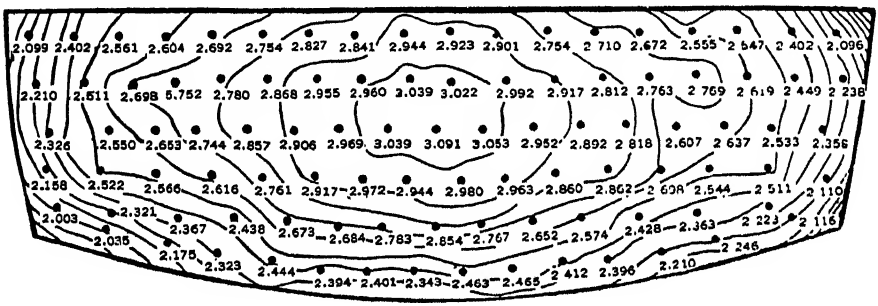


Fig. 568 A

the appearance of contours, and are known as velocity contours. As seen in the figure, the maximum velocity is near the middle of the channel and about one-third of the depth below the surface. If the area inclosed between two adjacent contours is multiplied by the average of the two, the total discharge thus obtained is very accurate.

569. **Methods of Measurement.**—The velocity may be measured by means of floats, Pitot tubes, or current meters. Of these, the most satisfactory is the method by current meters.

570. *Floats.*—To measure the velocity by means of floats, two parallel cords, from one to two hundred feet apart, are stretched across the stream, or parallel lines are marked by poles, at each cross-section. The rate at which the float (Fig. 570A) passes from

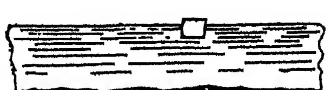


Fig. 570 A

the upper to the lower cross-section is observed and recorded. Several floats are timed and the average is taken.

Floats are preferably made of a material whose specific gravity is only a little less than water, so that they will be almost wholly submerged and be little affected by the wind.

If the velocity at any point below the surface is desired, a double float (Fig. 570B) is employed. This consists of a surface float

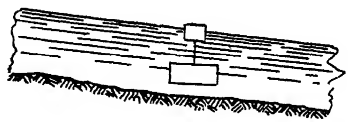


Fig. 570 B

which is small in order that the surface velocity may have little effect on it, and a large submerged float exposed to the action of the current. The lower float is connected to the surface float by a thin wire.

If the mean velocity in any vertical plane is desired, a weighted pole is used, extending from the surface almost to the bottom.

571. *Pitot tubes* (Fig. 571A) consist of two vertical glass tubes each open at the top and bottom and bent at the lower ends as shown. The lower ends are placed at right angles to each other. As shown in the figure, the tubes are placed so that the opening in one of the horizontal arms points upstream, and the other opening points to the side. The velocity of the water causes it to rise in the tube with the horizontal arm upstream, while in the other it remains at surface level. After the oscillations have ceased, valves are closed in the bottom of the tubes, and they are raised and the difference of height of water measured. This difference gives the

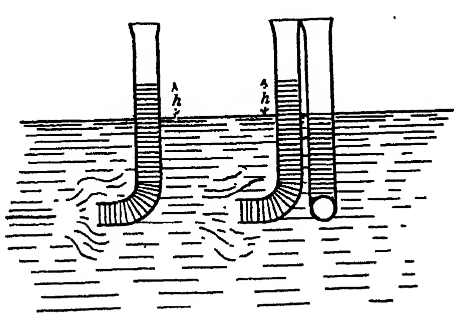


Fig. 571 A

velocity head, H_v, and the theoretical velocity, $V=\sqrt{2gH_v}$; but by experiment the velocity has been found to be about .84 of this amount; therefore, the formula used for determining the velocity by use of Pitot tubes is

$$V = 0.84\sqrt{2gH_v} \qquad (571A)$$

572. The *current meter* (Fig. 572A) has a wheel or propeller which is revolved by the current, and the number of revolutions is recorded by electric attachments or by a counting apparatus con-

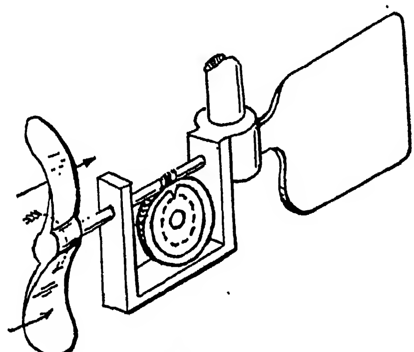


Fig. 572 A

nected to the shaft of the wheel or propeller. The meter is first rated by moving it at a uniform speed through still water. To measure a velocity, the meter is attached to a pole and lowered in the water to the depth at which the velocity is to be determined. In very deep water, the meter is suspended by a weighted cable. After a fixed time, depending upon the accuracy desired, the meter is raised, the number of revolutions read, and the velocity calculated. As the current meter is the most accurate and also involves but little computation, it is the most suitable method of measuring velocities.

PROBLEMS.

P. 501. A block of masonry 2′ x 3′ x 4′ 3″ weighs 3,720 pounds.

(a) What will it weigh in water?
(b) What is its specific gravity?

P. 502. A cube of a certain substance 4″ on edge floats in mercury with its upper side 1″ above the surface. What is the specific gravity of this substance? (Assume the specific gravity of mercury to be 14.)

P. 503. The specific gravity of white pine is 0.38. How high out of water would a block of white pine 2 ′ long and 1 ′ in square cross-section float?

P. 504. A cube of white pine measuring 1 ′ on an edge is held under water so that the top surface of the cube is parallel to and one foot below the surface of the water. Assume the specific gravity of white pine to be 0.38.

(a) Determine the total water pressure on each face of the cube.

(b) What force is necessary to hold the cube in the position described.

P. 505. A hollow sphere of 1 ′ diameter is held under water so that the top of the sphere is 1 ′ under the surface of the water. The sphere contains a vacuum. The material of the sphere is so thin that its weight may be neglected.

(a) Determine the pressure per square inch on the bottom of the sphere.

(b) What force is necessary to hold the sphere in the position described.

P. 506. A spar buoy of circular cross-section, with a diameter of 20″, is made of timber with a specific gravity of 0.8. It is held in a vertical position by a submerged block of stone one cubic foot in volume, weighing 160 pounds. The buoy must project 6′ above water. What is its length?

P. 507. A cylindrical stand pipe 3′ in diameter is filled with water to the height of 30′. An additional pressure of 40 pounds per square foot is applied at the surface. What is the total pressure at the base?

P. 508. What is the total pressure on the upper surface of an inclined isosceles triangle whose vertex is 9′ below the surface of the water, and whose base is parallel to and 18′ below the surface. The base of the triangle is 12′ and its altitude is 16′.

P. 509. (Figure P. 509.) The figure represents a vessel filled with water.

(a) What pressure must be applied to the piston P in order to hold the weight W in the position shown?

(b) Solve (a) when P is applied 2½′ instead of 3′ and *W* is applied 3½′ instead of 4′ above the center of the pipe.

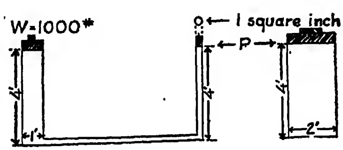


Fig. P. 509.

P. 510. A cast-iron water main 8″ interior diameter taps a reservoir at a depth of 150′ below the level of the water surface in the reservoir. How thick should the main be for a factor of safety of 10?

P. 511. A rectangular opening 2′ by 4′ (short dimension horizontal) in a reservoir dam has its center 80′ below the free surface and its plane is inclined at an angle of 45°. The opening is to be closed by a gate rotating on a horizontal axis. How far from the lower side of the gate should the axis be so that the water pressure would offer no resistance to opening?

P. 512. A rectangular opening in a dam is closed by a plank 10′ high and 1′ broad. The plank is held vertically in the opening by bearing against the studs A and B, at the top and bottom of the plank respectively. Find the pressures acting on A and B when the water level is 10′ above the center of the opening.

P. 513. A rectangular valve in a vertical dam is 8′ high. The bottom of the valve is 5′ above the bottom of the reservoir. Locate the position of the horizontal hinge in the valve so that the the valve will open automatically when the water in the reservoir rises above the depth of 30′.

P. 514. The center of a horizontal circular orifice, 2″ in diameter, is 20′ below the free surface of the water, so that the orifice opens upward. How high will the jet rise?

P. 515. The center of a circular orifice, 2″ in diameter, is 10′ below the surface of the water. What is the velocity of discharge and the quantity of discharge in gallons per minute?

P. 516. A standard tube of 8″ diameter opens into a reservoir 150′ below the free surface of the water. What will be the velocity of discharge in feet per second? How many gallons will it discharge per 24 hours?

P. 517. What must be the diameter of a standard tube to discharge 3,000,000 gallons per 24 hours, under a head of 100′?

P. 518. What head in feet is required to give a discharge of 3,000,000 gallons per 24 hours from a standard tube 4″ in diameter?

P. 519. What is the discharge in gallons per 24 hours over a weir 24′ long without end contractions, where the head is 4″.

P. 520. What is the discharge in gallons per 24 hours, over a weir 10′ long with end contractions, when the head of water on the weir is 6″?

P. 521. A pipe line with a standard tube entrance is 300′ long. The pipe is 1′ in diameter and delivers water under a head of 60′. What is the loss of head at entrance? ($\mu = 0.02$.)

P. 522. A pipe line with a standard tube entrance is 300′ long and 6″ in diameter and is under a constant head of 30′. The coefficient of friction is .010. Find.

(a) The loss of head due to velocity.
(b) The loss of head due to entrance.
(c) The loss of head due to friction.

P. 523. A pipe line, 300′ long, and 12″ in diameter, has its intake a standard tube under a constant head of 30′. Compute the loss of head at entrance; the loss of head due to friction; and the velocity of discharge, considering the pipe to be smooth.

P. 524. A pipe 200′ long, 1′ in diameter and with a coefficient of friction of 0.01, takes water from a reservoir with a head of 80′. Determine:

(a) The entrance head.
(b) The friction head.
(c) The velocity head.

P. 525. (Figure P. 525.) Water from the tank flows through the pipe. Determine the total head and divide it into the following:

(a) Entrance head.
(b) Friction head.
(c) Velocity head. ($\mu=0.01.$)

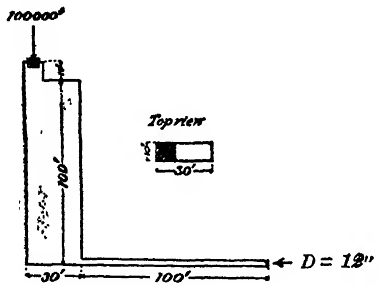


Fig. P 525

P. 526. A pipe line with standard tube entrance is 12″ in diameter and 500′ long and is under a head of 40′. Suppose the pipe is smooth, what is the velocity of discharge in feet per second and what is the discharge in gallons per 24 hours?

P. 527. A pipe line 300′ long delivers water under a head of 60′. The pipe is 1′ in diameter and the intake is a Borda tube. What would be the probable discharge in cubic feet per second? ($\mu=0.02.$)

P. 528. A pipe line 10,000′ long and 2′ in diameter delivers water under a head of 170′. What will be the probable discharge in gallons per 24 hours? ($\mu=0.02.$)

P. 529. The level of Lusk Reservoir is 59′ below the intake of the pipe line on Popolopen Creek. The diameter of the pipe is 20″ and its length is 32,000′. Assuming the pipe to be rough, what will be its discharge in gallons per day?

P. 530. In the preceding problem (P. 529) what is the velocity of discharge in feet per second?

P. 531. It is required to deliver 2,500,000 gallons of water per 24 hours at West Point from Popolopen Creek, from a point about 40,000′ distant, the available head being about 60′. What should be the diameter of the pipe in inches? ($\mu=0.01.$)

P. 532. A pipe line 300′ long delivers water under a head of 30′. The pipe is 1′ in diameter. ($\mu=0.02.$)

(a) Construct the hydraulic grade line.
(b) What is the loss of head at entrance?

P. 533. The velocity in a pipe line, 20″ in diameter, is 2.5′ per second; what is the mean hydraulic gradient? The coefficient of friction is .01.

P. 534. (Fig. P. 534). Water from the tank flows through the pipe. The pipe is 1′ in diameter and has a coefficient of friction of 0.006. Determine the discharge in cubic feet per second. (Use horizontal projections for lengths of pipe.)

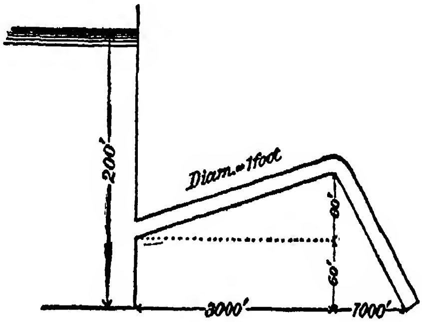


Fig. P. 534.

P. 535. An aqueduct is of circular cross section and is 8′ in diameter; it is lined with cement mortar; it runs half full; the slope is 2′ per mile. What will be its discharge in gallons per 24 hours?

(a) By Bazin's formula.
(b) By Kutter's formula.

P. 536. The Washington aqueduct is lined with brick and is of circular cross section 9′ in diameter; its slope is .00015. What is its discharge in gallons per 24 hours, if the depth of water is 3.5′?

(a) By Bazin's formula.
(b) By Kutter's formula.

P. 537. Deduce equation 555A.

PART II.

BUILDING MATERIALS.

A knowledge of the characteristics of building materials is necessary in order to obtain the best results in engineering construction. Materials suitable for certain purposes, in one structure, will not be at all suitable for almost the same purpose in another structure. Durability, possibilities of preservation against decay, power to resist unusual forces, and cost of installation, must all be considered.

Practical experience gained by engineers on construction work, and standard tests applied in the laboratory, are the principal guides in the selection of materials.

CHAPTER VI.

TIMBER.

601. **Timber,** from the Saxon word timbrian, to build, is a generic term applied to wood of a suitable size, and fit for engineering construction. The growing tree is called standing timber; the felled tree, which has been cut into proper lengths for the saw mill, is called rough timber or logs. When the logs have been squared, or cut into shape, either to be used in this form or cut into smaller pieces, the general term timber is applied to them. If cut from the trunk of the tree, they are known as square or round, hewn or sawed, according to the form of cross-section and mode of cutting; if cut from the branches and trunk, and of crooked shape, they are called knees and are used in ship building.

Timber which has been sawed into smaller pieces is called lumber, and is divided into classes known as joists, scantlings, strips, boards, planks, etc. When sawed to specified sizes, to fill a given bill of material, it is spoken of as dimension stuff.

VARIETIES.

602. The forests of our own country produce a great variety of timber for construction purposes. It is ordinarily divided into two general classes, soft woods and hard woods.

The first class includes all coniferous trees, like the pines, and also some few varieties of the broad-leaved trees; the second includes most of the timber trees that are non-coniferous or broad-leaved, like the oaks, etc.

The coniferous trees contain turpentine, and are generally distinguished by the straightness of the fiber and by the regularity of form of the tree. The timber made from them is more easily sawed or split along the grain, and much more easily broken across the grain, than that made from the hard woods. The hard woods or non-coniferous timber, contain no turpentine, and, as a class, are tough and strong.

Examples of Soft-Wood Trees.

603. **Yellow Pine.**—This tree is, perhaps, the most widely distributed of all the pines in this country, being found in all the States from New England to the Gulf of Mexico. In the Southern States it is called Spruce Pine. or Short-leaf Pine.

The heart-wood is fine grained and moderately resinous. Its sap-wood decays rapidly when exposed to the weather. The tree grows mostly in light clay soil and furnishes a strong and durable timber extensively used in house and ship building.

604 **Long-Leaf Pine.**—This tree is found from southeastern Virginia to the Gulf, and is the principal tree where the soil is sandy and dry. Inferior growths of it are frequently called Yellow Pine. It has but little sap-wood. The heart-wood is fine grained and compact, and has the resinous matter very uniformly distributed. The timber made from it is strong and durable, being considered superior to that of other pines. Its quality depends, however, on the kind of soil in which the tree grows, being less resinous in rich soils. Long-leaf pine is often classified as a hard wood.

605. **White Pine.**—This tree is found in Canada and New England, and along the Allegheny Mountains as far south as Georgia. It is frequently called Northern Pine. Its timber is light, soft, free from knots, slightly resinous, easily worked, and durable when not exposed to the weather. It is used in a great variety of ways for building purposes and for joiners' work.

606. **Fir.**—This tree, commonly known as Spruce, furnishes large quantities of timber and lumber which are extensively used throughout the Northern States. The lumber made from it has the defects of twisting and splitting on exposure to the weather and of decaying rapidly in damp situations. The common fir, the spruce fir, found in Northern California, and the Douglas Fir which grows to an enormous size, all furnish timber of great strength and size and excellent quality and are much used in building.

607. **Hemlock**—This is a well-known species, used throughout the Northern States as a substitute for pine when the latter is difficult or expensive to procure. It is very perishable in damp places or when subjected to alternate wet and dry periods.

608. **Cedar.**—The White Cedar, called Juniper, and the Cypress are celebrated for furnishing a very light timber of great durability when exposed to the weather; on this account they are much used for shingles and other exterior coverings. The shingles are light and may last for 40 years. These two trees are found in great abundance in the swamps of the Southern States.

Examples of Hard-Wood Trees.

609. **White Oak.**—This tree is largely used in ship-building, the trunk furnishing the necessary timber for the heavy frame-work, and the trunk and large branches affording an excellent quality of ship timber. Boards made from it are liable to warp and crack. It grows throughout the United States and Canada, but most abundantly in the Middle States. Proximity to salt air during the growth of the tree appears to improve the quality of the timber. The character of the soil has a decided effect on it. In a moist soil, the tree grows to a larger size, but the timber loses in firmness and durability.

610. **Live Oak.**—The wood of this tree is of a yellowish tinge; it is heavy, compact, and of a fine grain; it is stronger and more durable than that of any other species of oak, and on this account is considered invaluable for the purposes of ship-building. It is found in the South Atlantic and Gulf States and along the Pacific Coast.

611. **Post Oak.**—This tree resembles white oak, seldom attains a greater diameter than about fifteen inches, and on this account is suitable for posts, from which use it takes its name. It is found most abundantly in the Middle Atlantic and South Central States.

612. **Red Oak.**—Red oak is found in all parts of the United States. The wood is reddish, of a coarse texture, and quite porous. The timber made from it is generally strong but not durable in contact with the ground. It is used principally for furniture and trim.

613. **Black Walnut.**—The timber made from this tree is hard and fine grained. It has become too valuable to be used in building construction, except for ornamentation. It is extensively used in making gun stocks.

614. **Hickory.**—The wood of this tree is tough and flexible. Its great heaviness and liability to be worm-eaten have prevented its general use in buildings. Hickory is largely used in wagon-making, for tool handles, and generally where a strong and elastic wood is required.

STRUCTURE.

615. While color, weight, smell and other appearances give indications as to the variety of a specimen, the wood structure is the only reliable means of identification. All physical and mechanical properties of timber depend very much upon its structure. Thus, a comparison of the structure of oak and pine shows why oak is

heavier, stronger, and tougher; why it is harder to saw and plane; and why it is so much more difficult to season without injury.

616. **Classification.**—Trees are classified as exogenous and endogenous. Exogenous trees increase in diameter by the annual formation, between the old wood and the bark, of concentric rings of the new wood, which inclose the entire living portion of the tree. Trees of the exogenous group include practically all classes of timber important for engineering construction; they are the coniferous and the broad-leaved trees. Endogenous trees grow diametrically and longitudinally, principally the latter, by the addition of new wood fiber intermingling with the old. Trees of this second group include the palm, the bamboo, and the yuccas. These are used to a small extent for engineering construction in some of the Southern States, but they will not be discussed further.

617 **Pith, Annual Rings, and Bark.**—A cross-section of a sawed log of matured timber shows a small center of loosely-assembled, thin-walled cells called the *pith.* It is about one-eighth of an inch in diameter, never increasing in size after the first year. Surrounding the pith are concentric rings of wood which represent the layers of wood added each year. The thickness of the annual rings varies greatly in different trees of the same variety and in different parts of the same tree. These rings are inclosed in the weather protector, called the bark.

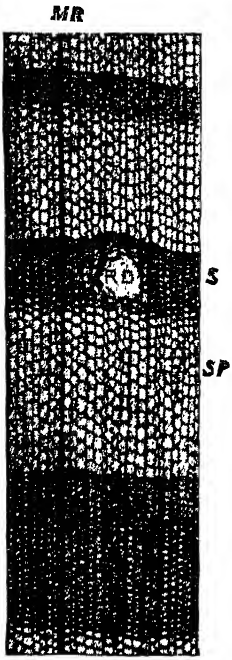
618. **Structural Elements.**—Wood is made up of *cells* and *vessels,* which are arranged with their long axes parallel to the axis of the tree or branch, and of *radial pith rays* which lie with their long axes perpendicular to the axis of the tree.

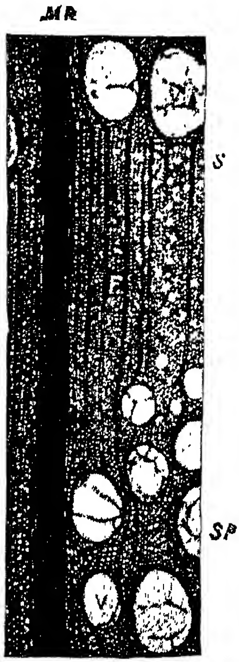
In cross-section, the *cells* (Fig. 618A) are roughly polygonal and usually appear to be rectangular with rounded corners. They are from .02″ to 2″ long and about .002″ in diameter. They are closed at the ends and the end walls of each cell are thin in comparison with its side walls.

The *vessels* are formed by the fusion of a number of cells. A single vessel often extends the length of the tree and sometimes has a diameter as large as .03″. Fig. 618B shows large vessels in oak in cross-section. The vessels in the outer portion of the tree serve as vertical water-supply lines.

In place of vessels, there exist in the conifers tubular spaces between cells, which are called resin ducts (Fig. 618A). These ducts possess no true walls like those of the vessels. The vertical ducts are interconnected with radial resin ducts to permit the passage of resin.

The *pith rays*, or medullary rays, consist of bundles of separate cells arranged with the long axis in a radial direction (Fig. 618C). They serve to distribute water and food between the bark and layers of wood.





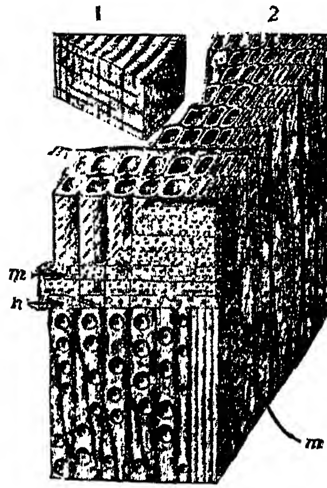


Fig. 618 C

Wood of Spruce, 1. Natural size; 2. Small part of one ring magnified 100 times. The vertical tubes are wood-fibers; m, medullary or pith ray n, transverse tracheids or pith ray

Fig. 618 A *Fig. 618 B*

Fig. 618 A Cross-section of Longleaf Pine, a Nonporous Wood. (Note: absence of pores; resin duct, RD; thick cell walls in Summer wood, S; thin walls in Spring wood, SP; narrow medullary ray, MR)

Fig. 618 B Cross-section of White Oak, a Ring Porous Wood. (Note: large vessels, V in Spring wood, SP; broad medullary ray, MR; also fibers, F)

619. Spring and Summer Wood.—If the annual rings of a conifer are examined, it will be found that each is alternately light and dark, one light and one dark ring representing a year's growth. The light ring is spring growth and is comparatively soft and weak; strength of the wood may, therefore, be judged by the proportion of summer to spring wood. It is also noted that the rings of summer wood are narrow near the pith and near the bark; in longitudinal section the rings decrease in thickness from the ground to the top; hence the strongest timber will come from the lower part the dark ring is summer growth and is dense and strong. The

of the tree, midway between the pith and the bark. The average amount of summer wood is 24 to 40 per cent of the entire tree.

620. **Sap-Wood** is the light, weak wood, next to the bark; the outer portion of it is the growing part of the tree. The heart-wood is the inner and darker portion and has no part in the growth of the tree; it is generally stronger and denser than the sap-wood. Heart-wood results from the gradual change of sap-wood due to the infiltration of chemical substances from the sap. The proportion of heart-wood depends upon the age of the tree; it forms about 60 per cent of an old long-leaf pine.

621. The conifers have a regularity of structure which makes them very dependable. The broad-leaved trees are more irregular, have more pith rays, and are more porous; consequently they are more difficult to work and are less dependable in engineering construction.

PHYSICAL CONSTRUCTION.

Defects.

622. From an engineering standpoint there are four important classes of defects in timber; irregular grain, knots, checks, and shakes. Wood is said to be straight grained when the wood elements are straight and run parallel to the axis of the tree trunk. Often, however, the elements spiral around the axis of the tree causing twisted grain. Wavy grain is caused by large undulations in the elements either on radial or tangential planes. Since straightness of grain is of great importance in wooden beams and tension members, careful examination should be made to eliminate twisted-grained and wavy-grained timber.

623. **Knots** are the beginning of branches which have been surrounded by the parent trunk. Generally the piths of the branch and trunk join and the annual rings in the branch are continuous downward with the rings in the trunk. The trunk fibers above the branch, however, do not continue in to the branch. The portion of the branch encased while living makes a sound knot; that encased after the limb dies will make a loose knot or a decayed knot. Knots greatly affect the strength and ease of working.

624. **Checks** are radial cracks produced by unequal shrinkage stresses set up in timber during seasoning.

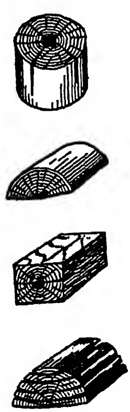
625. **Ring-shakes** are separations between adjacent annual rings, probably due to the bending of the tree by wind. *Heart-shakes*

are radial cracks beginning at the pith of the very old tree trunks, probably due to shrinkage of the heart-wood. Shakes and checks decrease the durability of the timber because they directly expose the interior wood to moisture and air. They may also greatly weaken the resistance of a wooden beam to horizontal shear.

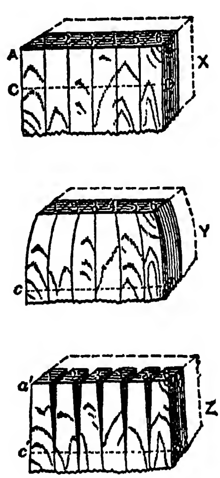
Density.

626. The density of woods varies with the amount of contained water and with differences in structure. The feature of the structure that is the most important in this regard is the average thickness of cell walls. The specific gravity of the wood substance alone of any species is about 1.55. Due to the porosity of structure, however, the specific gravity of wood varies from 0.11 for balsa wood to 1.29 for black iron wood.

627. **Shrinkage.**—Shrinkage is caused by the change in the thickness of the walls of the cells, resulting from a loss of moisture; the thicker the walls, the greater the shrinkage. As the side walls of the cells are thicker than the end walls, the lateral shrinkage is greater than the longitudinal; and as the cells of the summer wood have thicker walls than the spring wood, summer wood shrinks more than spring wood. Lateral shrinkage is somewhat checked by the pith rays, whose direction is perpendicular to that of the main cells of the timber. Longitudinal shrinkage amounts to very little; but



Effects of Shrinkage.



Formation of Checks.

Fig. 627 A

lateral shrinkage causes severe stresses in perpendicular planes, making cracks or checks (Fig. 627A). The radial shrinkage is less and more uniform than the tangential, since the summer and spring woods alternate in the former direction and are continuous in the latter direction. Thus the tangential shrinkage causes the radial cracks and the warping of the tangential boards.

MECHANICAL PROPERTIES.

628. From the structure of wood itself and from its growth in annual rings, it is apparent that it will not offer the same resistance to different kinds of stresses. It offers the greatest resistance to tensile, bending, and compressive forces which tend to elongate or crush the fibers. Less resistance is offered to a shearing force which acts in a plane perpendicular to the fibers or grain, and least of all to a shearing force which acts parallel to the fibers or grain.

629. **Tensile Strength.**—When a properly-shaped wooden stick is subjected to tensile forces acting parallel to the grain, it is found to have greater strength than can be developed under any other kind of stress. Wood tension members are, however, rarely used because the end connections always involve shear along the grain or compression along the grain. The resistance offered to the latter stresses is so much less than the tensile strength that economy will usually call for the use of steel for tension members. On this account, tensile strength is important to consider only in beams, and there only when the presence of defects on the lower side of the beam reduces the value of the under side in tension to or below that for the upper side in compression.

630. **Compressive Strength.**—When wood is subjected to compressive forces, acting parallel to the axis of growth (parallel to the grain), it is, in proportion to its weight, one of the strongest of structural materials. Columns and posts are therefore often made of it. The individual cells of wood act as so many hollow columns bound firmly together. The wood may fail through crushing of the cell walls or through lateral bending of the longitudinal elements. In most dry conifers, where the cells have comparatively thin walls, the former failure occurs. In wet or green wood and in hardwoods, which are composed of thick-walled cells and vessels, incipient failure is due to bending of these elements. Compressive strength (bearing) across the grain is only a small fraction of the strength parallel to the grain.

631. **Shearing Strength.**—Although shearing stresses are often

unimportant in steel and other materials, in a short, deep wooden beam the horizontal shear at the neutral axis may be sufficient to produce failure. Across the grain, wood has great shearing strength; with the grain, it has so little that it should never be used to withstand a shearing stress in that direction.

632. **Bending Strength.**—Unless the loading causes high shearing stresses or there are defects on the under side of the beam, the compressive strength of the wood will be the determining factor. The hardwoods as a class considerably exceed the conifers in static bending strength.

633. **Table of Mechanical Properties of Timber.**

		Breaking Unit Stress Lbs. per sq. in.	Safe Unit Stress Lbs. per sq. in.
	Oak	10,000	1,000
Tension, with grain......	White pine	7,000	700
	Yellow pine	12,000	1,200
	Oak	4,500	900
Compression, with grain..	White pine	3,500	700
	Yellow pine	5,000	1,000
	Oak	4,000	1,000
Shearing, across grain....	White pine	2,000	500
	Yellow pine	5,000	1,250
	Oak	6,000	1,000
Bending.................	White pine	4,200	700
	Yellow pine	7,200	1,200
	Oak	1,800	450
Torsion..................	White pine	....	....
	Yellow pine	1,200	300
Modulus of longitudinal	Oak	900,000	
elasticity..............	White pine	800,000	
	Yellow pine	1,200,000	

634. **Hardness.**—Hardness refers to the resistance of wood to indentation and scratching. These properties, together with the ability to wear without splintering, determine the wearing resistance of wood for floors and pavements. Almost all hardwoods are harder than the hardest of the conifers. Seasoning greatly increases the hardness.

635. **Toughness.**—Toughness is the capacity to resist blows or shocks. The spokes of an automobile, the handle of an axe, must be

tough. This property varies with the strength and flexibility. The hardwoods excel the conifers in toughness, although long-leaf pine is tougher than some of the hardwoods. Green wood, in general, is tougher than seasoned wood, because the occurrence of checking during drying usually more than nullifies the decrease of water content.

PRESERVATION.

636. Timber is weakened by dry rot, wet rot, marine borers, and insects. It is preserved against these by seasoning, painting, and special preservative processes, of which the most important are Creosoting, Burnettizing, and Kyanizing.

Weaknesses.

637. **Dry Rot.**—Rotting is caused by the attack of bacteria and fungi. Each of these has several varieties; but in general they thrive better under alternate conditions of moisture and dry air, and actually eat up the wood or destroy it by means of solvent chemicals which they secrete. If the timber is dry and kept dry, and is not freely exposed to the air, the bacteria and fungi can still work if there is any sap left in the wood; this is called "dry rot". In extreme cases, a stick of timber may be thoroughly weakened in its interior by dry rot while at the surface it may appear to be in perfect condition.

638. **Wet Rot.**—If exposed alternately to moisture and air, timber is very quickly weakened by the attacks of bacteria and fungi. This is called "wet rot". It may attack either live or dead timber, and is quite common in timber which has been felled and has not been seasoned. If, however, timber is thoroughly seasoned and is kept constantly wet in fresh water, it will last hundreds of years. Piles in London Bridge were sound after 800 years; the piles of Trajan's bridge over the Danube were found to be perfectly sound after 1600 years in the water.

639. **Marine Borers.**—The remarks made about timber placed in fresh water apply equally to salt water, as far as they relate to decay from rot. Timber immersed in salt water is, however, liable to the attacks of two of the destructive inhabitants of our waters, the *Limnoria terebrans* and the *Teredo navalis*; the former rapidly destroys the heaviest logs by gradually eating in between the annual

rings (Fig. 639A); and the latter, the well-known ship worm, converts timber into a perfectly honeycombed state by its numerous

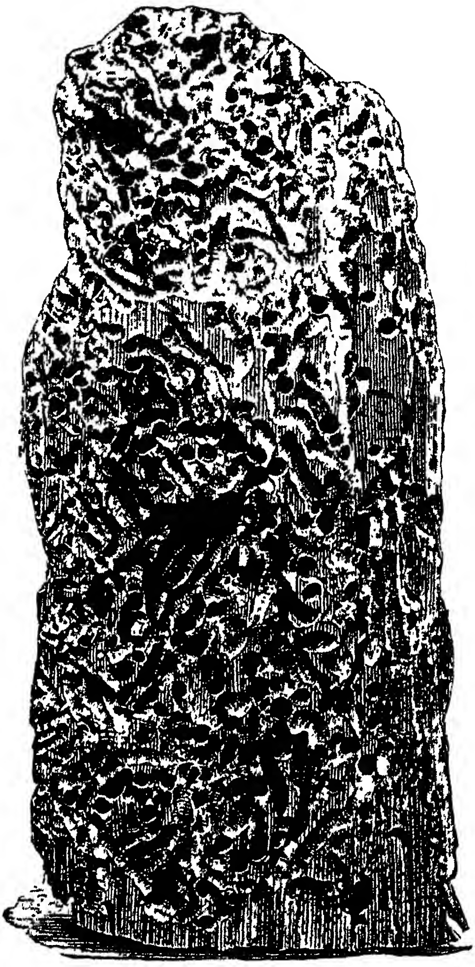


Fig. 639 A *Fig. 639 B*

Fig. 639 A Section of birch pile untreated after 7 years use at Portland. Maine. This pile was eaten by the Limnoria Terebrans

Fig. 639 B Piece of untreated timber after attack by Teredo Navalis

perforations (Fig. 639B). They both attack timber from the level of the mud, or bottom of the water.

640. **Insects.**—In the tropics, ants are very destructive; they eat out the inside of the wood leaving only the shell which often shows no evidence of the cavities beneath. In the temperate climates, two

insects, the powder-post insect and the pole borer, attack wood which is in contact with the ground.

Preservation.

641. **Seasoning.**—Timber is seasoned by expelling the moisture. It is not possible to expel all moisture; and even if it were expelled, the timber would reabsorb moisture at its first opportunity.

Natural seasoning consists in piling the timber so that the air can circulate freely and dry out the moisture; for good results it takes from two to four years. Water seasoning consists in placing the timber so that water will leach out the sap; this takes about two weeks but the results are not very good. Artificial seasoning consists in exposing the timber to hot air in a closed compartment; this seasoning, called kiln drying, takes from four days to two weeks, depending upon the temperature and the kind of wood. It is not as entirely satisfactory as natural seasoning. Steam is sometimes used instead of hot air, resulting in less cracking of the timber.

642. **Painting.**—After seasoning, it is desirable to keep the moisture out of the timber. Painting is one of the methods used for this purpose.

Paint consists of a base, a vehicle, and a solvent. The usual bases are red and brown oxide of iron, red and white lead, lampblack, and graphite. The vehicle is linseed oil, and sometimes cotton-seed oil or fish oil. The solvent is usually spirits of turpentine. The solvent makes the paint work more smoothly; the vehicle enables the paint to be spread and binds the particles together; while the base is the main part of the paint and is the body which covers the material being painted.

643. **Creosoting.**—Creosote oil, or dead oil, a coal-tar product, is the best known preservative. It fills up the cells, keeps out land insects and marine borers, prevents the growth of bacteria, and preserves the timber. In water, the creosote leaches out somewhat, but so does every other preservative. Creosote oil weighs about 8.8 pounds per gallon; and 10 to 12 pounds are required per cubic foot to protect the wood. The creosote oil is forced into the timber either by pressure or by boiling the timber in hot oil and then immersing it quickly in cold oil thus forcing the oil into the expanded cells.

644. **Burnettizing.**—This process consists in immersing wood for several hours in a 2 to 5 per cent solution of zinc chloride under a pressure of about 200 pounds per square inch. It is best used for

timber exposed alternately to wet and dry atmosphere, such as cross ties and damp floors. It is not satisfactory for structural timbers because the chemicals weaken the timbers.

645. **Kyanizing.**—Kyanizing consists in steeping the timber for several days in a solution of 1 pound of mercury bichloride (corrosive sublimate) to 100 pounds of water. It is used for bridge timbers and other structural pieces placed in comparatively dry places exposed to air and sunlight.

PURCHASE.

646. **Specifications.**—Standard specifications have been adopted in different sections of the United States. These are the result of experience with the lumber in each section, and should preferably be followed. For instance, if an agent on the Atlantic coast wishes to buy Oregon pine in the Northwest, he will preferably use the specifications prepared for that timber by the Lumber Associations in the Northwest.

647. Authoritative specifications are also prepared by the American Society for Testing Materials, the American Railway Engineering Association, and other large timber users.

648. **Sawing.**—Wood is sawed in two ways: bastard, when the cuts are all parallel to each other; quarter, when the log is first cut into quarters and then each quarter divided by cuts parallel to one face, or parallel to its faces alternately. Bastard sawing gives tangential sections; quarter sawing gives approximately radial sections. Quarter-sawed lumber lasts longer, and warps, splinters, and shrinks less.

649. Lumber is sold by the foot, *board-measure*, one foot B. M. being the equivalent of a board 1″ thick, 1′ wide, and 1′ long, equal to 1/12 cubic foot. Thus a stick of which the dimensions are 4″ x 10″ x 18′ would be sold for $4 \times 10 \times 18/12$ or 60 ft. B. M.

For more detailed information see:

"Materials of Construction," Johnson.
"Materials of Construction," Mills.
"Materials of Engineering," Moore.
"Architects and Builders Pocket Book," Kidder.

PROBLEMS.

P. 601. Show by a figure the part of a wooden beam in which it is most undesirable to have knots.

P. 602. How many 100 gallon drums of creosote oil would be required and how much would it cost at $0.07 a gallon to preserve 1000 railroad ties? A railroad tie is 6″ x 8″ x 8′.

P. 603. A bridge 120′ long and 20′ wide is to be floored with 5″ x 12″ plank. How many feet B. M. will be required?

P. 604. How many feet B. M. in the following bill of materials:

20 - 2″ x 4″ x 15′
45 - 1″ x 6″ x 20′
18 - 4″ x 4″ x 18′
6 - 6″ x 8″ x 20′

CHAPTER VII.

METALS.

701. Because of their strength, durability, and homogeneity, the useful metals and their alloys form a very important class of engineering materials. They may be safely exposed to conditions in which timber would fail. The principal metallic substances used in engineering construction are cast iron, wrought iron, cast steel, and structural steel.

IRON.

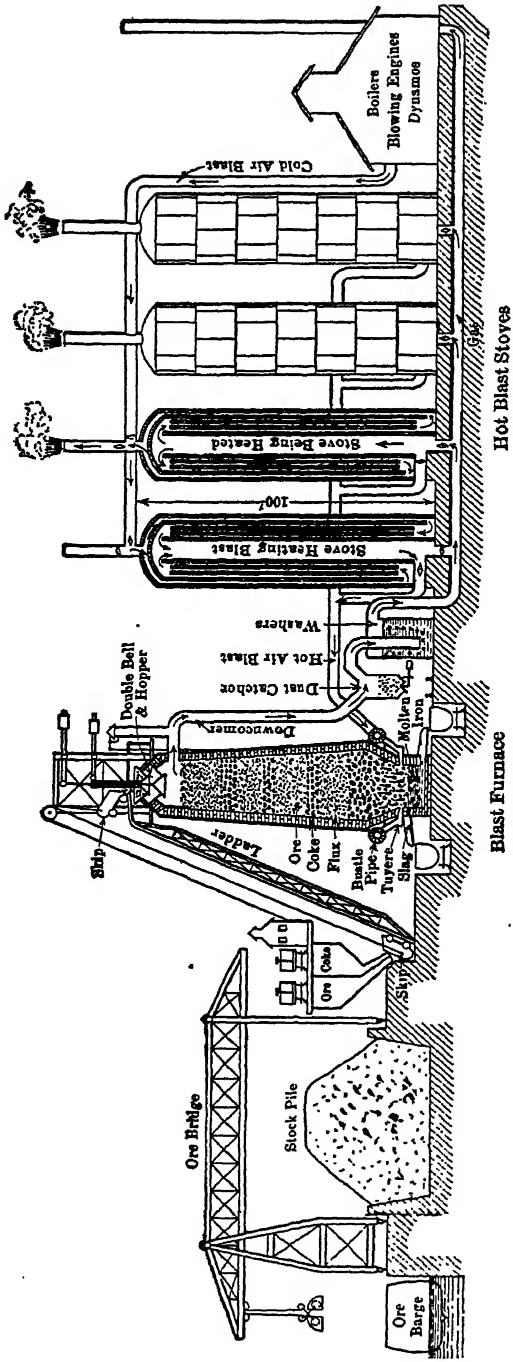
Economic Importance of Iron.

702. For engineering purposes iron is by far the most important of the metals. Its ores are very abundant in nature, which fact, together with the accessibility of the deposits and the comparatively low cost of extraction, makes it a cheap metal. The cost is about (1924) 1 cent a pound in the form of *pig iron*, which is the raw material used in the manufacture of the structural shapes. Production in the United States has now reached upwards of 40 million gross tons per year, valued roughly at $800,000,000. This is about one-half of the yearly production of the world.

703. **Iron Ore.**—Ore is the form in which a metal occurs in a deposit in the earth and from which it may be commercially recovered. Some metals which it is sought to recover are present in the native or metallic form, but iron always exists as a compound with other elements, usually as an oxide. Associated with the oxide is a varying amount of earthy matter.

704. **Pig Iron.**—The fundamental principle in the extraction of pig iron from the ores is one of deoxidation of iron oxide. Figure 704A shows a modern *blast furnace* plant, where the deoxidation is effected. Into the top of the furnace proper are put layers of fuel (usually coke), ore, and *flux* (usually limestone); a blast of air is pumped up through the bottom. The carbon in the fuel burns in the current of air to the final form of CO. The CO reacts with the iron oxide of the ore to form CO_2 and iron combined with carbon (pig iron). The lime in the flux combines with the earthy matter in the ore and the residual ash of the fuel to form waste matter or *slag*.

The operation is continuous. The furnace is charged with the mixture at the top often enough to keep it well filled. The iron



Sketch showing operation of Blast Furnace
Fig. 704 A

and slag at the bottom are molten and are drawn off periodically. The furnace may run this way steadily for 400 or 500 days, at the end of which time repairs to the brick lining and other parts are necessary.

705. Iron comes from the blast furnace plant in "pigs" semi-cylindrical in form, about 5″ wide, 36″ long, and weighing about 100 pounds. Further operations are necessary before the pig iron can be used. The results of these operations are the production of commercial cast iron, wrought iron, cast steel, and structural steel. The important difference between these four varieties of iron lies largely in the quantity of carbon. The lines of demarkation are not exact; cast iron and cast steel have much carbon and are brittle; structural steel has less carbon and is not brittle; wrought iron has very little carbon and is very malleable. The following table shows the principal characteristics:

Iron	Percent of Carbon	Properties
Cast Iron	5 to 2	Very Brittle
Cast Steel	2 to .25	Brittle
Structural Steel	.5 to .1	Malleable
Wrought Iron	.2 to .05	Very Malleable

Besides iron and carbon, the above contain different proportions of silica, sulphur, phosphorus and manganese, which modify the properties but do not change the classification.

706. These four varieties of iron form the most important group of materials in engineering construction. They have strength, durability, comparative cheapness, and uniformity, which latter quality is not possessed by wood.

CAST IRON.

707. **Cast Iron** is made by remelting pig iron. The molten metal is poured into cavities of the desired shapes formed in molding sand by the use of wooden patterns. The products of the molds are called "castings". On account of the ease of manufacture, cast iron is much used in engineering construction, in the form of hollow columns and pipes, which do not require the strength or toughness of steel or wrought iron.

708. **Varieties of Cast Iron.**—Cast iron is divided into six varieties, according to their relative hardness. This hardness seems to depend upon the proportion and state of carbon in the metal.

Manufacturers distinguish the different varieties by the consecutive whole numbers from 1 to 6.

No. 1 is known as *gray cast iron*, and No. 6 as *white cast iron*. They are the two principal varieties.

Gray cast iron, of good quality, is slightly malleable when cold, and will yield readily to the action of the file if the hard outside covering is removed. It has a brilliant fracture of gray, sometimes bluish gray, color. It is softer and tougher than white cast iron.

White cast iron is very brittle, resists the file and chisel, and is susceptible of high polish. Its fracture presents a silvery appearance, generally fine-grained and compact.

The Intermediate Varieties, as they approach in appearance to that of No. 1 or No. 6, partake more or less of the properties and characteristics of the extreme varieties.

Numbers 2 and 3, as they are designated, are usually considered the best for building purposes, as they are strong without being excessively brittle.

709. A medium-sized grain with a close compact texture indicates a good quality of iron. The color and lustre presented by the surface of a recent fracture are good indications of quality. A uniform dark-gray color with a high metallic lustre is an indication of the best and strongest iron. With the same color, but less lustre, the iron will be found to be softer and weaker. No lustre, with a dark and mottled color, indicates the softest and weakest of the gray varieties.

Cast iron of a light gray color and high metallic lustre, is usually very hard and tenacious. As the color approaches to white, and as the metal changes to a vitreous lustre, hardness and brittleness of the iron become more marked; when the extreme, a dull or grayish white color with a very high vitreous lustre, is attained, the iron is of the hardest and most brittle of the white variety.

710. **Specifications.**—The American Society for Testing Materials has adopted the following specifications for gray cast iron:

a. Gray castings shall be made by remelting the iron in the cupola furnace.

b. Sulphur shall be from less than .08% for light castings to less than .12% for heavy castings. Light castings have some sections less than ½″ thick; heavy castings have no section less than 2″ thick.

c. The minimum breaking strength of the "Arbitration Bar" (a round bar 1¼″ diameter and 15″ long) shall not be under 2500 pounds for light castings and 3300 pounds for heavy

castings. The loads are to be applied at the middle of the bars resting on supports 12″ apart; and the deflection must be not less than .10″ at rupture.

d. Tensile strength of light castings shall be 18,000 pounds per square inch; that of heavy castings 24,000 pounds.

711. Defects in castings are caused by improper composition of the metal and by carelessness or lack of skill in melting or pouring. Improper composition causes weakness and brittleness; carelessness in casting causes air holes and honeycombed cracks, flaws, and cold-shut. Air holes and honeycomb are caused by the retention of air and impurities in the molten metal while cooling; cold-shut is caused by the failure of the metal, poured through two or more holes, to cohere at the surface of contact. All of the defects may be determined by careful inspection, by hammering the casting, and by chemical analysis.

STEEL.

712. There are two principal methods of making steel: the open hearth process, and the Bessemer process. Of these two, the open hearth process is under better control and the steel is more uniform and of better quality. It is therefore preferred for most structural purposes.

713. **Open Hearth Process.**—The open hearth process makes molten steel out of pig iron and scrap steel. The materials of the charge lie in the basin-like bottom or hearth of a reverberatory furnace while air and flaming gas pass over the surface of them. The flame decarbonizes and desiliconizes the mixture as desired. The common practice now in the United States, in the "basic" open hearth process, is to use equal parts of pig iron and scrap. The proportion depends upon cost; the smaller the percentage of impurities the shorter is the operation, but scrap steel costs more than pig iron. The basic process requires about 11½ hours when equal parts of pig iron and scrap are used.

714. The special value of the basic process is that it will remove *phosphorus*. The other impurities will oxidize and stay in the slag, whether the slag be chemically acid or basic in character. Phosphorus, however, has a great affinity for iron, and if the slag is acid, the oxidized phosphorus will not stay in the slag but will recombine with the iron. In order to remove the phosphorus, the furnace must be lined with basic materials (magnesite brick being used); and an excess of quicklime, a strong base, must be added to the charge in order to make the slag basic. In the acid process the lining is of acid materials (silica brick) and no base is added.

715. The hearth, H, (Figs. 715A and B) contains the charge. The molten pig iron is poured through charging doors, T, from a ladle hung on an overhead crane (not shown in the figures). The

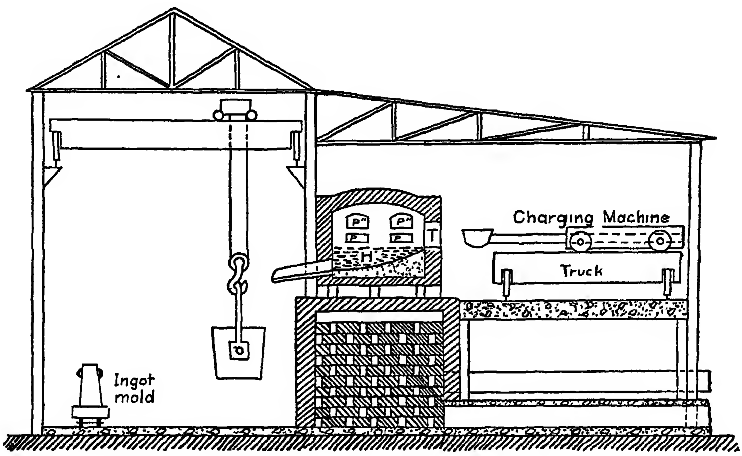


Fig. 715 A. Open-hearth Steel Plant.

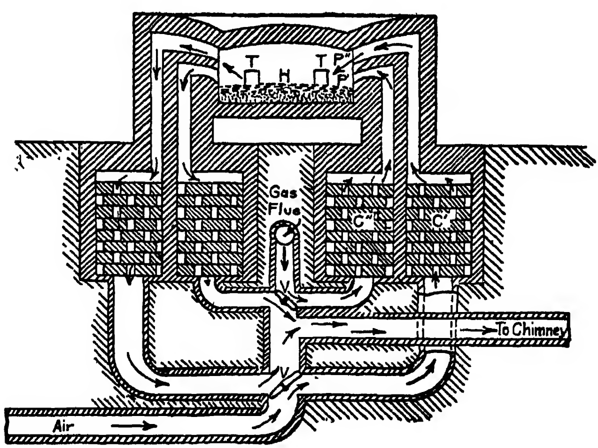


Fig. 715 B. Open-hearth Steel Furnace.

other ingredients of the charge (steel scrap, iron ore, and quicklime) are fed through these same doors by the charging machine (Fig. 715A), which is mounted on the truck.

The fuel, which is usually producer gas, enters through the gas flue shown in Fig. 715B, passes up through the brick checker-

work, C', and enters the furnace through the port, P'. Air passes up through the checkerwork, C'', and enters the furnace through the port, P''. The fuel and air ignite at the ports, burn over the hearth, and pass to the chimney through the opposite checkerworks, which take heat from the escaping gas. At intervals of about twenty minutes, the valves, V, are turned 90 degrees and the direction of flow of the gases is reversed. By this reversal of flow, the gas and air are pre-heated to a high temperature.

From time to time sample billets are poured, cooled, and tested to determine the state of progression of the refining and permit the correction of composition by further additions of materials. The carbon content is usually reduced below that finally desired. Ferro manganese, a pig iron rich in C and Mn, is then added in proper quantity. The process is quasi-continuous; that is, about one-eighth is drawn off at 1½-hour intervals and new materials added.

716. **Bessemer Process.**—In the Acid Bessemer process (the Basic Bessemer process, used abroad, is not considered in this text), molten pig iron is poured into a pear-shaped vessel, called a *converter* (Fig.

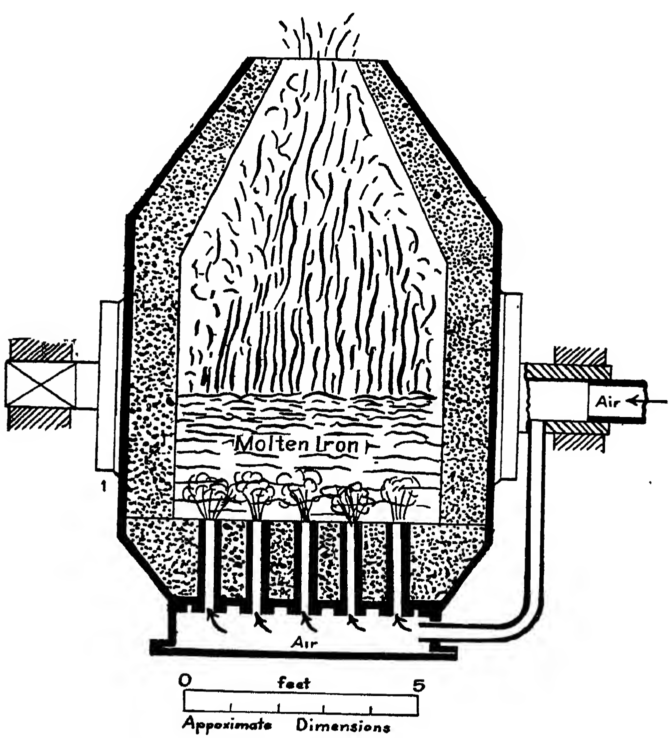


Fig. 716 A. Bessemer Converter.

716A). A blast of cold air is blown up through the charge and in 10 minutes oxidizes the foreign elements along with some of the iron. No fuel is used; the initial temperature of the pig iron is enough to start burning in such intimate contact with the air, and the heat generated by the chemical reaction further raises the temperature. The *acid Bessemer process* is the only one used in the United States. For the same reason that applies in the case of the acid open hearth process, the acid Bessemer process cannot remove phosphorus. Since it is not permissible for steel to contain more than 0.10% phosphorus and the quantity of ore of less than that content is limited in this country, practically no new furnaces use the Bessemer process, all using the open hearth process.

717. The blowing process is accompanied by a roaring flame issuing from the top of the converter. The flame changes its violence and color in such a manner that at the end, as the last carbon burns, the flame indicates that fact accurately. There is no time for testing samples, the blowing being continuous until the flame indicates the above; the steel is then recarbonized to the desired amount by adding molten ferro manganese to the converter before pouring.

718. Fig. 718A shows the relative order of the processes through which iron must go in a modern iron and steel plant in order to be transformed from ore to rolled steel shapes.

Cast Steel.

719. The molten metal is poured into molds made with patterns in the same general manner as cast iron. Cast steel develops the same weaknesses as cast iron but to a less extent. It is much less uniform than is structural steel.

720. **Specifications.**—The following are the specifications for cast steel adopted by the American Society for Testing Materials:

a. Ordinary castings shall not contain over .40 per cent of carbon nor over .08 per cent of phosphorus.

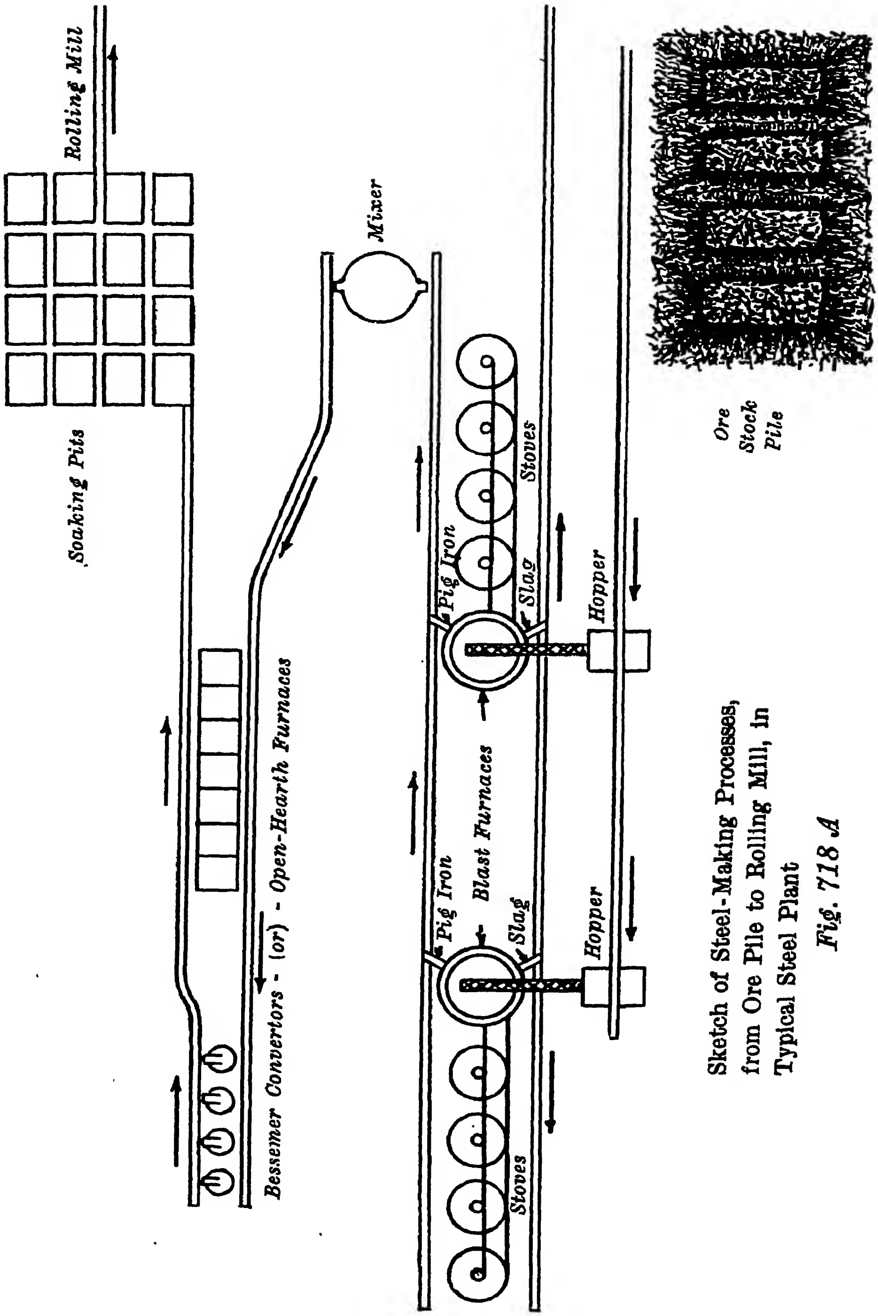
b. Tensile strength of medium castings shall be 70,000 pounds per square inch.

c. Large castings are to be suspended and hammered all over; no cracks shall appear.

d. A specimen $1\frac{1}{2}''$ in diameter shall bend cold 90° to 120° around a diameter of $1''$ without fracture.

Structural Steel.

721. In making structural steel, the molten metal is poured into molds to form ingots, which are taken to the rolling mill where



Sketch of Steel-Making Processes, from Ore Pile to Rolling Mill, in Typical Steel Plant

Fig. 718 A

they are reheated in the soaking pits and rolled into the desired shape. The rolling consists in running the hot metal between rolls which give the metal the desired shape; the hot metal is squeezed more and more as it passes from one set of rolls to another until it is finally in the form in which it is sold: rails, angles, I-beams, etc., as explained later.

722. **Specifications.**—Structural steel must fulfill the following minimum requirements:

Kind	Tensile Strength	Elongation
Rivet or very soft...	45,000 to 55,000 lbs.	28%
Soft, low, or mild...	52,000 to 62,000 lbs.	26%
Medium............	55,000 to 65,000 lbs.	25%

723. The following specifications are those required by the Chief Engineer of the Williamsburg Suspension Bridge over the East River, New York, for the structural steel: (Those of the American Society for Testing Materials are practically the same.)

"All steel shall be made in an open hearth furnace lined with silica.

"The finished steel shall not contain more than the following proportions of the elements named:

Phosphorus......	.07 of one per cent
Sulphur.........	.04 " " " "
Manganese.......	.50 " " " "
Silicon..........	.10 " " " "
Copper..........	.02 " " " "

"Specimens cut from the finished material shall have the following physical properties:

Description of Material	Strength in lbs. per sq. inch	Elongation per cent in 8″	Reduction per cent in area
Shapes and universal mill-plates..........	60,000 to 68,000	22	44
Shear-plates..........	60,000 to 68,000	20	44
Rivet-rods............	54,000 to 60,000	25	50

"All specimens cut from plates and shapes shall bend cold 180° around once the thickness of the specimen; when at or above red heat, 180° flat; and when quenched in water at a temperature of 80° F., 180° around three times the thickness of the specimen. Specimens cut from rivet-rods shall bend

180° flat when cold, when at or above red heat, or when quenched from a light yellow heat in water at a temperature of 60° F.

"The elastic limit of the steel shall not be less than one-half the ultimate strength.

"All bending tests shall show no signs of fracture on the outside of the bent portion.

"The fracture of all tension tests shall have a cup or angular shape and shall have a fine silky texture, of a bluish-gray or dove color, free from black or brilliant specks.

"All rolled or forged material shall be entirely free from piping, checks, cracks, and other imperfections, and shall have smooth-finished surfaces and edges.

"Rivets cut out of the work, when required by the engineer or his representative, shall be tough and show a silky texture without a crystalline appearance.

"Rigid tests will be made to guard against all red-shortness (excess of sulphur)."

Wire.

724. The process for making wire is similar to that for structural steel, except that the heated metal is rolled into the shape of rods. The rods are cooled and then drawn many times through holes in a steel plate. This gives them great tensile strength; the required tensile strength is generally 200,000 pounds per square inch.

Properties of Steel.

725. **Hardening, Tempering, and Annealing.**—Steel is *hardened* by heating it to a temperature of about 1300° F., called a yellow heat, and then plunging it into a bath of water or oil. It can then be *annealed* or made as soft as possible by reheating it to about 1300° F. and cooling it slowly. Hardened steel is *tempered* (toughened) by heating to a temperature below 700° F., called a dull red heat; the higher the temperature below 700° F., to which it is reheated the tougher it becomes.

726. **Welding** by hand consists in heating two pieces of metal, placing one on top of the other, and hammering them so that they unite. Soft steel can easily be welded, medium steel only imperfectly, and hard steel not at all by the ordinary processes. By using electricity, oxyacetylene torch, or thermit to melt the two pieces, it is practicable to weld hard steel pieces: no hammering is necessary, and no pressure except with electricity. A welded joint averages roughly half as strong as an unwelded solid bar of the same size, but may be from 30% to 100% as strong.

WROUGHT IRON.

727. Wrought iron, in a perfect condition, is really pure iron. Naturally, this condition cannot be attained. Certain grades of pig iron are placed in a reverberatory furnace, and melted. During a stage of the process, the metal is stirred by a workman in order to insure contact of all parts with the material forming the lining of the sides, which is rich in oxides. This process is called puddling. The metal is taken off in balls of convenient size, and these are hammered or squeezed so as to expel the slag. The material is then rolled into the desired shape.

728. **Appearances of Good Wrought Iron.**—The fracture of good wrought iron should have a clear gray color, metallic lustre, and a fibrous appearance. A crystalline structure indicates, as a rule, defective wrought iron. *Blisters, flaws,* and *cinder holes* are defects due to bad manufacture.

The strength of wrought iron is very variable, as it depends not only on the natural qualities of the metal, but also on the care used in the forging, and upon the greater or less compression of the fibers when it is rolled or hammered into bars.

729. **Specifications.**—The following are the specifications for wrought iron adopted by the American Society for Testing Materials:

a. The elastic limit shall be not less than 25,000 pounds per square inch.

b. The ultimate strength shall be not less than 48,000 pounds per square inch.

c. A cold bar of the poorest grade must be capable of being bent 180° around a bar twice its diameter or thickness.

OTHER METALS.

730. **Copper.**—After iron and steel, copper is the most important metal in engineering practice. It is used principally in sheets for roofing and in wire for electrical purposes. Copper is harder and more tenacious than any other metal used in engineering structures, excepting iron. Copper wire has a tensile strength of 36,000 pounds to 60,000 pounds. Copper is very durable under ordinary atmospheric conditions and does not rust.

731. **Tin.**—In engineering construction, tin is used in tin-plate which is sheet iron covered with tin. Good tin-plate is plated with pure tin, and appears bright and even. Poor tin-plate is plated with dark alloy, and is dark and rough.

732. **Zinc.**—Zinc is extremely durable when exposed to the action of the weather, and is therefore much used to coat sheet iron and iron wire. Galvanized iron is iron dipped in melted zinc.

733. **Lead.**—Lead is used in sheets and in lead pipes. It does not rust.

PROPERTIES OF METALS.

734. The metals, with the elements in combination with them, have properties which give them different values for engineering construction. Some are rust-proof, others are brittle or tough or ductile. The table below shows the principal ultimate values in pounds per square inch which determine their special uses for engineering purposes:

	Ultimate Tensile Str'gth	Elastic Limit	Net Compressive Str'gth	Net Bending Str'gth	Net Shearing Str'gth	Modulus of Elasticity	Remarks
Copper.......	32,000	10,000	40,000	22,000	30,000	10,000,000	Does not rust.
Cast Iron......	18,000	6,000	80,000	36,000	20,000	15,000,000	Very brittle.
Cast Steel.....	70,000	31,500	70,000	70,000	60,000	30,000,000	Brittle. Strong.
Structural Steel	70,000	35,000	50,000	60,000	50,000	30,000,000	Malleable.
Wrought Iron..	48,000	26,000	48,000	48,000	40,000	27,000,000	Expensive; very mal'ble
Lead.........	1,800					1,000,000	Does not rust; ductile.
Tin...........	3,500	1,800	6,000	4,000		4,000,000	Principally for plating.
Zinc..........	5,000	4,000	18,000	7,000		13,000,000	Does not rust; principally for galvanizing.

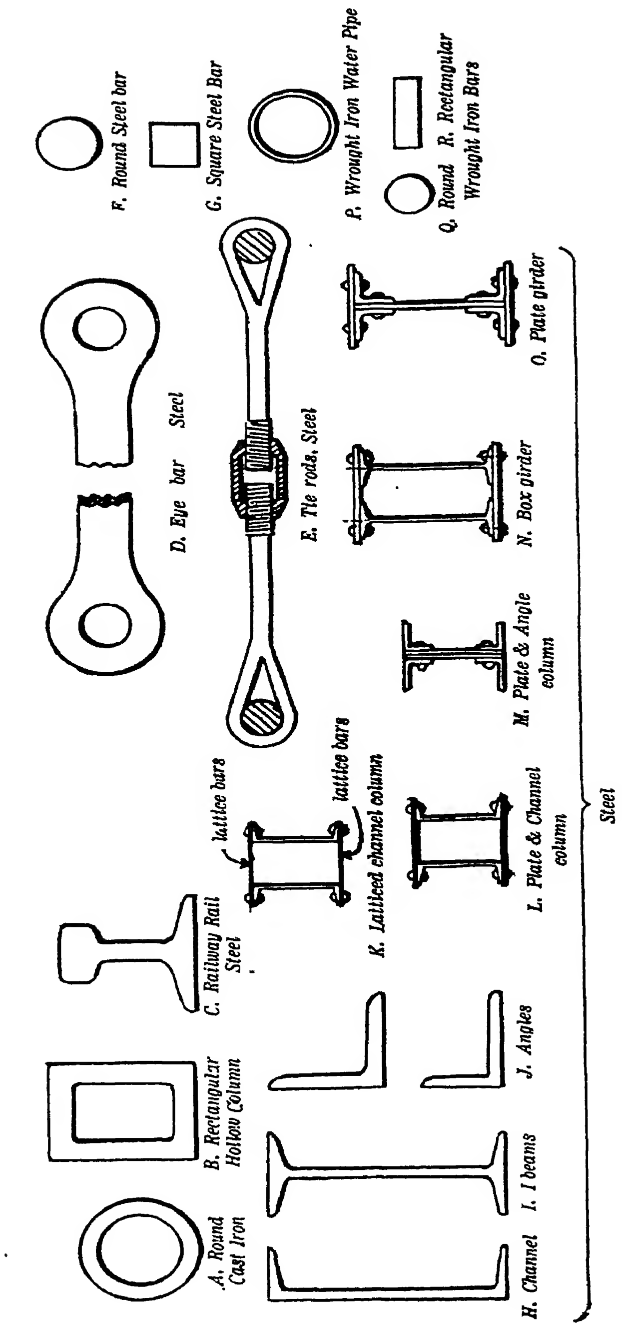
735. The weight of iron is about 450 pounds per cubic foot; copper weighs 556, zinc 428, and lead 710 pounds. The specific gravity, therefore, varies from 6.8 for zinc, through 7.2 for iron and 8.8 for copper, to 11.38 for lead.

PRESERVATION.

736. On coming from the mill, iron is covered with a scale, called *mill scale.* This should be removed before any paint is put on the metal, as the scale may later flake off and expose the metal. The mill scale is removed by (1) cold rolling, which, however, costs (1924) $2.00 per ton; (2) sand blast; (3) steel scrapers or wire brushes. The latter is generally the most satisfactory.

737. Many engineers prefer that the metal be not painted before leaving the mill, but be covered simply with a coat of pure linseed oil. The oil penetrates the surface and temporarily prevents rust; and it does not cover up any imperfections which, if hidden by paint, might not be noticed when the metal is put into place in a structure.

738. Paint is the final preservative for metals. The lead paints and the carbon paints are generally used; asphalt (tar) paints are



Figs. 740 A to R

also used; coal tar, or a special system of baking asphalt tar on metal is specially used for water pipes. One gallon of paint is sufficient to cover with two coats about 600 square feet of surface.

PURCHASE.

739. Metal is generally bought by the pound, or in large quantities by the ton. Standard commercial shapes are preferably selected, as the cost of these shapes is much less than the cost of making special shapes. This applies especially to structural steel; cast iron and cast steel are easily made into special shapes, the principal cost being that of making the special wooden pattern for the mold for casting.

740. Figures 740A to R show standard forms in which metals are manufactured for commercial use.

Cast Iron: hollow round columns (A), and hollow rectangular (B) columns.

Structural Steel: railway rails (C); eye bars (D); tie rods (E); round bars (F); square bars (G); channels (H); I-beams (I); angles (J); compound shapes, such as latticed channel columns (K), plate and channel columns (L), plate and angle columns (M), box girders (N), plate girders (O).

Wrought Iron: water pipes (P); round bars (Q); and rectangular bars (R).

For more detailed information see:

Iron and Steel, Thurston.
Materials of Construction, Johnson.
Materials of Construction, Mills.
Materials of Engineering, Moore.

PROBLEM.

P. 701. What is the carrying power of a beam 20′ long loaded at the center with a concentrated weight (a) if a 20″ at 90 pounds I-beam is used, (b) if the same amount of metal is rolled into a square section?

CHAPTER VIII.

NATURAL AND ARTIFICIAL STONES.

801. The term **Stone** or **Rock** is applied geologically to any aggregation of several mineral substances which form an essential part of the earth's crust. As a building material, stones may be either *natural* or *artificial.*

Natural Stones may be subdivided into three classes: the *silicious,* the *argillaceous,* and the *calcareous,* according as silica, clay, or lime is the principal constituent.

Artificial Stones are imitations of natural stones, made by consolidating fragmentary solid material by various means; they may be subdivided into classes as follows:

1st. Those in which solid materials are mixed together and consolidated by baking or burning: as *brick, tiles,* etc.

2nd. Those in which solid materials are mixed with some fluid or semi-fluid substance, which latter, hardening afterwards by means of chemical combinations, binds the former firmly together: as *ordinary concrete, patent stone,* etc.

3rd. Those in which the solid materials are mixed with some hot fluid substance which hardens upon cooling: as *asphaltic concrete,* etc.

802. The mineral substances which form stone, or which are prepared from stone, are used in construction in the following forms: Natural stone, Brick, Lime, Cement, Sand, Gravel, Broken stone, Mortar, Bituminous materials, Concrete, Reinforced concrete.

803. The qualities required in stone for building purposes are so various that no very precise directions can be given to meet exactly all cases. What would be required for a sea-wall would be unsuitable for a dwelling house. In most cases the choice is limited by the cost. The most essential properties of stone as a building material are *strength, hardness, durability,* and *ease of working.* These properties are determined by experience or actual experiment.

All stones, whether natural or artificial, must be selected with a view to their fitness for the particular duty which they are to perform in a structure.

NATURAL STONE.

804. **Composition.**—The important building stones classified according to their principal constituents, are:

a. Silicious, as quartz, granite, gneiss, trap, sandstone.
b. Calcareous, as limestone, marble.
c. Argillaceous, as slate, bluestone.

805. Classified according to manner of formation, they are:

a. Sedimentary or stratified, as sandstone, common limestone.
b. Igneous or unstratified, as granite, trap.
c. Metamorphic, as gneiss, marble, slate.

806. The stratified stones are easily separated along the planes of stratification, and are, therefore, more easily quarried than unstratified stones. A stratified stone should be laid with its planes of cleavage perpendicular to the maximum force to which it is subjected, as it thus offers the greatest resistance. If laid with its planes of stratification parallel to its exposed surface, it is not only weaker but it flakes off under the action of frost.

Some of the metamorphic stones, such as gneiss and slate, have surfaces or planes of cleavage similar to planes of stratification. They should therefore be treated as stratified stones.

Unstratified stones are quarried in blocks, which may be as large as their strength to resist the bending stresses involved in their movement allows; they are preferably used in structures where massive effects are desired.

The metamorphic rocks without planes of cleavage, e. g., marbles, are also equally strong in all directions, and may be treated as unstratified stones.

Physical Qualities.

807. **Specific Gravity.**—Natural stones range in specific gravity from 2.2 to 2.8. For average computations, the weight is taken as 160 pounds per cubic foot.

808. **Hardness.**—This property is ascertained by actual experiment and by a comparison made with other stones which have been tested. It is an essential quality in stone exposed to wear by attrition. Stone selected for paving, flagging and for stairs should be hard and of a grain too coarse to admit of becoming very smooth under the action to which it is submitted.

By the absorption of water, stones become softer and more friable.

809. **Freezing.**—Frost, or rather the alternate action of freezing and thawing, is one of the most destructive agents of nature with which the engineer has to contend. Its effects vary with the texture of the stone; those of a fissile nature usually split, while the more porous varieties disintegrate, or exfoliate at the surface. When stone from a new quarry is to be used, the best indications of its resistance to frost may be obtained from an examination of any rocks of the same kind within its vicinity which are known to have been exposed for a long period. Submitting the stone fresh from the quarry to the direct action of freezing would seem to be the best test, if it were not that there are some stones that are much affected by frost when they are first quarried, due to the moisture in the stone, which moisture is lost by exposure to the air, and never reabsorbed to the same amount.

810. **Strength.**—Among stones of the same kind, the strongest is almost always that which has the greatest density.

As stone ordinarily has very great crushing strength, it will be only in particular cases that the resistances to this strain need be considered, since the strength of stone, in this respect, is greater than is generally required of it. If its durability is satisfactorily proved, its strength, as a rule, may be assumed to be sufficient.

811. **Durability.**—In engineering practice, stones are subjected to chemical and mechanical agencies. The principal *chemical agents* are the *sulphuric* and *carbonic* acids in the air, resulting from the burning of gas, coal, etc. These acids attack every constituent of stone except silica. The silicious stones are therefore the most durable, and the limestones the least so. If the acids attack the cementing substances of the stone, the stone will crumble; if they attack the grains, a spongy, porous stone will result. The durability of a stone is, therefore, chiefly dependent upon its mineral constituents. Laboratory tests furnish some indication of the durability of a stone under the attacks of chemical agents, but the only thoroughly reliable test is to expose the stone for a long time to the conditions under which it is to be used.

812. The principal *mechanical agents* to which stones are subjected in engineering practice are *friction, concussion, winds, rains,* and *changes of temperature* especially near the freezing point. The effects of friction and concussion are seen in the paving stones; the wear of winds and rain is seen on the surfaces of buildings, especially in ornamental work where sharp edges are exposed to their action; the results of changes of temperature are seen especi-

ally in temperate climates where frost and ice crumble the stones, and in fires where the hot stones are drenched with cold water. The order of durability under ordinary conditions is usually: sandstone, granite, marble, ordinary limestone, porous sandstone.

Preservation.

813. Both natural and artificial stones are preserved, like wood, by coating them with an impervious covering. The different preservatives used for this purpose are *paint, boiled linseed oil, melted paraffin,* and a mixture of *alum* and *soft soap* dissolved in water. There are several special chemicals by which the stone is covered with a liquid which leaves an insoluble salt in the pores of the stone.

Quarrying.

814. If the engineer should be obliged to get out his own stone by opening a new quarry, he should pay particular attention to the best and cheapest method of getting it out and hauling it to the point where it is to be used. He will, if possible, open the quarry on the sides of a hill, and arrange the roads in and leading to it with gentle slopes to reduce the work of the animals or motor trucks. The stone near the surface, not being so good as that beneath, is generally discarded. The mass or bed of stone being exposed, a close inspection will discover the natural joints or fissures along which the blocks will easily part.

815. The drills used in quarrying are hand, churn, or steam drills. *Hand drills* are short blunt chisels driven in by blows from a hand-hammer or sledge. *Churn drills* are longer drills which make holes 3′ to 5′ deep by being continually dropped a short distance. *Steam* or *compressed-air drills* are driven by one of those agents. In quarries of sandstone, limestone, and marble, a *channeling machine* is often employed, which cuts a continuous groove instead of a series of holes. A *hollow revolving drill* or *diamond drill* is employed so as to obtain core specimens for the study of the character of the stone. *Explosives* employed in quarries are usually *slow-acting,* as the stone is thus less shattered.

Tests.

816. The tests of building stones are chemical and mechanical. The chemical tests determine the composition of the stone and the probable results of exposure to the conditions to which it will be subjected in the structure. The mechanical tests determine the

amount of absorption, the action of the stone when exposed to freezing temperature in a saturated condition, its action when exposed to a conflagration, and its crushing strength.

817. The **absorption test** consists in drying the specimen in an oven, weighing it and then immersing it for a long time in water. After removal from the water, it is exposed to the air until the surface-water is evaporated and then weighed again. The difference in weight before and after immersion gives the weight of the water absorbed.

818. The **freezing test** consists in repeatedly saturating a specimen with water and alternately freezing and thawing it; then finally weighing it to determine its loss by breaking off of material. The **fire test** consists in subjecting it to high heat and dashing it with cold water. The **crushing strength** is determined by crushing the specimen in a testing machine.

BRICK.

819. A **brick** is an artificial stone, made by moulding prepared clay into a form of the requisite shape and size, and hardening it, either by baking in the sun or by burning in a kiln or other contrivance. When hardened by the former process it is known as *sun-dried,* and by the latter as *burnt-brick,* or simply *brick.*

820. **Sun-dried Brick.**—Sun-dried brick have been in use from the remotest antiquity, having been found in the ruins of ancient Babylon. They were used by the Greeks and Romans, and especially by the Egyptians. At present they are seldom employed.

They were ordinarily made in the spring or autumn, as they dried more uniformly during those seasons; those made in the summer, drying too rapidly on the exterior, were apt to crack from subsequent contraction of the interior.

It was not customary to use them until two years after they were made.

Brick, known as *adobes,* made of earth hardened in a similar way, are found in parts of our country and in Mexico. They furnish a simple and economical mode of construction where the weights to be supported are moderate, and where fuel is very scarce and expensive. This mode, however suitable for a very warm, is not suitable for a temperate climate.

Manufacture.

821. The clay is mixed with water by hand or machine until it

forms a paste. This paste is placed in molds and taken to the drying floor, which is in the open air or in a drying house.

822. **Burning.**—The next stage of manufacture is the burning. The brick are arranged in the kiln so as to allow the passage of the heat around them. A very moderate fire is next applied under the arches of the kiln to expel any remaining moisture from the raw brick; this is continued until the smoke from the kiln is no longer black. The fire is then increased until the brick of the arches attain a white heat; it is then allowed to abate in some degree, in order to prevent complete vitrifaction; and it is thus alternately raised and lowered until the burning is complete, as ascertained by examining the brick at the top of the kiln. The brick should be slowly cooled; otherwise they will not withstand the effects of the weather. The length of time of burning varies, but is often as much as fifteen days.

823. **Classification.**—Common brick are classified according to the method of molding, into *hand-made, machine-made,* and *pressed brick.* Pressed brick are made by compressing partially dried brick in suitable molds under heavy pressure.

824. **Characteristics of Good Brick.**—Good brick should be regular in shape, with plane surfaces and sharp edges; the opposite faces should be parallel, and adjacent faces perpendicular to each other.

They should be free from cracks and flaws; be hard; possess a regular form, and uniform size; and, where exposed to great heat, be infusible.

They should give a clear ringing sound when struck; and when broken across, they should show a fine, compact, uniform texture, free from air bubbles, cracks, and pebbles.

825. **Size and Weight.**—There is no standard size in the United States, but the usual size is $8\frac{1}{4}''$ by $4''$ by $2\frac{1}{4}''$. Because of the shrinkage in burning, the size is not exactly uniform. Pressed brick weigh about 150 pounds per cubic foot, hard brick about 125 pounds, and inferior soft brick about 100 pounds per cubic foot. Common brick average about $4\frac{1}{2}$ pounds each.

Special Forms of Brick and Clay Products.

826. **Paving-brick.**—Paving-brick are used in street pavements, stable floors, etc. They are made of clay in the form of shale, which has a higher percentage of flux than common brick-clay, and are burned at a higher temperature than common brick, thus making

a compact semi-vitrified brick (paving-brick). A paving-brick should be hard and tough, and should absorb little water even after its surface is worn off.

827. **Fire-brick.**—Fire-brick are made of refractory clay which contains no lime or alkaline matter, and remain unchanged by a degree of heat that would vitrify and destroy common brick. They are baked rather than burnt, and their quality depends upon the fineness to which the clay has been ground and the degree of heat used in making them.

828. **Terra-cotta.**—Terra-cotta is a specially prepared clay which has been molded into ornamental forms and baked. It is made of different varieties of selected clays, mixed together with ingredients which make the mixture slightly fusible, formed in plaster molds, and then baked.

829. **Tiles** are a variety of brick, and from their various uses are divided into four classes, viz.: *building, roofing, paving,* and *draining* tiles.

830. **Glazed and Enameled Brick.**—These are special varieties of brick with a glazed or enameled surface, which may be made any desired color.

831. **Vitrified Sewer-pipe and Sewer-blocks.**—Sewer-pipe is made of selected clay; molded by machinery, dried, placed in a kiln, and gradually exposed to a high heat. At the proper temperature, coarse salt is thrown on the fire, forming a salt vapor which combines with the silica of the clay and forms a soda-salt (glass-like) coating on the sewer-pipe. Sewer-blocks are similar to sewer-pipes; they are used to replace the bottom courses in a brick sewer.

LIME.

832. Lime is made by burning limestone to about 900° C. so as to drive off the carbon dioxide. The chemical reaction is

$$CaCO_3 \text{ (limestone)} + \text{heat} = CaO \text{ (lime)} + CO_2$$

The lime (CaO) will take up water, thus

$$CaO + H_2O = Ca(OH)_2 \text{ (slaked lime)}$$

This is the form in which it is used. Water and sand are added to the lime, thus forming a *mortar.* This is used in engineering construction. On being exposed to the air, CO_2 is absorbed from the air, and the water evaporates; so that the final uniting material in the structure is again limestone, practically as in the

beginning except now it has sand in it and has been worked into suitable position and shape, and also acts to unite the adjacent materials. Chemically, the action is

$$Ca(OH)_2 + CO_2 = CaCO_3 \text{ (limestone)} + H_2O \text{ (evaporated)}$$

Manufacture.

833. Although it is possible to make lime simply by burning limestone in the open air, it is usually made in a special receptacle,

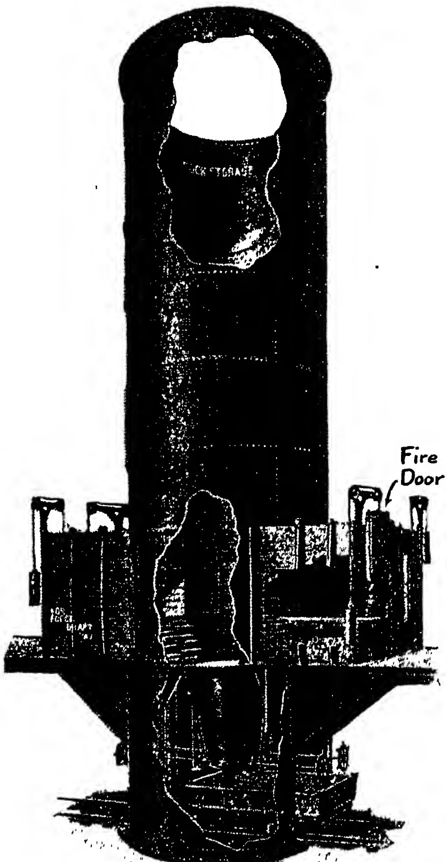


Fig. 833 A. Keystone Separate-feed Lime Kiln.

called a *kiln* (Fig. 833A), which is a large cylindrical chimney ordinarily lined with fire-brick. There are two methods of manufacture, the intermittent and the continuous.

In the intermittent process, a complete charge is burned at one time, and all of it is removed from the kiln before a new charge is added. The fire is at the bottom of the kiln, and is usually separated from the limestone by an arch of brick.

In the continuous process, there are two forms: (1) the fuel is placed near the bottom and around the circumference of the limestone chamber (Fig. 833A) so that the limestone is changed to lime as it passes by and is burned; (2) alternate layers of fuel and limestone are placed in the chamber, and the fire at the bottom passes successively from one layer of the fuel to the other, burning the limestone so that it is lime when withdrawn at the bottom.

Properties.

834. When water is added to lime, the change from CaO to $Ca(OH)_2$ is accompanied by great heat. Therefore, lime must be stored in protected places, as fires have often been caused by the slaking .of lime due to the presence of water. Lime will often absorb water from the air and slake itself, though the moisture is absorbed so gradually that very great heat does not often result.

835. Lime in storage will, while absorbing moisture from the air, at the same time absorb CO_2, so that the limestone is again formed. In a limited time this absorption will not extend more than a few inches from the surface; therefore, it is possible to use the greater part of the lime after throwing away the hardened surface.

836. The process in construction work of reformation of limestone from the $Ca(OH)_2$ and sand (mortar) is slow, as it is a continuous process of absorption of CO_2 from the air and evaporation of water; and it becomes slower and slower as the outer parts become stone and there is less facility for evaporation and absorption. For mortar placed between brick, it may take years to change the central portions into stone.

837. **Hydraulic lime** is made from certain varieties of silicious and argillaceous limestone. It has qualities very much like those of cement, as it hardens under water, and hardens throughout, and not simply gradually from the surface inward.

CEMENT.

838. Cement is made by burning certain natural or artificial combinations of calcium carbonate with alumina, magnesia, and silica; iron and other elements usually being present in small

quantities. Cement differs in manufacture from lime in its constituents, the temperature of burning, and in that it must be ground to a powder as it will not break up of itself under action of the air. Its vital difference, as compared with lime, is that it will set under water. It absorbs water less quickly from the air.

839. Cements are classified as: natural cement, Portland cement, and slag cement.

Natural cement is obtained by burning a natural rock which has the proper ingredients in approximately the correct proportions. The temperature is about 1200° C., which is below that of incipient fusion. Natural cement is also called quick-setting, light, American, and Rosendale.

Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum. The amount of Portlant cement used each year is 100 times that of both the other classes, being about 140,000,000 barrels (1923). Portland cement is next to timber and steel in importance as a structural material. It is by far the most important of all masonry materials; it is used in monolithic concrete, in foundations, footings, piers, abutments, dams, retaining walls, etc., also in reinforced concrete in walls, floors, roofs, bridges, tunnels, subways, highways, etc.

Slag cement is obtained by mixing proper proportions of slaked lime and granulated blast-furnace slag; the mixture is not burned. This cement will set under water. It is also called *puzzolan* cement, from the town of Puzzuoli near Naples, Italy, where it was first discovered as puzzolana, a naturally burned mixture of volcanic origin.

Manufacture of Portland Cement.

840. **Mixing.**—American made Portland cement has an average chemical composition by weight of

CaO	62.0%
SiO_2	22.0%
Al_2O_3	8.0%
Fe_2O_3	2.5%
MgO	3.0%
SO_3	1.5%
Miscellaneous	1.0%
	100.0%

841. Fig. 841A shows the complete process of manufacture. For mixing, it is seen that the different materials are (1) dried in the drying kilns, (2) pulverized in the ball mills, (3) weighed to

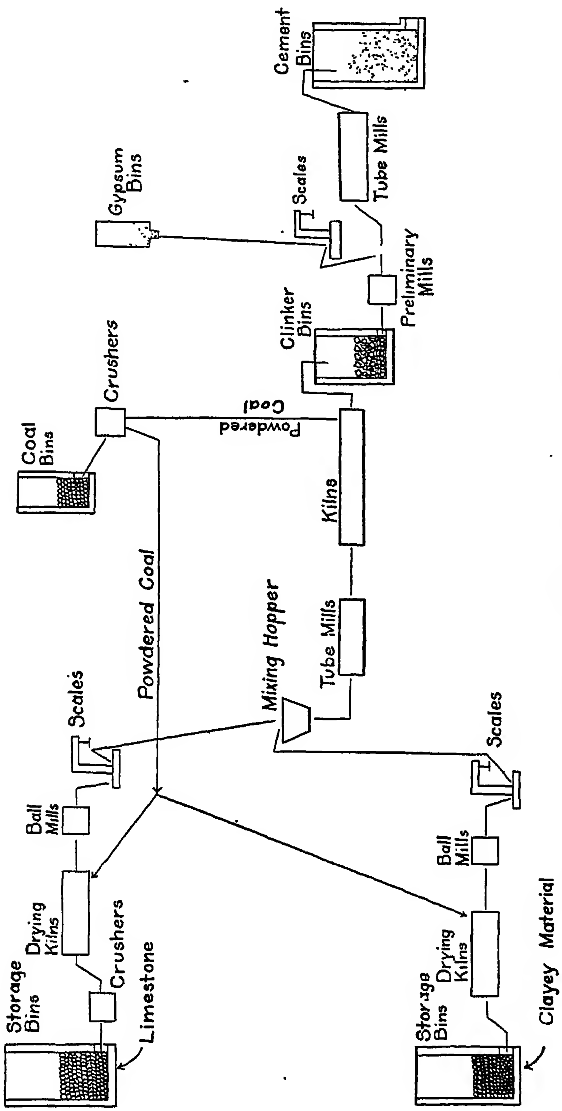


Fig. 841 A. Process of Manufacture of Portland Cement.

the proper proportions in the weighing scales, and then put together in the mixing hopper in these proper proportions. The mixing hopper may use the wet process whereby the mixture is

turned out in a wet thoroughly-mixed paste, or it may use the dry process whereby the ground materials are thoroughly mixed together so as to produce an intimate blending. From the mixer, the materials pass to the tube mills, where the mixture is ground to a fine powder.

842. **Calcination.**—The powder is burned to a clinker in the kiln, which is a rotary tubular furnace, lined with fire brick, and arranged to rotate on rollers. The furnace is not horizontal but has a slope of about 5%, so that the material fed in at the high end is gradually worked to the bottom and burned on the way.

843. **Grinding.**—The hot clinker is cooled in storage bins. Then it is ground into small pieces in the preliminary mills. Next, 2% to 3% by weight of gypsum (calcium sulphate) is added in order to strengthen the cement by making it slow-setting. Finally, the mixture is ground to powder in tube mills similar to those which ground the mixture for the kiln. After this last grinding, the cement is put into storage bins, ready for placing in sacks or barrels.

Properties.

844. The standard specifications for concrete and reinforced concrete (1924) of The American Society for Testing Materials require the following tests of Portlant cement: *chemical analysis, soundness, fineness, tensile strength, specific gravity,* and *time of setting.*

Chemical Analysis.—The specifications state that the following limits shall not be exceeded:

Loss on ignition, per cent	4.00
Insoluble residue, (HCl is principal solvent), per cent	0.85
Sulphuric Anhydride (SO_3), per cent	2.00
Magnesia (MgO), per cent	5.00

Soundness.—Unsoundness is due to presence of free lime which expands on contact with water and causes disintegration. This is tested by making on a glass plate a pat of neat cement, which is a small cone or spherical segment 3″ in diameter and ½″ thick at the center, and exposing it to moist air for 24 hours. The pat is then placed 1″ above water which is gradually raised to the boiling point, and kept there for 5 hours. The pats should remain attached to the

plates, and should show no signs of cracking or expanding or otherwise disintegrating.

Fineness.—The adhesive power of cement depends upon its fineness. The specifications require that 78% of Portland cement shall pass through a No. 200 sieve which has 200 wires to the inch, each wire being .002″ in diameter, thus making a sieve with 40,000 openings per square inch, each hole being about .0029″ square.

Tensile Strength.—For the purpose of making this test, a cement mortar of 1 part cement and 3 parts standard sand is molded into briquettes (Fig. 844A) with a cross-section of 1 square inch at

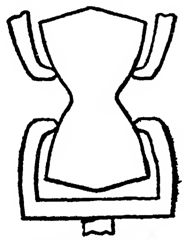


Fig. 844 A

the smallest section. The briquettes must at the end of 1 day in moist air and 6 days in water, have a tensile strength of 200 pounds per square inch, and at the end of 1 day in moist air and 27 days in water, have a tensile strength of 300 pounds per square inch.

Specific Gravity.—This test is required for the purpose of determining whether a Portland cement has been adulterated, as adulterating materials have a less specific gravity than pure Portland cement. The specific gravity must be not less than 3.10.

Time of Setting.—Cement must not set either too rapidly or too slowly. Therefore, two limits are prescribed, known, respectively, as the *initial set* and the *final set.* The initial set is attained when a neat cement paste will just bear without indentation a wire 1/12″ in diameter, supporting a weight of $\frac{1}{4}$ pound; the final set is attained when the paste will bear without indentation a wire 1/24″ in diameter, supporting a weight of 1 pound. The specifications require not less than 60 minutes for the initial set, and not more than 10 hours for the final set.

SAND, GRAVEL, AND BROKEN STONE.

845. **Sand** should consist of clean, hard, uncoated grains free from impurities, such as mud and organic matter. Sand from pits

is usually sharp and clean; sand from rivers and lakes and the sea is usually not sharp and often not clean. Sand is principally used in making mortar and concrete.

846. **Gravel** is an aggregate of small rounded stones having a diameter of about $\frac{1}{4}''$ to $1\frac{1}{2}''$. It is employed principally in making concrete; also, somewhat, in making roads. Pit gravel is generally better than river gravel.

847. **Broken Stone** is obtained by crushing stone to a size usually not greater than 2″. When needed in large quantities, it is broken by a stone crusher, and passed through an inclined revolving screen with holes of the proper size. The tougher the stone, the better the product. Broken stone is used in concrete and in making roads.

MORTAR.

848. **Mortar** is made by mixing water, sand, and slaked lime or cement. In making lime mortar, water is first added to the lime to slake it; that is, make it into $Ca(OH)_2$. In making cement mortar, it is not necessary to add water first; all the ingredients may be mixed at one time. The proportions are usually about 1 part of lime or cement to 2 or 3 of sand. The binding material is the lime or cement; the sand is simply an inert material which fills voids and makes the mortar cheaper without materially weakening it.

Mortar is used to bind stone together, as in walls. Also it is used as surfacing for sidewalks and for the outside of concrete monoliths and exposed joints.

BITUMINOUS MATERIALS.

849. The most important single bituminous material is asphalt, which is native bitumen. It is found in stone and in liquid form, but the most important asphalts are the Bermudez and Trinidad asphalts which are in the form of pitch and are about 90% pure bitumen.

Bituminous materials are used in mortars to surface roads; as binders to cement the crushed rock of roads; and as road surfacing in the form of crushed asphaltic rock which is rolled and binds itself together. Also, since the bituminous materials are waterproof, they are used in coating water tanks and reservoirs.

CONCRETE.

850. **Concrete** is artificial stone produced by mixing cement and water with materials such as sand and broken stone. Concrete

weighs about 150 pounds per cubic foot. When concrete is strengthened by the addition of steel as explained later, it is called *reinforced concrete.* Plain concrete is used when only compressive loads are carried, as in walls, arches, dams, roads, and sidewalks.

The quality of concrete depends upon:

(1) Quality of ingredients.
(2) Correct proportioning of ingredients.
(3) Mixing.
(4) Proper protection of concrete during hardening.

Ingredients of Concrete.

851. The inert materials used in concrete are classed as *fine aggregate* and *coarse aggregate.* A fine aggregate is one whose particles pass through a screen with ¼″ openings. A coarse aggregate is one whose particles are larger than ¼″ in diameter (and usually less than 1½″ in diameter). Ordinarily, sand is the fine aggregate used in concrete. Gravel, broken stone, cinders, slag, burned bricks, and similar materials are used as coarse aggregates. The maximum size of the coarse aggregate varies with the structure; in foundation work the maximum size should not exceed 3″, in reinforced concrete work it should not exceed 1½″. Concrete can be only as strong as the aggregate; therefore, soft broken stone, as broken limestone, is unsuitable as a coarse aggregate in concrete.

852. The presence of clay or vegetable matter is objectionable in an aggregate. Clay coats the particles of sand and prevents the cement from getting in contact with the grains and thus decreases the strength of the concrete. Vegetable matter retards or prevents hardening. One half of one per cent of vegetable matter may reduce the strength of concrete 40%. Sand suspected of containing organic matter should be carefully tested before being used. When it is impossible to obtain aggregates free from undesirable materials, they should be thoroughly washed before any are mixed with cement.

Proportioning of Ingredients.

853. The strength and the cost of a concrete structure are affected by the proportioning of the ingredients. The kind of strength required will depend upon the nature of the engineering structure. Walls should resist compressive stresses, dams should resist percolation of water through them, and roads should resist abrasion.

Generally speaking, the densest concrete is the strongest against compression, abrasion, and passage of water.

The densest mixture, that is, the one with the least void space, is formed by a mixture of such proportions that the finer particles just fill the interstitial space of the size next larger. Density of concrete means the ratio of the absolute volume of the solid particles of cement and aggregate to the volume of the resulting concrete, and varies from 0.7 to 0.9. For a given portion of cement, well graded aggregates give a denser concrete than an aggregate in which the particles are more nearly the same size. In massive concrete construction, such as break-waters and dams, large rocks, when available, are thrown in the concrete, thereby making less cement necessary.

854. In proportioning ingredients, the following points should be remembered:

a. That the greatest density and, therefore, the greatest strength and impermeability, is secured when the cement fills the voids in the sand and the cement mortar fills the voids in the coarse aggregate.

b. That for a given aggregate the density will vary directly with the amount of cement used, up to the point where all of the voids are filled. If more cement is added, the density of the concrete decreases.

855. Cement is the most expensive of the concrete ingredients. It is, therefore, the duty of the engineer to so proportion his aggregate that the cement needed is a minimum. The ideal mixture is one giving maximum density as described above. In practice, however, the cement used is slightly greater than that needed to exactly fill the voids in the sand, and the volume of cement mortar used is greater than the voids in the coarse aggregate. The percentage of voids varies greatly, depending upon the grading of the materials. The voids in coarse aggregate graded from ¼″ to 1½″ range from 35% to 50% of the volume of the aggregate.

For common sand, the voids range from 28% to 40%. A well-graded mixture of fine and coarse aggregate will have voids as low as 12%. The voids of small particles of nearly equal size total more than those of large particles of equal size.

856. The volume of voids in material is determined by filling a vessel, whose cubic contents are known; then adding water until the water overflows the vessel. The volume of the water poured into the vessel is the volume of the voids in the material when dry

and loose. The volume of the voids will be less when concrete is rammed in place.

857. The following methods are used to proportion the ingredients of concrete:

a. By voids in the aggregate.
b. By trial mixture.
c. By mechanical analysis.
d. By arbitrarily selected volumes.

Proportioning by Voids in the Aggregate.—Samples of fine and coarse aggregates are obtained in the condition they will occur in the work. The volume of the voids is determined for both the fine and coarse aggregates. The volume of the fine aggregate should be equal to the volume of the voids in the coarse aggregate plus 5% to 10%. This excess is necessary due to the fact that particles of sand separate the coarse aggregate. The volume of cement to be used must be slightly in excess of the volume of voids in the fine aggregate. This method of proportioning is infrequently used because the voids in the aggregate determined by experiments, may vary greatly by reason of differences in compactness, due to differences in methods of handling the materials.

Proportioning by Trial Mixtures.—This method of proportioning is based on the assumption that the best and strongest mixture is the densest, the ratio of cement to the total aggregate being fixed. A trial mixture is made by arbitrarily selecting a proportion of fine aggregate to coarse aggregate. The cement, water, sand and coarse aggregate are weighed separately, then mixed. The mixture is placed in a cylinder and tamped, and the height of the surface noted. The concrete is removed, the cylinder cleaned and another proportion of fine aggregate to coarse aggregate selected. The ingredients are weighed and mixed, placed in the cylinder, and the height again measured. This is repeated until a mixture is found which gives the smallest height. Since the total weight is the same for each batch placed in the cylinder, this mixture will be the densest one. Care must be taken to see that the proportion of water is such that the resulting dense concrete will be a smooth working mixture which can be easily placed in forms. This method of proportioning gives accurate results when carefully carried out.

Proportioning by Mechanical Analysis. — This is a scientific method of proportioning the material available. The aggregate is separated into its various sizes by a nest of sieves, the sieve with

the largest opening being at the top of the nest and the one with the smallest at the bottom. The material held on each sieve is weighed and the percentage by weight which passes each sieve is determined. With this information, a curve is plotted, the abscissae being the diameters of the particles in inches, and the ordinates, the percentage by weight which passes each sieve. From this curve the engineer can determine what sizes should be added or omitted in order to secure a well-graded material. By adding or screening out certain sizes of particles, he is able to obtain nearly ideal materials so far as grading is concerned.

Proportioning by Arbitrarily Selected Volumes.—In general, the voids in the coarse aggregate are somewhat less than half of the total volume. It has become common practice in engineering to use one-half as much sand as the volume of broken stone. The cement proportion is varied with the strength required. A 1:2:4 mixture is 1 sack (1 cu. ft.) of Portland cement, 2 cu. ft. of sand, and 4 cu. ft. of coarse aggregate. If the voids in this coarse aggregate are greater than usual, the amount of sand is increased and the proportions are such as 1:2:3 or 1:2½:4. In case the volume of voids in coarse aggregate is low, such proportions as 1:2:5 or 1:3:7 are used. *The method of proportioning by arbitrarily selected volumes is more widely used than any other method.* Where high compressive stresses must be carried, as in columns, a 1:1½:3 mixture is used; in building construction for slabs, tee beams, etc., a 1:2:4 mixture is used; in larger masses of concrete such as dams, where stresses are not high, a 1:3:6 mixture is commonly used.

858. As stated above, arbitrarily selected volumes is the method in general use. The tables below (taken from a pamphlet issued by the Koehring Co., manufacturers of concrete equipment), give usual mixtures:

Table 858A.—Recommended Mixtures and Maximum Aggregate Sizes.

Mixture and Character of Work	Recommended Maximum Size of Aggregate in Inches
1:1:1 Mixture for	
The wearing course of two-course floors subject to heavy trucking, such as occurs in factories, warehouses, on loading platforms, etc.	½
1:2:3 Mixture for	
Reinforced concrete roof slabs	1
One-course concrete road, street, and alley pavements	3
One-course walks and barnyard pavements	1½
One-course concrete floors	1½
Fence posts	¾
Sills and lintels without mortar surface	¾
Watering troughs and tanks	1
Reinforced concrete columns	1
Mine columns	¾
Construction subjected to water pressure, such as reservoirs, swimming pools, storage tanks, cisterns, elevator pits, vats, etc	1
1:2:4 Mixture for	
Reinforced concrete walls, floor beams, columns, and other concrete members designed in combination with steel reinforcing	1
Concrete for the arch ring of arch bridges and culverts	1½
Foundations for engines causing heavy loading, impact and vibration	3
Concrete work in general subject to vibration	1½
Reinforced concrete sewer pipe	½
1:2½:4 Mixture for	
Silo walls, grain bins, coal bins, elevators and similar structures	1½
Building walls above foundation, when stucco finish will not be applied	1½
Walls of pits or basements, exposed to moisture	1½
Manure pits	1½
Dipping vats, hog wallows	1
Backing of concrete block	¾
Base of two-course road, street and alley pavements	3
1:2½:5 Mixture for	
Walls above ground which are to have stucco finish	1½
Base of two-course walks, feeding floors	1½
Bridge abutments and wing walls, culverts, dams, small retaining walls, when not reinforced	2
Basement walls and foundations where water tightness is not essential	2
Foundation for small engines	2
1:3:6 Mixture for	
Mass concrete—large retaining walls, heavy foundations and footings	3
1:1½ Mixture for	
Inside finish of water tanks, silos, and bin walls, where required, and for facing walls below ground when necessary to afford additional protection against the entrance of moisture	To pass through No. 8 Screen.
Back plastering of gravity retaining walls	To pass through No. 8 Screen.

Table 858A —Continued.

Mixture and Character of Work	Recommended Maximum Size of Aggregate in Inches
1:2 Mixture for	
Facing block and similar concrete products	¼
Wearing course of two-course walks, floors subjected only to light loads, barnyard pavements, etc.	¼
1:2½ Mixture for	
Scratch coat of exterior plaster (cement and stucco)	To pass through No. 8 Screen.
Fence posts when coarse aggregate is not used	¼
1:3 Mixture for	
Intermediate and finish stucco coats	To pass though No. 8 Screren.
Concrete block when coarse aggregate is not used	¼
Concrete brick	¼
Concrete drain tile and pipe when coarse aggregate is not used	¼
Ornamental concrete products	¼

Table 858B.—Concrete, per Sack of Cement.

Mixtures			Materials			Concrete Cu. Ft.	
Cement	Sand	Pebbles or Stone	Cement in Sacks	Sand Cu. Ft.	Pebbles or Stone Cu. Ft.	Mortar	Concrete
1	1.5		1	1.5		1.75	
1	2		1	2		2.1	
1	3		1	3		2.8	
1	1.5	3	1	1.5	3		3.5
1	2	3	1	2	3		3.9
1	2	4	1	2	4		4.5
1	2.5	4	1	2.5	4		4.8
1	2.5	5	1	2.5	5		5.4
1	3	5	1	3	5		5.8

Table 858C.—Materials, per Cubic Yard of Concrete.

Mixtures.—Quantities of Materials to make 1 cubic yard of Concrete						
Cement	Sand	Pebbles or Stone	Cement in Sacks	Sand cu. ft.	Stones or Pebbles cu. ft.	Result
1	1.5		15.5	23.3		1 cu. yd.
1	2		12.8	25.6		"
1	3		9.6	28.8		"
1	1.5	3	7.6	11.4	22.8	"
1	2	3	7	14	21	"
1	2	4	6	12	24	"
1	2.5	4	5.6	14	22.4	"
1	2.5	5	5	12.5	25	"
1	3	5	4.6	13.8	23	"
1	3	6	4.2	12.6	25.2	"

To illustrate the use of Table 858C, let it be required to find how much cement, sand, and pebbles will be needed to build a one-course floor 30′ by 24′ 5″ thick.

Multiplying the area (30 by 24) by the thickness in feet gives 300 cubic feet, and dividing this by 27 gives 11-1/9 cubic yards as the required volume of concrete. A one-course floor should be of 1 : 2 : 3 mixture (see Table 858A). Table 858C shows that each cubic yard of this mixture requires 7 sacks of cement, 14 cubic feet of sand, and 21 cubic feet of gravel or stone. Multiplying these quantities by the number of cubic yards needed (11-1/9) gives the quantities required (eliminating fractions) as 78 sacks of cement, 156 cubic feet of sand, and 233 cubic feet of pebbles or stone. As there are four sacks of cement in a barrel, and 27 cubic feet of sand or pebbles in a cubic yard, we shall need a little less than 20 barrels of cement, 6 cubic yards of sand and 9 cubic yards of pebbles or stone.

Table 858D.—Quantity of Cement Required per Cubic Foot and per Cubic Yard of Concrete for Various Mixtures in Terms of Sacks and Barrels.

1 Cu. Ft. Concrete	Sacks of Cement	1 Cu. Yd. Concrete	Bbl. of Cement
1:1:1	.5404	1:1:1	3.375
1:1½:3	.2808	1:1½:3	1.895
1:2:4	.2220	1:2:4	1.498
1:2½:5	.1848	1:2½:5	1.247
1:3:6	.1570	1:3:6	1.060

Table 858E.—Materials Required for 100 Square Feet of Surface for Varying Thickness of Course.

Thickness Mix	1 in.			2 in.			4 in.			5 in.		
	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.
1:2	3.9	0.29	....	7.9	0.58	....	....	....	....	....	....	....
1:1:1	4.2	0.15	0.15	8.3	0.31	0.31	....	....	....	....	....	....
1:1:1½	3.7	0.14	0.20	7.3	0.27	0.41	....	....	....	....	....	....
1:1½:2½	2.6	0.14	0.24	5.1	0.28	0.47	....	....	....	....	....	....
1:1½:3	....	....	....	....	....	....	9.4	0.52	1.04	11.7	0.65	1.30
1:2:3	....	....	....	....	....	....	8.6	0.64	0.95	10.8	0.80	1.19
1:2:4	....	....	....	....	....	....	7.4	0.55	1.10	9.3	0.69	1.37
1:2½:4	....	....	....	....	....	....	6.9	0.64	1.02	8.6	0.80	1.23
1:2½:5	....	....	....	....	....	....	6.2	0.57	1.14	7.7	0.72	1.47
1:3:6	....	....	....	....	....	....	5.2	0.58	1.16	6.5	0.73	1.45

Thickness Mix	6 in.			7 in.			8 in.			9 in.		
	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.
1:1½:3	14.0	0.78	1.56	16.4	0.91	1.82	18.7	1.04	2.08	21.1	1.17	2.34
1:2:3	12.9	0.95	1.43	15.0	1.11	1.67	17.2	1.27	1.90	19.3	1.43	2.14
1:2:4	11.1	0.82	1.64	12.9	0.96	1.92	14.8	1.10	2.19	16.7	1.23	2.47
1:2½:4	10.3	0.95	1.53	12.0	1.11	1.78	13.8	1.27	2.03	15.5	1.43	2.29
1:2½:5	9.2	0.86	1.72	10.8	1.00	2.00	12.3	1.14	2.29	13.9	1.29	2.57
1:3:6	7.9	0.87	1.74	9.2	1.02	2.03	10.5	1.16	2.32	11.8	1.31	2.61

Table 858F.—Materials Required for 100 Square Feet of Sidewalk and Floors for Varying Thickness of Course.

CONCRETE BASE.

Proportions	1:2:3			1:2:4			1:2½:4			1:2½:5		
Thickness	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.	C. Sks.	Sd. Cu. Yds.	St. Cu. Yds.
2½″	5.4	0.40	0.60	4.6	0.34	0.68	4.3	0.40	0.63	3.9	0.36	0.72
3″	6.5	0.48	0.72	5.6	0.41	0.82	5.2	0.48	0.77	4.6	0.43	0.86
3½″	7.5	0.56	0.84	6.5	0.48	0.96	6.0	0.56	0.89	5.4	0.50	1.00
4″	8.6	0.64	0.95	7.4	0.55	1.10	6.9	0.64	1.02	6.2	0.57	1.14
4½″	9.7	0.72	1.07	8.3	0.62	1.23	7.7	0.72	1.14	6.9	0.64	1.28
5″	10.8	0.80	1.19	9.3	0.69	1.37	8.6	0.80	1.27	7.7	0.71	1.43
5½″	11.8	0.88	1.31	10.2	0.76	1.50	9.5	0.87	1.40	8.5	0.78	1.57
6″	12.9	0.96	1.43	11.1	0.82	1.64	10.3	0.95	1.53	9.2	0.86	1.72

WEARING COURSE.

Thickness Inches	1:1		1:1½		1:2	
	Cement Sacks	Sand Cu. Yds.	Cement Sacks	Sand Cu. Yds.	Cement Sacks	Sand Cu. Yds.
½	3.0	0.11	2.4	0.13	2.0	0.15
¾	4.5	0.16	3.6	0.19	2.9	0.22
1	6.0	0.22	4.8	0.26	3.9	0.29
1¼	7.5	0.27	6.0	0.33	4.9	0.36
1½	9.0	0.33	7.2	0.40	5.9	0.43
1¾	10.5	0.39	8.4	0.46	6.9	0.50
2	12.0	0.45	9.6	0.53	7.9	0.58

Mixing Concrete.

861. Concrete may be mixed either by hand or machine. The method to be used on any particular work depends upon the amount of concrete to be mixed, the availability of labor, and the possibility of obtaining a concrete mixer. Machine-mixed concrete is usually cheaper and of a more uniform quality than hand-mixed concrete.

Up to a certain point, the strength of concrete increases as the amount of water is decreased. However, it is necessary to sacrifice strength and use more water in order to get a workable concrete, one that will take the shape of the forms and completely fill them.

862. The *slump test* is employed frequently to determine the proper consistency of concrete; the concrete is poured in at the top of a tapered form 12″ high, 4″ in diameter at the top, and 8″ in diameter at the bottom. The form is raised immediately, and the concrete should not slump over 3″, if it is to be used in pavement floors and large foundations which permit tamping; and not more than 7″ if it is to be used in thin walls where reinforcing is used, or in thin water-tight structures. In general, concrete should have a jelly-like consistency; a very watery concrete should never be used.

863. **Hand Mixing.**—Concrete is made by hand on a platform large enough to conveniently carry the largest batch to be mixed. First the sand is measured and spread in an even layer on the mixing platform. Next, the cement is measured and spread over the layer of sand; and by means of shovels, this mixture is turned over at least twice. The coarse aggregate is then measured and spread over the cement-sand mixture, and all the ingredients are turned over at least two more times. A hole is made in the center, and water enough to produce a mushy mixture (usually between 8% and 15% of water) is gradually added while men turn over the mixture with shovels. Mixing continues until the cement, sand, and coarse aggregate have been uniformly mixed.

864. **Machine Mixing.**—If mixed by machine, the proper proportions of aggregate, sand, and cement are dropped from an elevated platform into a revolving mixer, into which the water may be introduced through a hollow axle. The amount of water and the number of revolutions necessary for complete mixing are determined by experiment. When the concrete is thoroughly mixed, it is dropped into cars or carts which are run beneath the mixer. This method is employed on large works.

Placing Concrete.

865. As the hardening action resulting from the combination of cement and water begins very soon after a batch of concrete is mixed, concrete should be deposited as quickly as possible after mixing, at most within 30 minutes. For small structures, the concrete is carried in cars or in open chutes. When chutes are used, the concrete is hoisted from the mixer to the top of a tower and loaded in a hopper from which it is fed to the chute. Care must be taken when concrete is placed by means of chutes that the mixture is not too wet, as the coarse aggregate in a wet mixture flows faster than the cement, sand, and water, and the resulting concrete is not good.

Concrete is placed in layers 6″ to 12″ in thickness, tamped to remove air bubbles and to produce a dense mixture. If concrete is placed on or against concrete already set, the hardened concrete surface, which in very wet concrete construction becomes covered with a chalk-like substance called "laitance", is cleaned and roughened, the surface wetted, and a coating of 1:1 cement mortar put on. This insures proper bond between the two concrete surfaces. In mass concrete which is to stand compressive stresses only, it is necessary only to clean the old surface.

866. **Forms.**—In order to obtain satisfactory concrete work, the forms (Figs. 866A and B) should be durable and rigid and so well braced that bulging or twisting cannot occur. The joints should

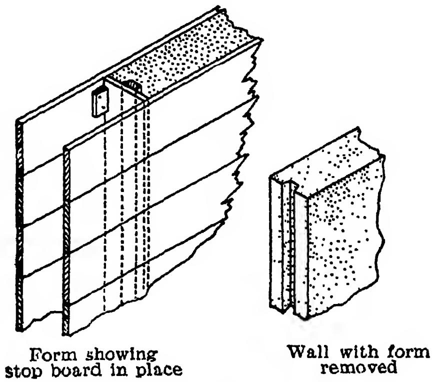


Fig. 866 A

be made tight enough to prevent any material leakage of the liquid mass, as such leakage allows escape of mortar, thereby decreasing the strength and marring the appearance of the finished work.

Forms should have sufficient strength to support properly the loads which they are called upon to carry.

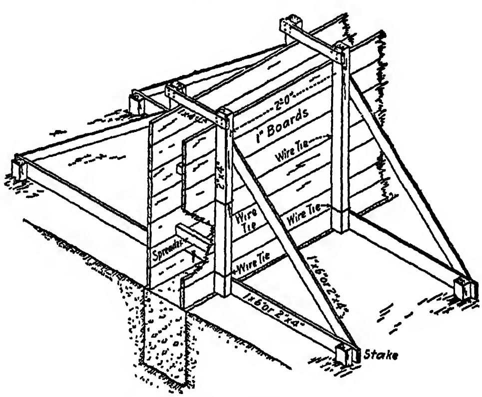


Fig. 866 B

The forms in which reinforced concrete beams are constructed are commonly of wood and are fastened together by wooden clamps and bolts. Steel concrete forms, when they can be used repeatedly, are economical. For circular columns, steel forms are generally used.

Protection of Concrete.

867. **During Hardening.**—Care must be taken after concrete is placed to see that is does not dry out rapidly. Normally, concrete gains strength with age; however, when it drys out too rapidly, it gains little with age. This is due to the fact that shrinkage cracks are produced, and the chemical composition of the concrete is poor because the cement is not completely hydrated. Complete hydration is secured by keeping the forms in place and the concrete surface wet for at least two weeks. Newly placed concrete is sometimes covered in dry warm atmosphere by wooden frames or canvas. After the initial set, the concrete is sometimes covered with damp earth or damp sand. Tests made at the University of Illinois on concrete cylinders showed that at the age of five years air-stored specimens were only about one-half as strong as specimens of like consistency stored in damp sand.

During Cold Weather.—Concrete should not be placed during cold weather unless means are taken to prevent freezing during the early hardening period. Freezing prevents proper hydration of the cement and impairs the strength of the concrete. When it is necessary to lay concrete during cold weather, the stone, sand, and water should be heated before mixing. After the concrete is placed the heat should be retained by covering the concrete with canvas, straw, burlap, or sawdust. In very cold weather, it may be necessary to keep the covered concrete warm by using artificial heat from steam coils or stoves.

868. **Fire Proofing Qualities.**—Concrete has a low conductivity of heat due to two causes: (1) the presence of combined water, and (2) porosity. The water of crystallization being chemically combined is not given off at the boiling point. A part of this water goes off at about 500° F., but the dehydration is not complete until 900° F. is reached. This vaporization of water at the surface absorbs heat and protects the interior. The layer of changed materials is then a poorer conductor of heat than before, so the process of dehydration goes on very slowly. The air spaces in concrete due to porosity also offer considerable resistance to the passage of heat, it being well known that air spaces are a most efficient protection against conduction. Experiments and large fires have shown that sharp corners of beams and columns are more susceptible to attack than wide, flat surfaces such as slabs.

Strength of Concrete.

869. **Compression.**—The strength of concrete depends upon density, age, quality of ingredients and amount of water used. The ultimate strength in compression in pounds per square inch of various mixtures 28 days old is shown in the table below. The test specimens were made in the form of cylinders 8″ in diameter and 16″ long. Portland cement was used.

Aggregate	Proportions: 1 : 2 : 4	1 : 3 : 6
Granite, trap rock	2200	1400
Gravel, hard limestone, hard sandstone	2000	1300
Soft limestone, soft sandstone	1500	1000
Cinders	600	400

Shearing Strength.—The shearing strength of concrete is about one-half the compressive strength. Experiments made at the University of Illinois on specimens 60 days old gave the following results:

	Proportions: 1 : 2 : 4	1 : 3 : 6
Ultimate shearing strength in pounds per sq. in.	1290	1090
Ultimate compressive strength in pounds per sq. in	2430	2290

However, the American Society for Testing Materials takes the allowable shearing stress for concrete as 2% of the ultimate compression, which gives very small allowable shearing stresses.

Tensile Strength.—Concrete, being a brittle material, has practically no strength in tension. In reinforced concrete design, it is assumed that concrete takes no tension, as explained in discussion of reinforced concrete. However, if concrete is unusually well made, its tensile strength is actually about 1/10 its compressive strength.

Effect of Age on Strength.—The strength of concrete increases with age. The relative amount of increase in strength of concrete, from 7 days to 6 months, for two common proportions of mixture, is shown approximately in the table which follows. (The table gives the average values of the compressive strength in pounds per square inch based on tests at the Watertown Arsenal of unusually good concrete):

	Proportions: 1 : 2 : 4	1 : 3 : 6
7 days	1565	1311
1 month	2399	2164
3 months	2896	2522
6 months	3826	3088

Effect of Water Content on Strength.—It has been shown by experiment that the strength of concrete increases with the amount of water used up to a certain proportion, and then decreases. The proper quantity of water to give maximum strength is used in the manufacture of artificial stone with special factory equipment. This is possible due to the fact that there are facilities for tamping and pressing. In average building and road construction, in order to have a workable mixture, more water is used than is required in an ideal mixture of maximum strength.

REINFORCED CONCRETE.

870. Reinforced concrete is concrete in which metal, preferably steel, is imbedded to increase its strength. When the steel in a structural member is designed to take all the stress, the concrete serving only to make the structure fire proof or to enhance its appearance, the member is not to be considered as composed of reinforced concrete, but rather of steel protected by concrete.

871. Owing to its small tensile and shearing strengths, simple concrete is not suitable for use in the construction of structural forms which are subjected to forces of flexure, as floor beams, slabs, girders, etc.; or in the construction of those subjected to tension only, as watertanks. For the same reasons, although it is fairly strong in compression, it is not suitable for use in long columns.

The requisite strength, in tension and compression, can, however, be secured by imbedding steel or wrought iron rods in concrete; these will assist the concrete in bearing the longitudinal and shearing stresses to which it is subjected. Simple concrete thus strengthened is called reinforced concrete.

872. Reinforced concrete makes a good structural material, under suitable conditions, because of the following qualities in the combination:

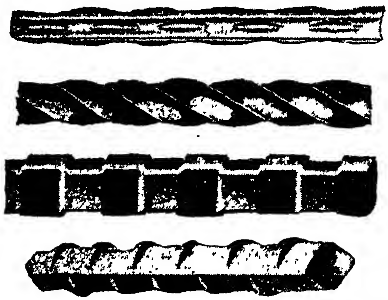
a. Cement mortar preserves steel; it forms an impervious coating.

b. Both concrete and steel have practically the same coefficient of expansion for changes of temperature.

c. Cement mortar adheres to the metal with such strength that considerable force is necessary to separate them.

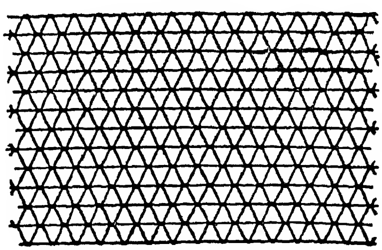
d. The combination is cheaper, and resists atmosphere and fire better than steel alone; it is stronger than concrete alone, and cracks less.

873. **Character of Reinforcement.**—The reinforcing steel in reinforced concrete construction is mostly in the form of rods or bars



Deformed bars
Fig. 873 A

of round or square cross-section. These vary in size from $\frac{1}{4}''$ to $1\frac{1}{2}''$ as a maximum size for heavy beams. Larger sizes are sometimes used for columns. There are various forms of special bars



Triangle mesh reinforcement
Fig. 873 B

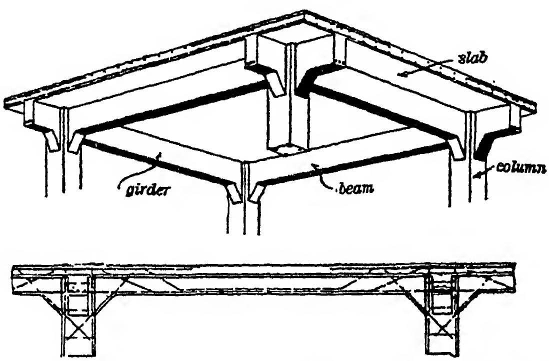
on the market. The Ransome bar is made by twisting cold square rods. Other forms are illustrated in Figs. 873A and B. Various forms of steel mesh are employed in floor slabs.

874. **Spacing of Bars.**—Reinforcing rods should be spaced at sufficient intervals to insure concrete enough between the rods so that full stress on the rods will not rupture the concrete between them. The lateral spacing of parallel bars should be not less than three diameters, or three times the side of a square bar. The distance from the side of the beam to the center of the nearest bar should be not less than 2 diameters. If rods are located in more than one horizontal layer, a clear space of not less than 1″ should be provided.

Below or above extreme layers of rods, the thickness of concrete should be in girders not less than 2″; in beams, 1½″; and in floor slabs, 1″.

Types of Reinforced Concrete Construction.

875. **Floor Slabs.**—Floor slabs are usually thin (about 4″ being an average thickness) and their span is usually limited to about 6′ to 8′, although larger spans may be used. The slab may be supported by steel beams or may be cast as part of the whole floor system as indicated in Fig. 875A. It is to be noticed that the re-

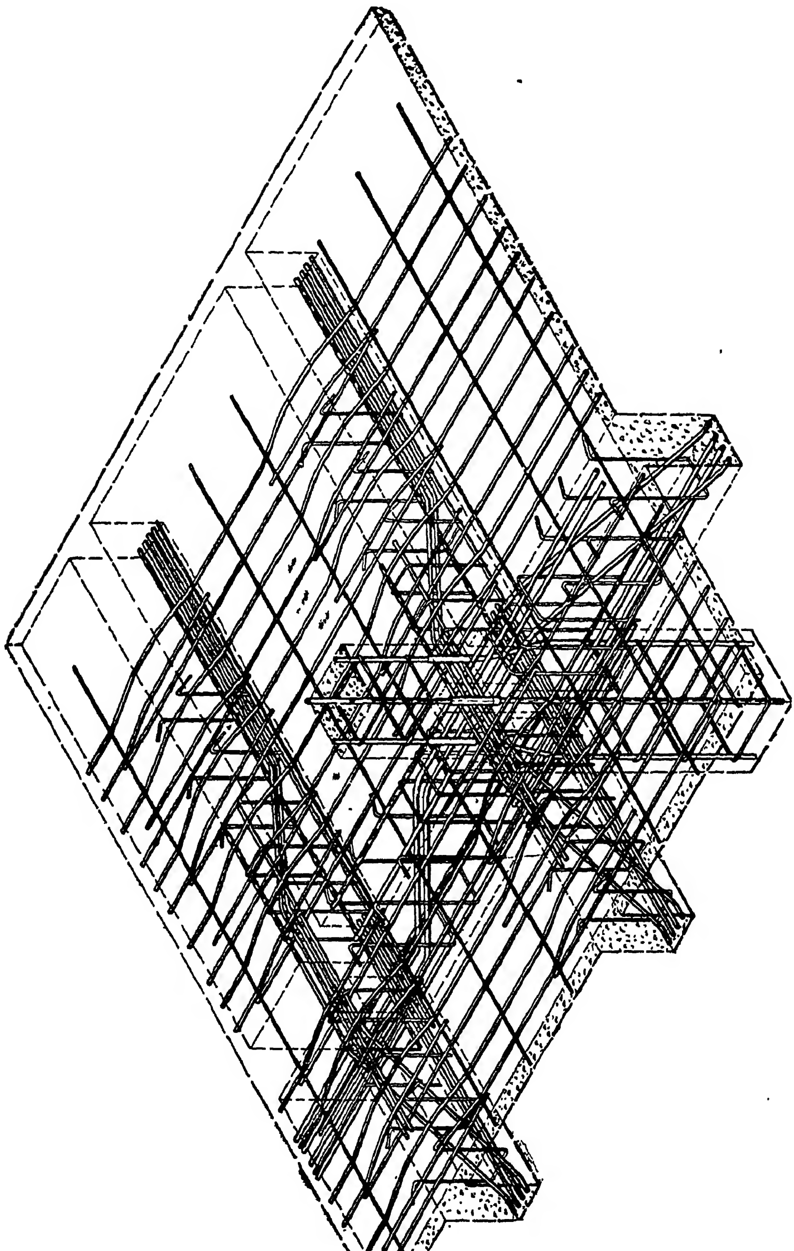


Flat slab, all concrete design, reinforced in both directions

Fig. 875 A

inforcing is not straight through the slab, being near the lower surface at the middle of the span and near the upper surface where it passes over the beam. Reinforced concrete slabs are essentially wide beams supported on girders, or in case of the so-called flat slab construction, directly on the tops of the columns. Slabs supported on all four edges, or on the tops of columns, are reinforced longitudinally and traversely (see Fig. 875B on next page).

876. **Beams.**—Beams support the slabs and are in turn usually supported by girders which carry the loads to the columns or walls. The beams and girders are reinforced by placing the rods near the lower surface, but since the stresses for which the rods are provided decrease toward the ends of the beam, it is possible to bend up some of the rods diagonally and in this way help to provide reinforcement against shearing stresses. When the beams or girders form part of a floor system, it is customary to carry the rods over into the adjoining beam or girder, thus causing the beam to act as a continuous beam.



Isometric view of design of floor system
Fig. 875 B

When a force is applied to a reinforced concrete beam, cracks develop on the tension side. These cracks do not become noticeable, even on very close examination, until a strain has developed corresponding to a tensile stress beyond the ultimate strength of concrete. The steel forces the concrete to elongate uniformly throughout, so that a crack will open up very slowly and remain invisible for some time.

A reinforced concrete beam for working loads is usually stressed on the tension side beyond the ultimate tensile strength of plain concrete, but enough steel is imbedded usually on the tension side so the full allowable compressive strength of the concrete can be utilized on the compressive side.

It would appear that the cracks on the tension side of the beam would expose the steel to disintegrating agencies; but, fortunately, the lime present in concrete has the property of absorbing carbon dioxide, thereby effectually protecting the steel when covered with even a thin film of cement mortar.

The formulas now in most general use for the design of reinforced concrete beams neglect entirely the tensile strength of the concrete.

Reinforced Concrete Design.

877. In addition to the nomenclature given in the introduction to this text, the following special nomenclature is used in reinforced concrete design in this text:

A_s = area of steel reinforcement.

b = breadth of beam.

D_n = depth to neutral axis.

D_s = depth to axis of steel reinforcement.

E_c = modulus of elasticity of concrete.

E_s = modulus of elasticity of steel.

f_c = unit compressive stress at extreme fiber of concrete.

f_s = unit tensile stress in steel.

l_c = lever arm of resisting couple.

l_s = spacing of stirrups.

M_c = resisting moment of concrete.

M_s = resisting moment of steel.

n = elasticity ratio $= \dfrac{E_s}{E_c}$.

p = per cent of steel = $\frac{A_s}{bD_s}$.
S_c = total compressive stress.
S_t = total tensile stress.
s_b = unit bond stress.

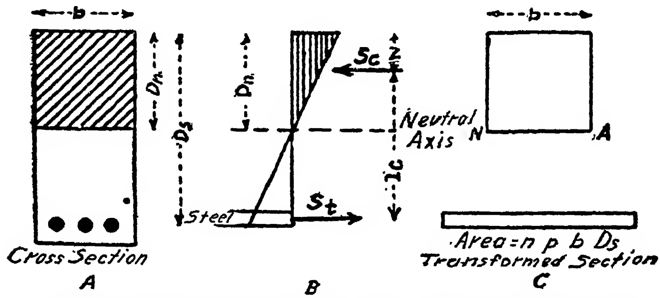
878. In reinforced concrete design, the following assumptions are made:

a. The adhesion of concrete to steel is perfect within the elastic limit of steel. This is necessary in order that the steel may be stressed to its elastic limit; otherwise, the concrete and steel will not act as one piece.
b. Both steel and concrete have no initial stress. In more complicated designs, the steel and concrete can be given an initial stress, but this is not considered.
c. Planes of cross-section before bending will remain planes after bending. This is the same assumption as made in paragraph 258 in deducing the equations of bending moments, and is the result of experiments.
d. The planes of cross-section have no motion of translation along the neutral fiber. This is based on experiments.
e. The concrete takes no tension. Actually, the concrete does take tension; but this is small, and the assumption that concrete does not take tension is an error on the safe side.
f. The modulus of elasticity of concrete is constant. If concrete could be made homogeneous, this would be correct; actually it is not correct, but the modulus of elasticity is assumed sufficiently low to make up for weaknesses in construction.

879. The steel is placed where it will receive the maximum unit fiber stress. In a beam, this is on the extreme surface fiber, but in order to obtain the proper adhesion and prevent rusting of the steel, it is necessary to place the steel a short distance within the beam. Also, since the area of the steel is small and the differences in the fiber stresses are very small, it is usual to take the area of the steel in single reinforcement as having the same fiber stress throughout and equal in intensity to that of its center fiber.

In accordance with this idea, it is usual to base calculations on a "transformed section" as shown in Fig. 880C, which is simply a section of the beam in which the concrete has no stress on the tensile side of the neutral fiber and an amount of concrete equivalent to the steel is concentrated theoretically in one point at the center of the steel reinforcement and takes its place.

880. **Position of Neutral Axis in a Beam.**—(Fig. 880.) Let it be required to find the position of the neutral axis in a rectangular beam with single reinforcement. In homogeneous beams, the neutral axis passes through the center of gravity. To use this prin-



Beam resting on end support with single reinforcement

Fig. 880 A

ciple in determining the position of the neutral axis of a reinforced concrete beam, it is necessary to consider the steel as replaced by a sufficient area of concrete (concentrated at the same distance from the neutral axis) to produce the same effect as the steel. Since the unit strain (elongation) is the same for the concrete and the steel, and since, from paragraph 204, the modulus of elasticity varies directly with the unit stress and inversely with the unit elongation, the area of concrete substituted will be equal to the area of the steel multiplied by the quotient of the modulus of elasticity of steel divided by the modulus of elasticity of concrete. Expressing this in an equation, we have

$$\text{Area of concrete substituted} = \frac{E_s A_s}{E_c} = nA_s = npbD_s. \qquad (880\text{A})$$

The ratio of the moduli of elasticity of steel and concrete is usually taken as 15, corresponding to values of 30,000,000 and 2,000,000 pounds per square inch.

Substituting the concrete for the steel, we now have a homogeneous beam and the neutral axis passes through the center of gravity. Knowing that the sum of the moments about the neutral axis must equal zero, we have

Compression area × lever arm = Tension area × lever arm (880B)

Expressing this in an equation and remembering that only the substituted area is to be considered in tension, we have

$$bD_n \frac{D_n}{2} = npbD_s\,(D_s - D_n) \qquad (880\text{C})$$

Thence, $$D_n = D_s(\sqrt{2pn+(pn)^2}-pn).$$

In this equation, it is necessary to know only the percentage (p) of steel, the ratio of moduli of elasticity of steel and concrete (n), and the dimensions of the beam (lD_s). By substitution and solving for D_n we obtain the distance of the neutral axis from the top.

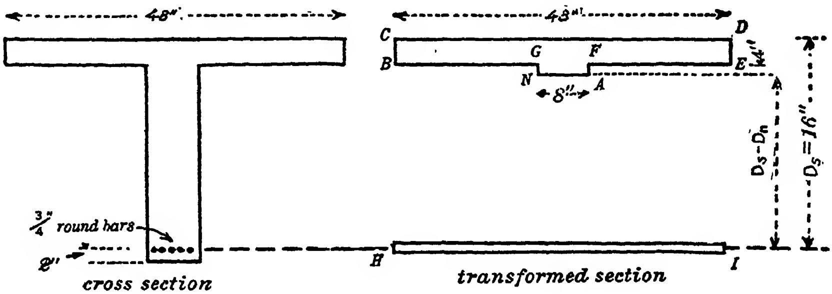


Fig. 881 A

881. **Problem.**—Find the position of the neutral axis of the T-beam in Fig. 881A, assuming $n = 15$.

Solution.—

Area $B\ C\ D\ E$ $= 48 \times 4 = 192$ sq. in.; lever arm $= D_n - 2''$.
Area $N\ G\ F\ A = 8 \times (D_n - 4)$; lever arm $= \frac{1}{2}(D_n - 4)$.
Area $H\ I = 5 \times .4418 \times 15 = 33.135$ sq. in.; lever arm $= 16 - D_n$

Substituting in equation 880B, we have

$$192\ (D_n - 2) + 8\ (D_n - 4)\ \tfrac{1}{2}\ (D_n - 4) = 33.135\ (16 - D_n).$$
$$D_n = 4.05'' \ +.$$

Neglecting $N\ G\ F\ A$, and substituting in equation 880B, we have

$$192\ \ (D_n - 2) = 33.135\ \ (16 - D_n)$$
$$D_n = 4.06''$$

Ordinarily the area $N\ G\ F\ A$ is so small comparatively that it can be neglected without making any material error in calculation.

882. **Percentage of Steel in Tensile Reinforcement.**—From the definition of the modulus of elasticity, we have

$$E_c = \frac{f_c}{\lambda_c} \text{ or } \frac{f_c}{E_c} = \lambda_c \tag{882A}$$

and

$$E_s = \frac{f_s}{\lambda_s} \text{ or } \frac{f_s}{E_s} = \lambda_s \tag{882B}$$

Also, since we assume that planes of cross-section before bending remain planes after bending, and since under this assumption the elongation is proportional to the distance from neutral axis, we have

$$\frac{\ }{\lambda_s}=\frac{D_n}{D_s-D_n} \qquad (882C)$$

Combining these three equations, we have

$$\frac{\lambda_c}{\lambda_s}=\frac{\frac{f_c}{E_c}}{\frac{f_s}{E_s}}=\frac{D_n}{D_s-D_n} \qquad (882D)$$

$$\frac{f_c}{f_s}\times\frac{E_s}{E_c}=\frac{D_n}{D_s-D_n} \qquad (882E)$$

$$\frac{f_c}{f_s}n=\frac{D_n}{D_s-D_n} \qquad (882F)$$

$$\frac{D_n}{D_s}=\left(\frac{nf_c}{f_s+nf_c}\right) \qquad (882G)$$

Since the algebraic sum of the horizontal components of all the forces acting on a cross-section must be equal to zero, we have $S_t-S_c=0$, or $S_c=S_t$. The total compressive force, S_c, acting on the cross-section in Fig. 880B is the shaded area multiplied by the breadth of the beam, or $S_c=(f_c/2)lD_n$. The tensile stress is assumed to be uniformly distributed over the cross-section of the steel; therefore, $T=A_sf_s=plD_sf_s$.

Equating these two, we have $\frac{f_c}{2}bD_n=plD_sf_s$. (882H)

Thence $$\frac{D_n}{D_s}=\left(\frac{2f_s\,p}{f_c}\right) \qquad (882I)$$

Equating the two values of D_n/D_s in equations 882G and 882I, we have

$$\frac{nf_c}{f_s+nf_c}=\frac{2f_s\,p}{f_c} \qquad (882J)$$

$$p=\frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)} \qquad (882K)$$

This formula shows that for a given concrete and ratio of working stresses, the percentage, p, has the same value for all sizes of beams. For concrete of ordinary proportions and with ordinary working stresses in both steel and concrete, $n=15$, $f_c=650$, and $f_s=16{,}000$; hence the economical percentage of steel is 0.77% or approximately 0.8%.

883. Arm of the Resisting Couple.—The point of application of the compressive force, S_c, is at the center of gravity of the shaded force triangle shown in Fig. 880B. Therefore,

$$l_c = D_s - \tfrac{1}{3} D_n \tag{883A}$$

884. Moments of Resistance of the Beams.—The bending which can be borne by the beam is dependent upon the allowable unit stress in the concrete and in the steel. These are taken as separate, and two equations result.

Considering concrete, and taking the center of moments at the center of the steel, we have

$$M_c = S_c l_c = \frac{f_c}{2} D_n b l_c \tag{884A}$$

Considering steel, and taking the center of moments at the point of application of the resultant of the compression stresses in the concrete, C, we have

$$M_s = S_t l_c = f_s b D_s ? l_c \tag{884B}$$

If the bending moment is known, the above equations will give the unit stresses which develop in the concrete and the steel.

The allowable unit stresses of the concrete and steel are usually taken as 650 and 16,000 pounds per square inch.

885. **Problem.**—A reinforced concrete beam has dimensions and reinforcement as given in Fig. 885A. Assuming f_s=16,000 pounds per square inch, f_c=650 pounds per square inch, and n=12, find the moment of resistance of the beam.

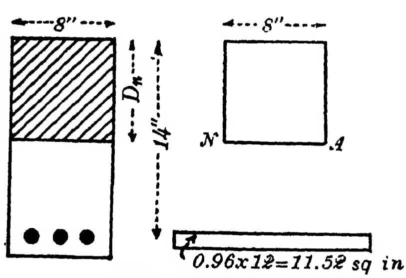


Fig. 885 A

Solution.—Using equations 884A and 884B, we have

$f_c = 650$
$f_s = 16{,}000$
$b = 8$
$D_s = 14$
$D_n =$?, can be determined from moments

$l_c = ?$, can be determined from equation 883A, when, D_n is known
$p = ?$, can be determined from areas of steel and concrete
$M_c = ?$
$M_s = ?$

Solving for D_n: Taking moments around neutral axis, we have

$$8 \times D_n \times \frac{D_n}{2} = 11.52\ (14 - D_n)$$

Solving for D_n, we obtain

$$D_n = 5.068''$$

Solving for l_c: We have, by substituting 5.068″ for D_n in equation 883A,

$$l_c = 14 - \tfrac{1}{3}\ (5.068) = 12.311''$$

Solving for p: $p = \frac{A_s}{bD_s} = \frac{0.96}{8 \times 14} = 0.0086$

Substituting the above values in equations for M_c and M_s we have

$$M_c = \frac{650}{2} \times 5.068 \times 8 \times 12.311 = 162{,}200 \text{ inch pounds.}$$

$$M_s = 16{,}000 \times 8 \times 14 \times 0.0086 \times 12.311 = 189{,}700 \text{ inch pounds.}$$

886. **Problem.**—Find the maximum stresses in the concrete and steel of the reinforced concrete beam shown in Fig. 886A. Neglect dead weight.

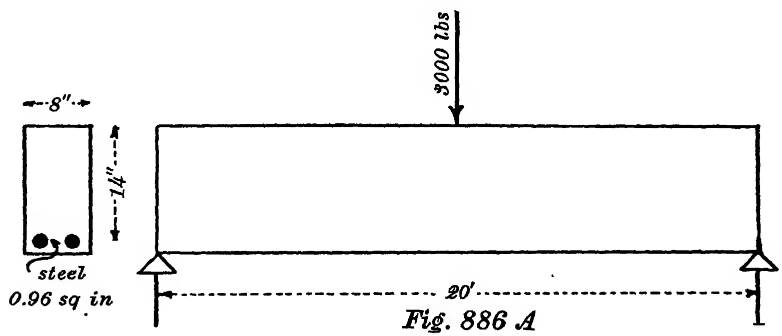


Fig. 886 A

Solution.—Using equations 884A and 884B, we have
$b = 8$
$d = 14$
$D_n = ?$, can be determined by moments.
$l_c = ?$, can be determined from equation 883B, when D_n is known
$p = ?$, can be determined from areas of steel and concrete

M_{fm} (in place of M_c and M_s) is at middle of beam and $= 1500 \times 10 \times 12 =$ 180,000 inch pounds.

The beam has the same cross-section as in the preceding problem, and n is again assumed as 12; therefore, in solving for D_n, l_c and p, we obtain
$D_n = 5.068''$
$l_c = 12.311''$
$p = 0.0086$

Substituting the above values in equations 884A and 884B, we have

$$180{,}000 = \frac{f_c}{2} \times 5.068 \times 8 \times 12.311.$$

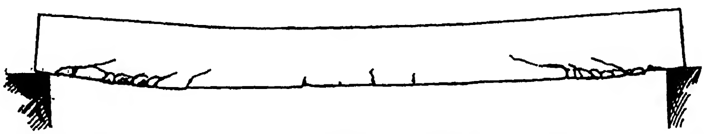
$$180{,}000 = f_s \times 8 \times 14 \times 0.0086 \times 12.311.$$

Transposing and solving, we have

$f_c = 720$ pounds per sq. in.

$f_s = 15{,}200$ pounds per sq. in.

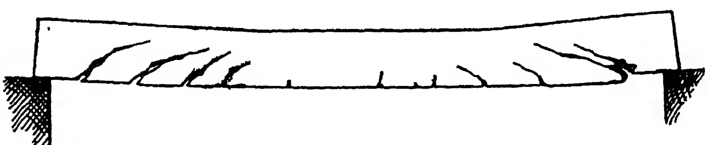
887. **Shearing Stress.**—A reinforced concrete beam may fail by diagonal tension, as shown in Figs. 887A and 887B. This is pre-



Horizontal reinforcement and loose stirrups. Method of failure when tested to destruction. Medium load. Sudden failure due to slipping of horizontal rods. Shear of concrete on horizontal plane above bars but no diagonal cracks.

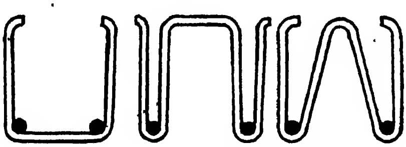
Fig. 887 A

vented by reinforcing with vertical stirrups (Fig. 887C), or by bending up the reinforcing bars (Fig. 887D), or by both at the



Horizontal reinforcement only. Method of failure when tested to destruction. Light load. Sudden failure caused by ends of reinforcement slipping and horizontal shear diagonal cracks in concrete.

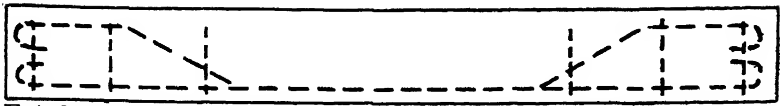
Fig. 887 B



Stirrups

Fig. 887 C

same time. If this stirrup or diagonal reinforcing is not provided, a beam may fail before the concrete and steel have been stressed to their allowable limits in compression and tension.



Reinforced concrete beams showing vertical stirrups and bent up rods

Fig. 887 D

888. Fig. 888A shows a reinforced concrete beam resting on end supports and carrying a uniform load. The various stresses acting

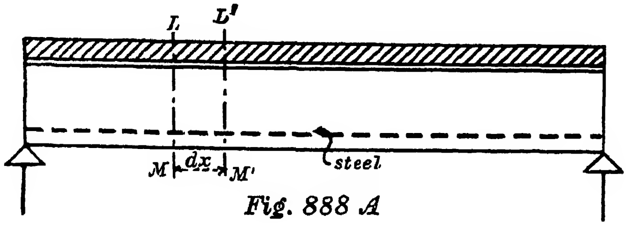


Fig. 888 A

on the planes LM and $L'M'$ are shown in Fig. 888B. It is assumed that the uniform load on the differential section between LM and $L'M'$ is so small that it may be neglected.

Let S_v = intensity of either the horizontal or vertical shear at any point in the neutral surface of the beam.

From paragraph 238, we know that the unit horizontal shear on any molecule is equal to the unit vertical shear on the molecule. Also, since the tensile strength of the concrete is neglected, we know that

$$S_c = S_t \text{ and } S_c' = S_t'$$

The total shearing stress on any horizontal plane between the steel and the neutral plane $= S_t - S_t'$.

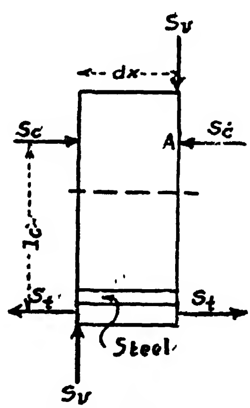


Fig. 888 B

Assuming that b is the breadth of the beam, and dx is the length of the differential section, we have

$$bdx = \text{area of a section sheared,}$$

and,
$$\text{Unit shearing stress} = s_v = \frac{S_t - S_t'}{b\,dx} \tag{888A}$$

Since the forces acting on the differential section are in equilibrium, the algebraic sum of the moments of all the forces about any point, as A, is equal to zero. Hence

$$S_v\,dx + S_t'\,lc - S_t l_c = 0 \tag{888B}$$

$$S_t - S_t' = \frac{S_v\,dx}{l_c} \tag{888C}$$

Substituting in equation 888A, we obtain

$$s_v = \frac{S_v\,dx}{l_c} \div b\,dx = \frac{S_v}{bl_c} \tag{888D}$$

In a homogeneous beam the shear on a cross-section varies as a parabola with the maximum at the neutral axis. In a reinforced

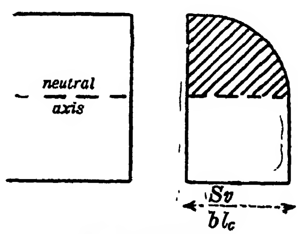


Fig. 888 C

concrete beam the shear varies as a parabola over the compression area only. (Fig. 888C.) Since concrete is assumed to take no tension, and the steel to take all, the shear is constant between the neutral axis and the steel.

The resultant of the vertical and the horizontal shear produces diagonal tension. Since the maximum unit shear occurs at the neutral axis, the maximum diagonal tension occurs at the same place; it has the same intensity and acts at an angle of 45° with the neutral axis.

889. Stress in Web Reinforcement.—If s (Fig. 889A) be the distance between vertical stirrups, then the diagonal area between

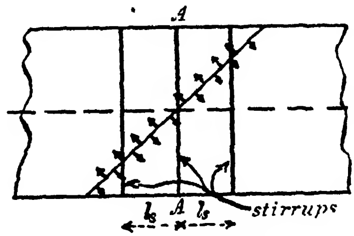


Fig. 889 A

two stirrups is bl_s cos 45° = $bl_s\sqrt{2}$. The total diagonal stress between two stirrups is $s_v bl_s\sqrt{2}$. The vertical component, which is taken up by the stirrups, is

$$s_v\, bl_s \frac{\sqrt{2}}{\sqrt{2}} = s_v\, bl_s.$$

Represent the total stress in the stirrups at AA by S.

Then $$S = s_v bl_s \qquad (889A)$$

Substituting the value of s_v from equation 888D, we have

$$S = \frac{S_v bl_s}{bl_c} = \frac{S_v l_s}{l_c} \qquad (889B)$$

With a one-loop stirrup (two verticals), each vertical has a total stress of $S/2$; with a two-loop stirrup, each vertical has a total stress equal to $S/4$.

In engineering practice, it is usual to assume that the concrete takes one-third of the stress; therefore, the stress in the stirrup is expressed as

$$S = \frac{2S_v l_s}{3l_c} \qquad (889C)$$

The area of the steel to take up this stress in a length l_s is, therefore,

$$A_s f_s = \frac{2S_v l_s}{3l_c} \qquad (889D)$$

Or, $$A_s = \frac{2S_v l_s}{3f_s l_c} \qquad (889E)$$

890. If the stirrups are placed or bars are bent up at angles of 45°, they will be as shown in Fig. 890A. The value of S then becomes

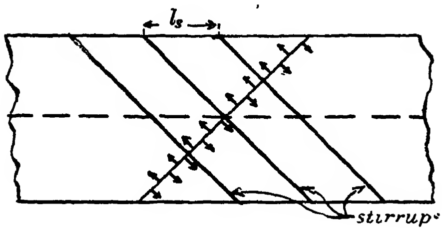


Fig. 890 A

$$S=\frac{S_v b l_s}{b l_c \sqrt{2}}=\frac{S_v l_s}{l_c \sqrt{2}}=\frac{3S_v l_s}{4 l_c} \text{ approx.} \qquad (890A)$$

Assuming concrete to take one-third of the stress, the equation becomes

$$S=\frac{S_v l_s}{2l_c} \qquad (890B)$$

The area of steel to take up this stress is

$$A_s=\frac{S_v l_s}{2f_s l_c} \qquad (890C)$$

891. For simple beams the need for web reinforcement is greatest at the support, where the vertical shear is a maximum, and decreases as the center is approached. Hence, the interval between stirrups at the supports should be less than at a point near the center. The spacing of vertical stirrups in that portion of the beam which requires reinforcing is never greater than $\frac{1}{2}d$. Since the bending moment of a beam decreases from the center towards the supports, it is customary to bend up the longitudinal reinforcement near the ends where it is not required to carry the tension due to flexure. In continuous beams, the bent-up rods take the negative moment which exists at intermediate supports.

892. **Bond.**—Usually the entire stress carried by the steel of a reinforced concrete beam is transmitted to the reinforcing bars by bond or adhesion between the steel and concrete. Stress is also conveyed to the steel of a reinforced concrete column in like manner. Experience has shown that the bond is reliable and permanent, and that plain bars may be used in most structures with success. Deformed and twisted bars are used to a large extent in structures

where the stress between the steel and the concrete exceeds the safe working adhesion of the plain rod. The indented surfaces of deformed bars produce a mechanical bond in addition to the adhesion already mentioned.

For ordinary smooth, round, or square bars the adhesive strength may be taken as 200 to 300 pounds per square inch. A rough surface gives a greater adhesive value than a smooth surface. Consequently, a thin film of rust on the reinforcing metal should not cause rejection. Steel with loose or scaly rust should not be allowed; however, it may be used if it is first cleaned with a stiff-wire brush or given a prickly bath of sulphuric acid solution (1 part acid to 6 parts water) and then dipped into clean water. Oiling or painting of reinforcing steel reduces the adhesion and hence should not be permitted.

893. The adhesion should be equal to the stress which is transmitted by the rod; therefore, the rod must be imbedded for such a length that the area of the surface imbedded multiplied by the unit bonding stress equals the cross-sectional area of the rod multiplied by the allowable unit tensile stress in steel.

Expressed in an equation, we have, letting L equal length to which rod is imbedded, l_b equal the unit bonding stress, and r equal the radius of the rod,

$$2\pi r L l_b = A_s f_s = \pi r^2 f_s \qquad (893A)$$

Or

$$L = \frac{r f_s}{2 l_b} \qquad (893B)$$

It is a common practice to bend the ends of reinforcing rods into hooks consisting of either curved or right-angle bends. (Fig. 893A.)

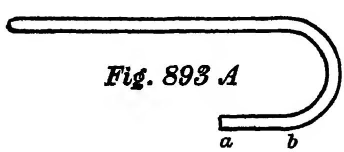


Fig. 893 A

Experiments on the efficiency of such hooks have shown that the ultimate bonding strength was greatly increased. When curved ends are used they should consist of bends through 180° with short lengths of straight rods beyond as shown at ab in Fig. 893A. Short square hooks upon the ends of bars are not of great value.

For more detailed information see:

Concrete Engineers Handbook, Hool & Johnson.
Concrete, Plain and Reinforced, Taylor & Thompson.
Reinforced Concrete Constructions, Hool.
Materials of Construction, Johnson.
Portland Cement Association Pamphlets.

PROBLEMS.

P. 801. How many fence posts 3″ x 3″ at the top, 5″ x 5″ at the bottom, and 7′ long can be made from one sack of cement? How much sand and gravel will be needed?

P. 802. Required the quantity of materials to construct a basement wall 12″ thick, 6′ 5″ high above footing, for a house 25′ x 40′ outside dimensions. The footing is 1′ 6″ wide and 6″ thick. Concrete proportions are 1 : 3 : 5.

P. 803. Required the quantities of materials to construct a concrete floor for a basement. Interior dimensions of the basement 23′ x 38′. Floor 5″ thick all over, with 4″ base of concrete proportioned 1 : 2½ : 5, and 1″ wearing course composed of cement mortar proportioned 1 : 2.

P. 804. A small dam 50′ long is to be built of concrete. It will have a trapezoidal section 5′ wide on top and 10′ wide on the bottom, height 12′. What mixture of concrete should be used and how much cement, sand, and gravel will be required?

P. 805. A one-course concrete floor 6″ thick is to be built. It is 18′ x 24′. What mixture should be used and how much cement, sand, and gravel is required?

P. 806. It is proposed to build a swimming pool 100′ long and 40′ wide, the walls and bottom to be 12″ thick. It is to be 12′ deep at one end and 5′ at the other. It will be surfaced inside, the coat being 1″ thick. How much cement, sand, and gravel are required?

P. 807. Find the position of the neutral axis of the T-beam shown in Fig. P. 807.

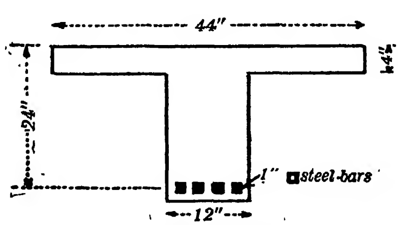


Fig. P 807

P. 808. Find the position of the neutral axis of a rectangular reinforced concrete beam, $b=10''$ and $d=16''$, reinforcement three 1″ square bars.

P. 809. Find the position of the neutral axis of a reinforced concrete beam in which $b=14''$ and $d=18''$, the reinforcing being four 1″ round rods.

P. 810. A rectangular reinforced concrete beam with $b=8''$ and $d=12''$ is reinforced by three steel ¾″ round bars; $E_s=30{,}000{,}000$, $E_c=2{,}500{,}000$, $f_c=650$ pounds per sq. in, $f_s=16{,}000$ pounds per sq. in. What is the moment of resistance of the beam?

P. 811. What would be the moment of resistance of the beam in Problem P. 810 if d were 16″?

P. 812. Find the value of f_c and f_s when the beam given in Problem P. 810 is subjected to a bending moment of 120,000 inch pounds.

P. 813. What will be the moment of resistance of a reinforced concrete beam whose breadth is 12″ and depth, d, is 16″? The beam is reinforced with three 1″ square rods. $E_s=30{,}000{,}000$, $E_c=2{,}000{,}000$, $f_c=550$ pounds per sq. in., $f_s=16{,}000$ pounds per sq. in.

P. 814. What would be the maximum unit stress in the steel and concrete in the beam given in Problem P. 813 if it were subjected to a bending moment of 400,000 inch pounds?

P. 815. A reinforced concrete beam resting on end supports is 20′ long. It is 12″ x 18″ and the center of the steel is 15″ below the top of the beam. It is loaded at 5′ from the left end with 2500 pounds and at the center with 3000 pounds. What percentage of steel is required in order that the unit stress in the concrete shall not exceed 600 pounds per sq. in? $n=15$.

P. 816. A rectangular reinforced concrete beam 14′ long carries a load of 700 pounds per linear foot. If $b = 10''$ and there is .6% of steel, what value has d? ($f_c=600$ pounds per sq. in., $f_s=16{,}000$ pounds per sq. in., $n=15$)

P. 817. A reinforced concrete beam 18′ long resting on end supports is 12″ x 20″. It carries a uniform load, including its own weight, of 1000 pounds per linear foot. If 30 pounds per sq. in. is the unit allowable shearing strength of concrete, find the size and spacing of the stirrups.

CHAPTER IX.

TESTING OF MATERIALS.

901. **Purpose of Testing Materials.**—In designing any structure it is necessary for the engineer to know the physical properties of the materials of which the structure is to be built. The engineer works simply with an allowable or safe unit stress but this unit stress must be determined with accuracy in order that the work may be of the required strength and still be economical in material. The allowable unit stress being a function of the ultimate unit stress, it becomes necessary to determine the latter. This is done by testing the material in a laboratory equipped with standard testing machines.

902. **Research Tests.**—The tests made to determine the physical properties of a material are called *research tests,* and are made for the most part in the laboratories of the U. S. Bureau of Standards and several of our larger universities and by some manufacturing concerns. From the results of a large number of these tests the allowable unit stresses are determined. The engineer has then simply to determine that the material delivered to the work meets the specifications which have been laid down by the designer and which were the basis of the design.

903. **Acceptance Tests.**—The tests of the delivered materials to determine whether they are acceptable are called *acceptance tests.* The actual tests made are similar to those made in research work, but the number necessary is not so great, and the test is usually abbreviated. This employment of the results of tests, as a basis for the acceptance of materials, at the work, has become very common, and is becoming more and more important as the methods of manufacture improve and our knowledge of the materials of construction increases.

904. **Specifications and Inspection.**—The specifications which form part of the contract covering the building of a structure include requirements as to the strength, limiting sizes, finish, and chemical ingredients of the various materials to be used. In this way it is provided that defective material, or material which does not reach a certain standard of quality, shall not be used on the work. The American Society for Testing Materials publishes each year specifications, which have come into quite general use as standards throughout the country, for practically all materials.

905. The inspection of the delivered materials consists in examining them for visible defects and checking the correctness of dimensions as well as the selecting of samples to be tested in the laboratory. The tests made may be chemical, to see that the proper elements and none others are present; microscopic, to determine the structure of the material, especially in the case of metals; or physical, to determine the strength of the material, its hardness, toughness, or ductility.

906. **Selection of Specimens.**—In order that the test results may be of value, it is necessary that the sample or specimen tested be representative of the material. The specifications usually prescribe the manner in which specimens will be selected. In the case of metals, the test pieces may be cut from the finished product or, in the case of castings, special molds of the desired size may be poured. When cutting a piece from a casting, it is to be remembered that the outside is stronger than the interior due to the lesser shrinkage strains while cooling. Where the metal has been rolled, the thinner parts are stronger because they have received more work under the rollers than the thicker sections.

907. In selecting samples of brick, tile, or other similar materials, the pile, or piles, should be divided into a number of approximately equal parts and a sample taken from each. The samples thus selected can then be reduced in number by a similar method. With burnt-clay products, the color, state of burning, and ring under the hammer, should be considered in order that the samples taken may be truly representative of the material.

908. Stone from a quarry should be selected from each stratum that is being worked, remembering that stone near the surface has weathered and therefore partially changed in strength and that the stone adjacent to blast holes has been weakened by the explosion.

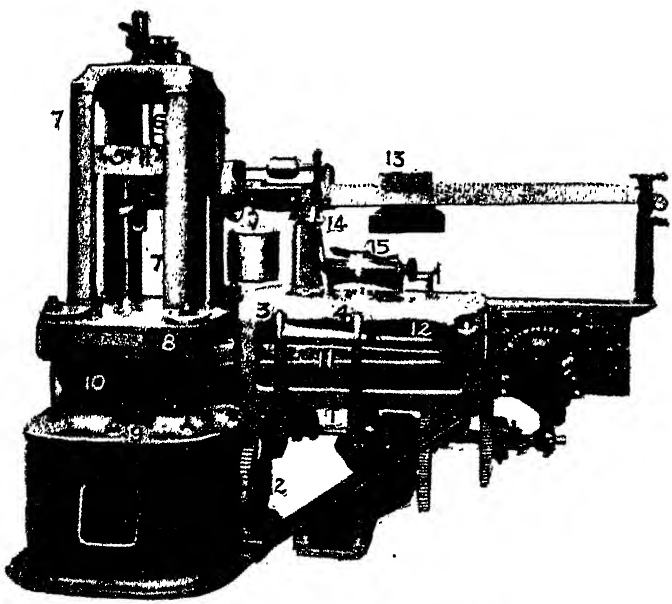
909. Timber specimens should always be of sufficient size to offset any local irregularity in the structure of the wood due to knots, seasoning, or other causes which would make the results of tests on a small specimen not truly representative of the timber in structural sizes.

TESTING MACHINES.

910. **Universal Testing Machine.**—The most common machine used in testing materials is so arranged that it can make tension,

compression, and flexure tests, and is therefore known as a universal machine. There are two general types used in this country, (according to the way the movable crosshead is operated), the screw-gear machine, and the hydraulic machine. Testing machines may also be classified according to whether the specimen is placed vertically or horizontally. The vertical position is in general use for testing of small specimens, but the parts of the machine and the specimen are not as accessible as in the horizontal type.

911. **Olsen Testing Machine.**—A common universal machine of the screw-gear type is the Olsen (Fig. 911A), which is built in



Olsen's Automatic Universal Testing Machine
Fig. 911 A

capacities from 30,000 to 400,000 pounds. Power is furnished by a direct-connected motor through the shaft, 1, to the gear, 2, and a system of gears shown connected to the screws, in Fig. 911B. The gears rotate the screws, which cause the pulling head or crosshead, 5, to move either up or down, according to the direction of drive. The clutch levers, 3 and 4, are used to control the speed at which the pulling head moves. The machine is shown with a tension specimen, 6, in place, the pull on which is carried down

through the columns, 7, to the weighing table, 8. The weighing table is supported on knife-edges connecting with a system of levers, (Fig. 911B), and 10, 11, and 12 (Fig. 911A). This system of

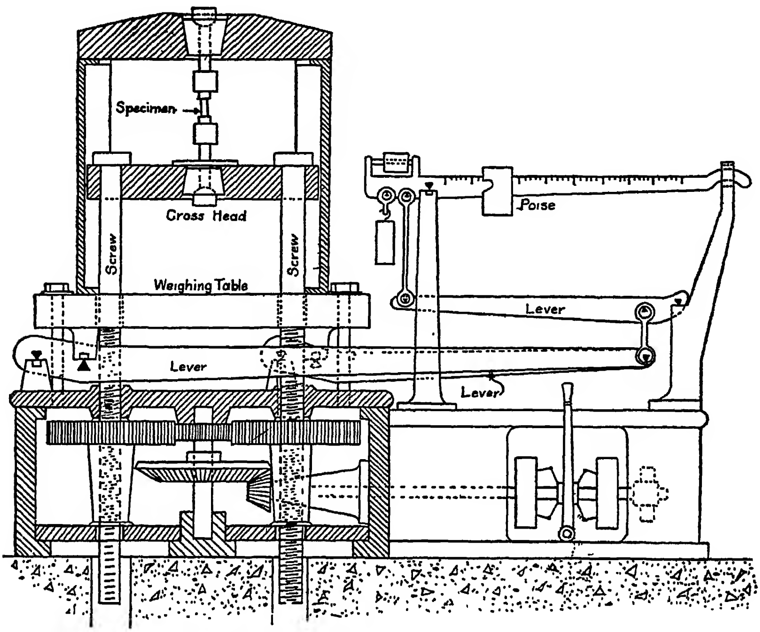


Fig. 911 B. Screw-power Universal Testing Machine

levers has the effect of reducing the force exerted until it can be balanced by the poise, 13, on the scale beam. The poise may be moved by hand by means of the hand wheel, 14, and a screw running along the scale beam, or it can be controlled automatically by the cone drive, 15, which is disconnected electrically as soon as the scale beam drops.

In making compression tests the specimen is placed between the pulling head, 5, and the weighing table, 8. Flexure tests and shear tests are made in a similar manner, attachments being provided to direct the force on the specimen in the proper manner.

912. **Emery Testing Machine.**—This machine (Fig. 912A) is an example of the hydraulic type. The load is applied to the specimen, 2, by the hydraulic ram, 1, and transmitted to the yoke, 3, which causes the hydraulic support, 4, to bear against the vertical beam, 5. The pressure on the liquid (oil) in the hydraulic support is transmitted to the chamber, 7, where it operates a system of levers

attached to the scale beam. This method of weighing the load is more sensitive than that used in the screw-gear type of machine.

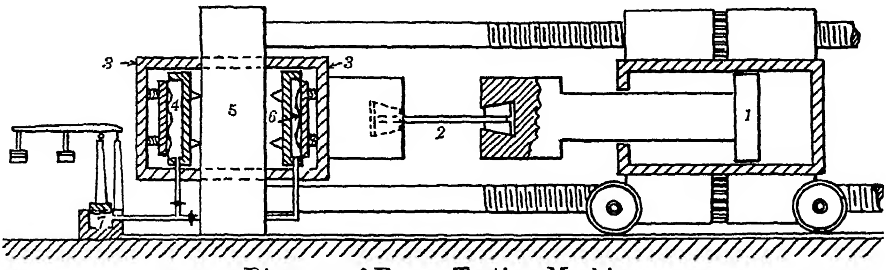


Diagram of Emery Testing Machine
Fig. 912 A

The U. S. Bureau of Standards at Washington, D. C., has a machine of this type, with a compression capacity of 2,300,000 pounds, which will take a specimen 33′ long.

913. **Attachments for Bending and Shear Tests.**—The apparatus shown diagrammatically in Fig. 913A is used in conjunction with universal testing machines in making bending tests. In

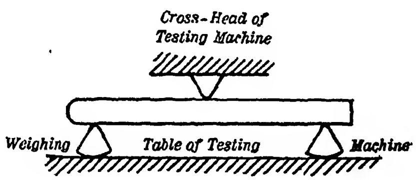
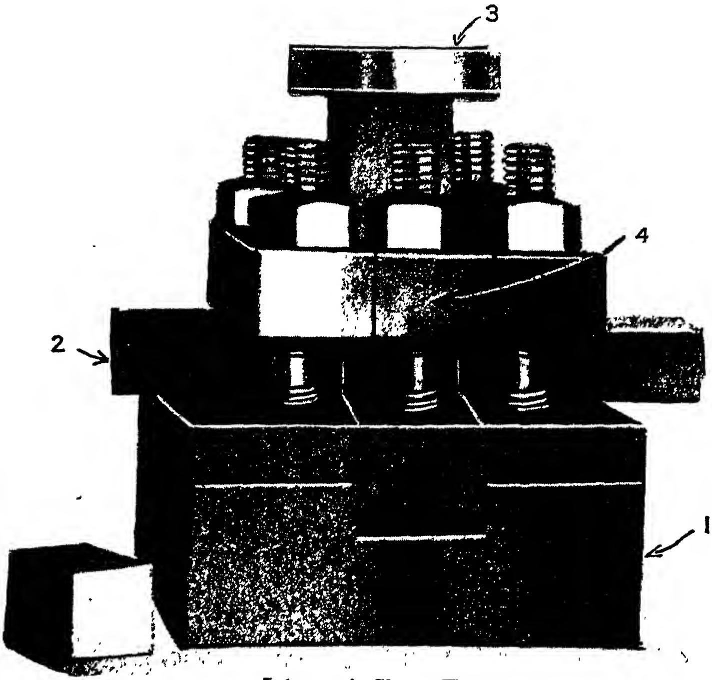


Fig. 913 A

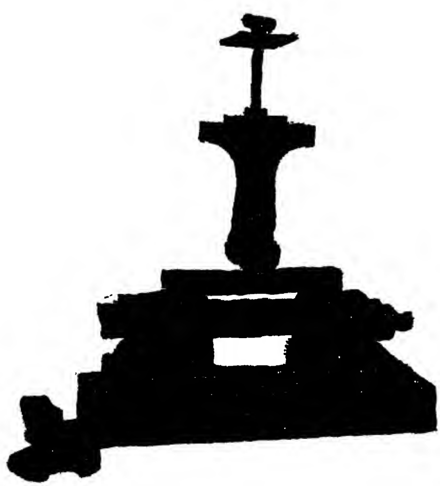
laboratories where any amount of this work is done special equipment is usually provided. These machines hold the specimen in the manner shown in Fig. 913A, the load being applied through the upper knife-edge while one of the lower knife-edges is fixed and the other attached to the weighing device.

To test for shear, a shear block (Fig. 913B) is used. The bed, 1, is placed on the weighing table, the pulling head exerting the required force through the plunger, 3, which moves the block, 4, against the specimen, 2.

914. **Cold-bending Machine.**—The cold-bending test can be made in a universal machine by use of the apparatus shown in Fig. 914A which is a Riehlé Brothers bending attachment. The upper part of the tool is forced down on the specimen by the movable head, the bend being made around it.



Johnson's Shear Tool
Fig 913 B



A Bending Attachment
Fig. 914 A

The Olsen cold-bending machine is shown in Fig. 914B. This machine clamps the specimen at the left end, bending it around a central pin by the action of a pin starting from the right and controlled by an electric motor.

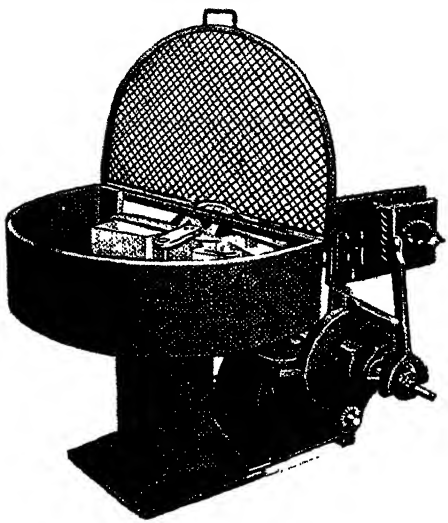


Fig. 914 B
Olsen's Improved Cold-bending Machine

915. **Cement Briquette Testing Machine.**—In order to test the tensile strength of cement a standard briquette is made, which, after proper drying, is tested in a machine of the general type shown in Fig. 915A. The briquette is placed in the clamps, *g*, enough fine shot having been placed in the holder, *b*, to balance the weight, *w*. The trigger, *t*, is then released which operates a valve allowing the shot to drop gradually into the holder, *c*. This unbalances the lever on top, and the weight, *w*, moving down, exerts a force on the clamps, *g*, which increases until rupture takes place. As soon as the specimen ruptures and the clamps, *g*, separate, the rod, *r*, closes the shot valve. The force which was required to break the briquette is proportional to the amount of shot which dropped into *c*. The balance at *c* is graduated to read the load on the briquette usually to the nearest 5 pounds.

916. **Impact Testing Machine.**—Two general types of machines are used for impact tests: the pendulum type, and the drop-hammer type. The former is used for testing small specimens and consists

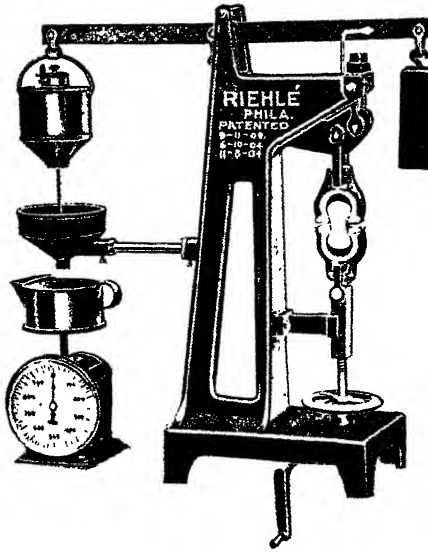


Fig. 915 A
Automatic Briquette Testing Machine

of a heavy pendulum which is swung from a known height and allowed to break the specimen. The resistance of the test piece is then computed from the energy remaining in the pendulum after it passes through the specimen, the height to which the pendulum swings after breaking the test piece being a measure of this residual energy.

The drop-hammer type of machine is shown in diagrammatic form in Fig. 916A. In this machine the electric magnet, *M*, is attached to the hammer, *H*, by closing the switch. Power then applied through the pulley and hoist at *D* raises the hammer to a predetermined height indicated by the pointer attached to *H* on the scale. The specimen is placed on the adjustable support blocks, *B*, as shown in the figure. The hammer is then dropped by pulling the switch, *S*, which demagnetizes the magnet. As the hammer drops, the pencil, *P*, which is attached to it, records a graph as shown in Fig. 933A on a sheet of paper attached to the roll, *R*, which is driven at a known constant speed by the pulley shown at its base.

TESTS.

Tension Tests.

917. **Importance of the Tension Test.**—In general, the tension test is the one most commonly made on metals, since it can be per-

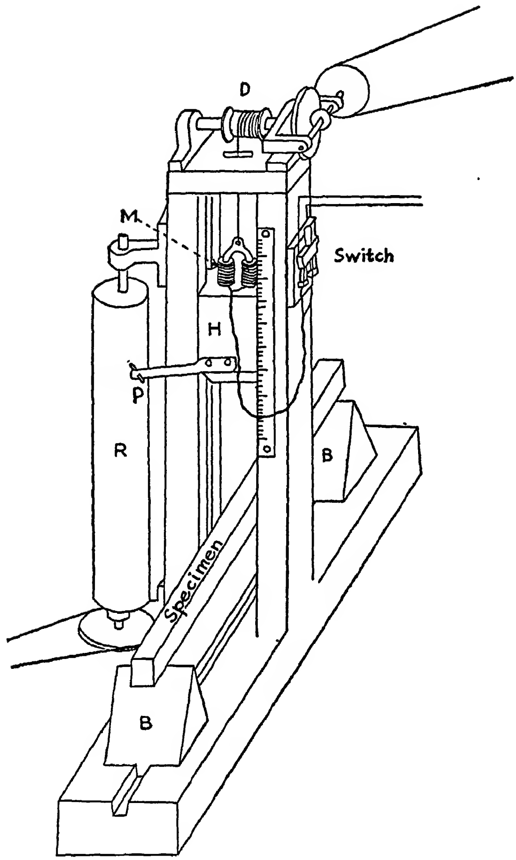


Fig. 916 A

Drop Type of Impact Testing Machine

formed easily, and shows more completely the true homogeneity and strength of the material. The ruptured tension specimen presents broken surfaces which are of value in determining the structure of the metal and its condition. For these reasons the tension test has come to be considered as the best single test to make of a metal.

In the case of wood, the tension test is not of great importance, since wood in tension parallel to the grain, is so much stronger than in compression that the latter stress governs in design.

918. **Tension Specimens.**—The form of the test specimen, it has

been found, affects the results obtained in tension tests. For this reason efforts have been made to standardize the preparation of specimens, and the specifications resulting should be followed in order that the tests made may be of uniform application. In Fig. 918A are shown two forms for specimens which are in accordance with recent practice, although sizes and shapes vary with the material and the purpose of the test. The type of grips with which the testing machine is equipped determines whether a smooth or threaded end is to be used. The latter end is preferable, however, since it gives a firm grip in the machine, yet reduces to a minimum the compression produced by the grips.

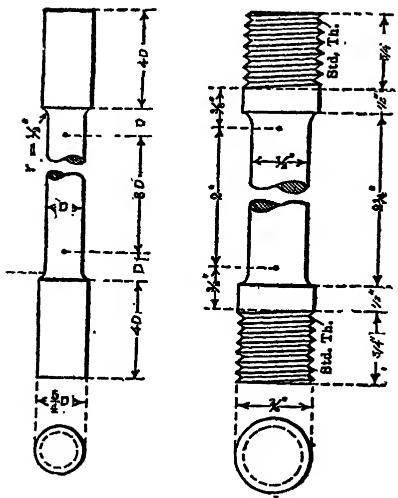


Fig. 918 A

919. **Form of Results.**—The results of a tension test are plotted on cross-section paper to form a stress-strain curve as described in

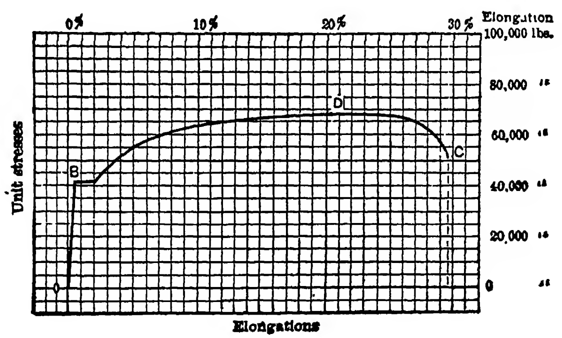


Fig. 919 A

paragraph 207. From this curve (Fig. 919A), the following significant results are obtained:

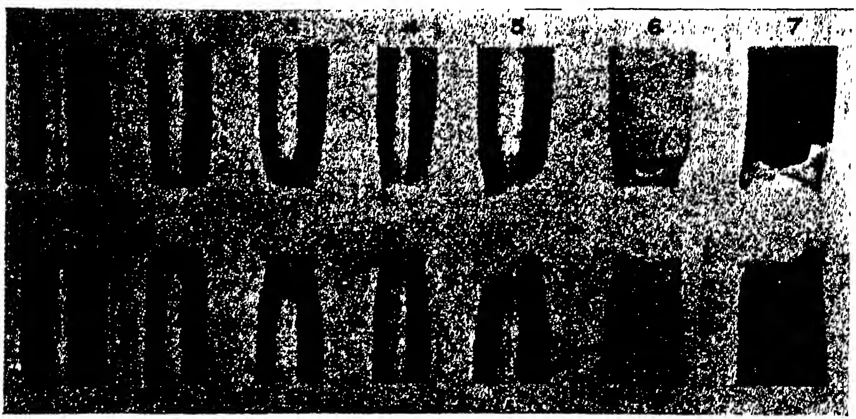
a. The *ultimate strength* which is found by dividing the load in pounds at the point of rupture, *D*, by the original area, in square inches, of the test specimen.

b. The *elastic limit* which is found by dividing the load in pounds at the yield point, *B*, or the elastic limit, by the original area in square inches of the specimen.

c. The *percent elongation* which is determined from the ratio of the original length and the amount of elongation.

d. The *modulus of elasticity* which is obtained by dividing the unit stress in pounds at the elastic limit by the corresponding unit elongation.

920. **Examples of Rupture.**—In Fig. 920A are shown several ruptured metal specimens which are characteristic. Cast iron gives a square break in tension, as shown in 1, and presents a crystalline



Characteristic Fractures of Metals in Tension
Fig. 920 A

surface rupture. Steel of a very high carbon content is apt to rupture in a similar manner showing a very fine crystalline structure. Specimen 2 is also high carbon steel which in this case shows a fine-cup and flat-cone fracture. Soft and medium steels break with a full-cup and cone fracture as in 3, the appearance of the break being silky; if, however, the specimen is eccentrically loaded, the rupture occurs as shown in 4 and 5. A common type of rupture for a medium steel bar is shown in 6, while a wrought iron bar shows a ragged fibrous break as in 7.

Compression Tests.

921. In compression tests it is necessary to take into consideration the ratio of slenderness of the specimen. The testing of columns is complicated by many elements such as: the method of

attaching the ends, position of the load, make-up of the column, and its proportions. Standard texts on testing of materials and many reports on specific tests are obtainable which cover this type of compression test, hence we shall consider only tests on short prisms.

The compression test is of primary importance in the case of wood, since, as stated in paragraph 916, compressive strength governs this material in design. The compression test is also of primary importance in the case of materials like stone, concrete, and clay products, which have such a low tensile strength that they are generally used only as compression members.

922. **Compression Specimens.**—Two general forms of specimens are used in making compression tests. For materials which rupture on inclined planes it is common to use a cylinder whose height is twice its diameter; for all others, the cube, or a prism proportioned as the cylinder above, is used. The cylindrical form has the advantage of eliminating the corners which on cubes are apt to give incorrect results due to weaknesses produced at these points in preparing the specimen. For this reason the cube or rectangular form of test piece is gradually being replaced by the cylindrical. For concrete, as an example, the common practice is to use a cylinder 6″ in diameter by 12″ high.

It is important that the faces of the specimen remain parallel during the compression test and that the specimen be centered in order that the line of force may pass through its axis. To aid in keeping the faces parallel, spherical seated blocks are used which adjust automatically as the test progresses. With such blocks in use the importance of centering the specimen is evident.

923. **Result of Compression Test.**—With the elastic materials the results obtained in a compression test are similar in form to those obtained under tension. The yield point, the ultimate strength, and the modulus of elasticity can be quite accurately determined. A curve similar to a tension curve can be plotted. With softer materials the critical points are not so accurately marked and it is difficult to determine the exact point at which rupture takes place.

Concrete, stone, brick, and other brittle materials may or may not show an elastic limit and the load at the first sign of failure becomes of importance. With these materials the compression test is the most important just as the tension test is the most important with metals.

Bending Tests.

924. The transverse test of materials can be quickly made and is comparatively an inexpensive test. It is used to develop and correct formulas used for flexure and deflection of beams, and in the case of structural shapes, to determine the constants found in the engineering handbooks. With brittle materials this test gives an index of the strength, toughness, and stiffness which is of value in determining their properties. The flexure test is also used to determine the elastic resistance.

925. **Bending Specimens.**—For tests on timber, cement mortar, and malleable iron, the specimen in general use is rectangular in section. The span varies with the material, and specifications for transverse tests usually give the span a relationship to the depth of the specimen in order that the test results may be uniform. With cast metals it is customary to use a cylindrical section with a span 15 times the diameter. For structural steel the specimen should be cut from the rolled product in order that it will be geometrically similar and the condition of rolling will be the same as in the product being tested.

926. **Results of Bending Tests.**—The results obtained from transverse tests are similar to those described under tension. Curves are often plotted, showing the results, in which the loads in pounds are used as ordinates, and the deflections in inches, as abscissas. The critical points can be indicated on the curves thus drawn.

Shear Tests.

927. **Object of Shear Tests.**—Shear as a stress appears in conjunction with bending except in the case of rivets and very short pins. In wooden beams, strength in shear may be a governing factor in design, and hence important: whereas in metals, it enters into the design of rivets, pins, and very short beams, only, and therefore is of secondary importance. Its determination, however, does aid the designer and should therefore be made for structural materials.

928. **Shear Specimens.**—It is common practice to use a specimen with a rectangular cross-section, although cylinders are often employed. For tests of wood, the specimen should be prepared so that the test is not made tangent to the annual rings, since wood has very little strength in a plane tangent to these rings. Care should be exercised in placing the test piece in the shear tool in order that

the stress developed may be purely shear or accompanied by as little cross-bending as possible. In order to eliminate cross-bending the sliding planes of the shear tool should be as close together as possible and the specimen should be rigidly fixed in place.

929. **Results of Shear Tests.**—It is impossible to determine the elastic limit and modulus of elasticity of a material from the shear test because of the inability of the observer to obtain data during the test. The ultimate strength is, therefore, the only real result obtained, and torsion tests have to be resorted to in order to obtain working values for the other significant results.

Cold-Bending Tests.

930. The cold-bending test consists in sharply bending a specimen through an angle of 180°. It is used to determine a measure of the ductility of a metal, and, when specimens are nicked to insure breaking, to determine the structure and condition of the metal. The angle to which a specimen can be bent without rupturing is taken as a measure of its ductility.

This test is used very commonly in shops and can be made with very simple equipment, which is a great advantage, but does not permit of the standardization accomplished in the other tests.

Impact Tests.

931. **Objects of Impact Tests.**—When a material is to be subjected to shock, as railroad rails or car wheels, it becomes necessary to determine how it will withstand these suddenly applied forces. Especially in railroad work and machine design the whole subject of impact is becoming of ever increasing importance due to increases in the loads and the speeds at which they move.

932. **Accuracy of Impact Tests.**—Due to the impossibility of standardizing impact tests on the machines now available, the results obtained cannot be applied with the same accuracy as, for example, is the case with the results of tension tests. The principal inaccuracies develop from the mechanical difficulties of providing a rigid and standard anvil and hammer, and the lack of parallelism between the faces of the specimen and the anvil and hammer while the machine is operating. However, while the results obtained are not so accurate as could be desired, they are, nevertheless, of great value in determining the physical properties of the various materials under impact forces.

933. **Results of Impact Tests.**—As stated in paragraph 916, the impact machines in general use are self-recording, furnishing a diagram of the general form shown in Fig. 933A. In this diagram

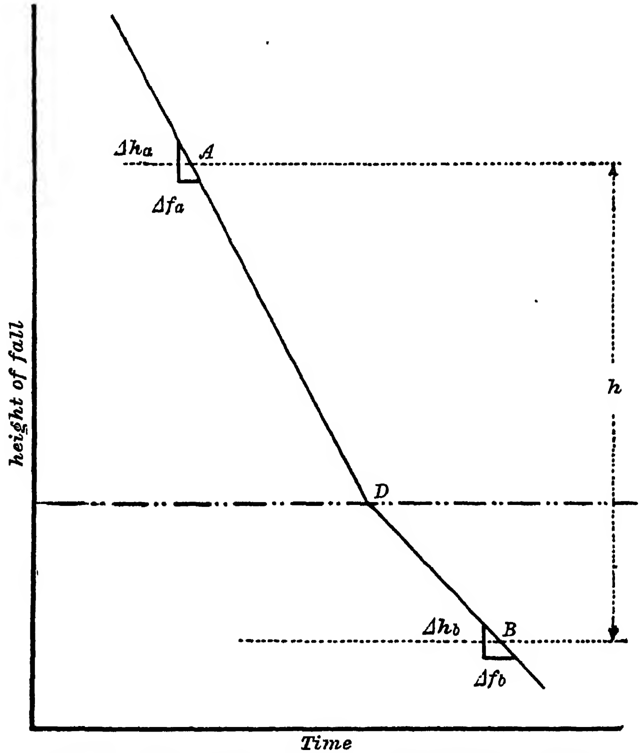


Diagram from drop type of impact testing machine

Fig. 933 A

time is plotted on the X axis and height of fall on the Y axis. The point, D, gives the location of the hammer as it strikes the specimen. The part of the curve above D represents then the free fall of the hammer before striking the specimen, while that below D represents its free fall after it has broken the specimen. Both parts of the curve are parabolas.

Energy is required to rupture the specimen, which energy is absorbed from the falling hammer, thereby changing its speed as shown by the abrupt change of slope in the curve at D. If the actual velocities of the hammer, before and after rupturing the test piece, can be determined, the amount of energy required to break the specimen can be computed.

Select any two points as A and B and let h be the vertical distance between them. Let W represent the weight of the hammer. Then since the velocity, V, at any point of the curve is represented by the slope at that point we have, representing increment by $\triangle$,

Velocity at $A = V_a = \frac{\Delta h_a}{\Delta f_a}$ and the velocity at $B = V_b = \frac{\Delta h_b}{\Delta f_b}$.

Neglecting friction and the losses due to the machine, the kinetic energy of the hammer at A is

$$\frac{W (V_a)^2}{2g} \tag{929A}$$

If the hammer had fallen freely to B it would have absorbed between A and B energy represented by Wh and its total kinetic energy at B would then have been

$$\frac{W (V_a)^2}{2g} + Wh \tag{929B}$$

But the velocity at B is V_b, and we know therefore that the actual kinetic energy in the hammer at B is

$$\frac{W (V_b)^2}{2g} \tag{929C}$$

The difference then between expressions 929B and 929C gives the energy which was given up in rupturing the specimen, which is

$$\left[\frac{W (V_a)^2}{2g} + Wh\right] - \frac{W (V_b)^2}{2g} \tag{929D}$$

For more detailed information see:

Materials of Construction, Johnson.
Materials of Engineering, Moore.
The Strength of Materials, Ewing.
The Testing of Materials of Construction, Unwin.

PROBLEMS.

P. 901. A steel tension test specimen is 8″ long and 1.5″ in diameter. If Fig. 919A shows the results of the test of this specimen, find (a) the ultimate strength, (b) the elastic limit, (c) the percent elongation, and (d) the modulus of elasticity. What allowable unit stress would you recommend for this steel?

P. 902. Work problem P. 901, the specimen being 10″ long and 1.75″ in diameter.

P. 903. Find the ultimate strength, the coefficient of elasticity, and the elastic limit of a specimen 3″ long, 0.505″ in diameter, which was tested with the following results:

Applied Load Pounds.	Elongation per Inch. Inches.
200	0
1000	0.000133
4000	0.000667
8000	0.001367
12000	0.002067
13600	0.002467
14000	0.006133
16000	0.0133
18000	0.0200
20000	0.0333
22000	0.0500

PART III.

COMPONENT PARTS OF STRUCTURES.

The science of construction consists mainly in giving to every part of a structure the proper construction and the proper degree of strength with the least material necessary for the purpose. If any part of a structure is not strong enough, the structure will fail; if any piece is made stronger than is necessary, the superfluous weight of this piece will in general be transmitted to some other part, and the latter, in consequence, will be required to sustain a greater load than it should. Hence the proper construction, and the proper size and distribution, of the different parts of the structure should be carefully determined before the parts are combined in the complete structure.

CHAPTER X.

FRAMING.

1001. A **frame** is an arrangement of beams, bars, rods, etc., made for sustaining strains. The art of arranging and fitting the different pieces is called **framing,** and forms one of the subdivisions of the **art of construction.** The object to be attained in framing is to arrange the pieces, with due regard to *lightness* and *economy of material,* so that they shall best resist, *without change of form in the frame,* the stresses to which the latter may be subjected.

The principal frames employed by engineers are those used in bridges, centers for arches, coffer-dams, caissons, floors, partitions, roofs, and staircases.

The materials used in their construction are generally wood and steel. The latter, in addition to superior strength, possesses an advantage over wood in being susceptible of receiving the most suitable form to resist the stresses to which it may be subjected.

1002. The **joints** are surfaces at which the pieces of a frame are connected; they are of various kinds, according to the relative positions of the pieces and to the forces which the pieces exert on each other.

The following general rules should be observed in the construction of joints for frames of timber:

a. Arrange the joints and fastenings so as to weaken as little as possible the pieces which are to be connected.

b. In a joint subjected to compression, place the abutting surfaces as nearly as possible perpendicular to the direction of the force.

c. Give to such compressive joints as great a surface as practicable.

d. Proportion the fastenings so that they will be equal in strength to the pieces they connect.

e. Place the fastenings so that there will be no danger of the joint giving way by the fastenings shearing or crushing the timber.

In modern construction, it is good practice to reinforce an important wooden joint with metal fastenings, such as nails, bolts, pins, straps, stirrups, plates, and caps. These are easily procured, and greatly increase the strength of the joint.

Figs. 1002A to S show timber joints, some with and some without metal reinforcement.

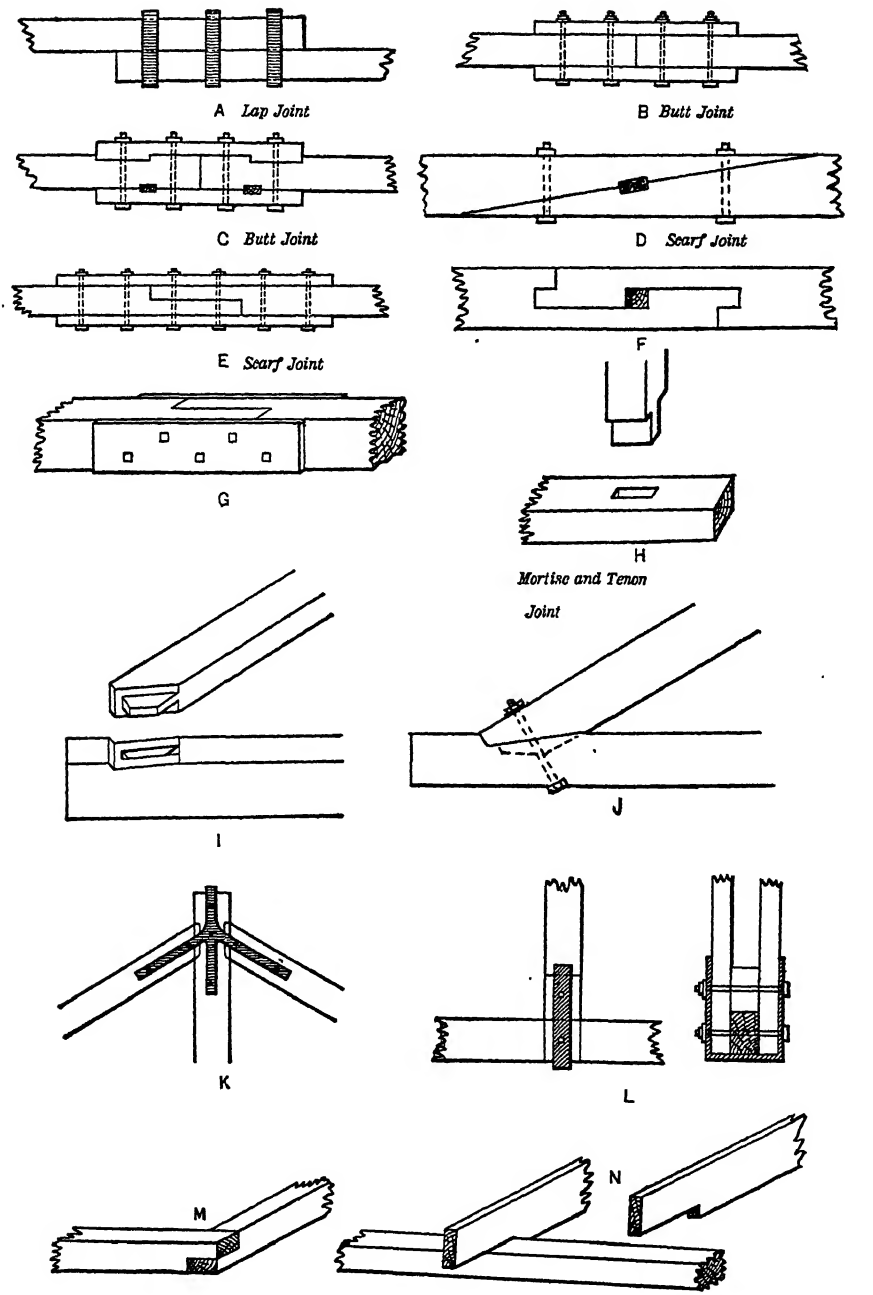


Fig. 1002

Fig. 1002 continued over

Fig. 1002 continued

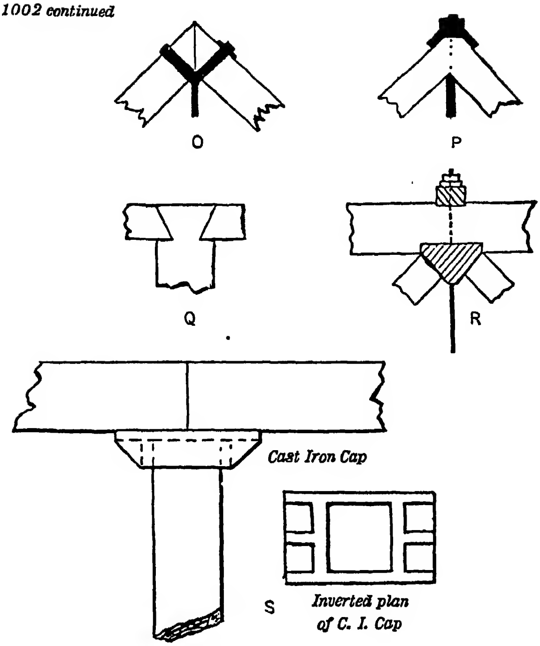


Fig. 1002

1003. With respect to the method of connection, joints are classified as:

Lap joints, as shown in Fig.		1002A.
Butt " " " "	"	1002B, C, E, G.
Scarf " " " "	"	1002D, E, F, G.
Mortise and tenon joints	"	1002H, I, J, K.
Dove-tail joints	"	1002Q.

1004. Joints may be classified, with respect to the forces to which they may be subjected, as tensile, compressive, etc. However, many joints resist more than one kind of stress, and the same joint may be listed under two or more classifications.

1005. **Tension.**—Many of the joints shown in Figs. 1002A to S are suitable to resist tension. In Figs. 1002K, O, and P, the metal fastenings materially strengthen the joints.

1006. **Problem.**—Design a scarf joint similar to the one shown in Fig. 1002F connecting two timbers 8″ × 12″ in cross section and sufficiently strong to resist a tensile stress of 15,000 pounds.

Solution.—The timbers tend to shear off longitudinally in a plane parallel to the direction of the axial tension.

Using equation 236A $\qquad a_s = \frac{F}{A}$.

Making proper substitutions, we have,

$$150 = \frac{15,000}{A}$$

$$A = 100 \text{ sq. in.}$$

As the width of the surface of shear is 12″, the length of the timber from the joint to the end must be at least: $\frac{100}{12} = 8.33''$.

1007. **Compression.**—Joints shown in Figs. 1002B, C, E, and I are suitable to resist compression. The fish plates in Figs. 1002B, C, and E would be more effective if placed on all four sides. The joints, Figs. 1002 J, R, and S are also effective, greater strength being obtained by the addition of the metal pieces.

1008. **Bending.**—In a joint like that in Fig. 1002G, it is found that the resistance to bending is greatest when the fish plates are on the side, as shown. However, joints should be avoided in pieces subjected to bending.

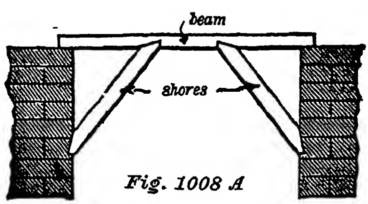


Fig. 1008 A

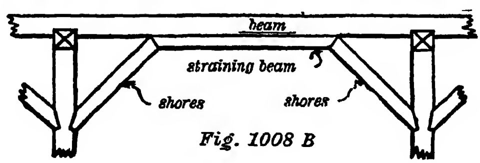


Fig. 1008 B

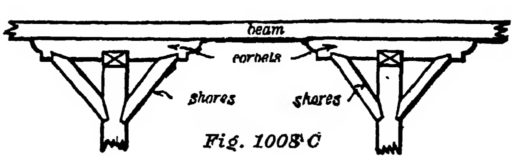


Fig. 1008 C

If the span is so great that the beam may not support the load, it is best to strengthen it by shores, corbels, or shores and straining beams, as shown in Figs. 1008A, B, and C.

BUILT-UP BEAMS.

1009. When a beam or girder of great depth is required, we do not obtain it by merely laying one beam flat upon another, as we would thus secure only as much strength as the two beams would have if separate, and not as much strength as that of a solid beam as large as the two. But if we prevent the two beams from sliding on one another, by inserting transverse blocks or keys, or by indenting them into one another and then bolting or strapping them firmly together to create friction,

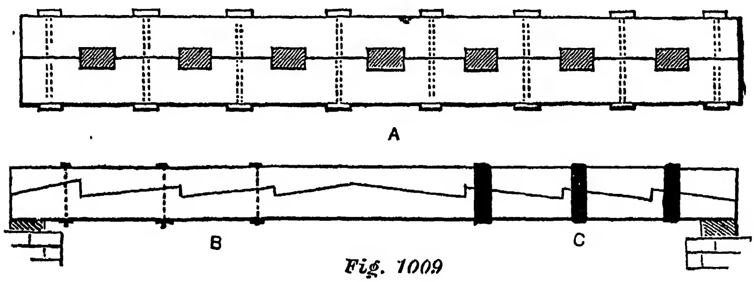


Fig. 1009

we obtain nearly the strength of a solid beam of the total depth; which strength varies as the square of the depth. (Figs. 1009A, B, C.) Beams so constructed are called **built-up beams.**

When a beam is built of several pieces in length as well as in depth, the pieces should break joints with each other. The layers below the neutral axis should be strengthened by the scarf or fish joints used for resisting tension, and the upper pieces should have the ends abut against each other, using plain butt joints.

It is difficult to obtain a large beam without defects. Therefore many builders prefer using a built-up beam of selected timber rather than a single solid one. Moreover, the strength of the built-up beam can be relied upon, although it cannot be stronger than the corresponding solid beam would be if perfectly sound.

1010. **Problem.**—A wooden beam 18′ long, 8″ deep, and 6″ wide, supported at the ends, is made up of two pieces, each 4″ thick and 6″ wide, placed one on the other and held together by round wooden pins 1″ in diameter. A weight of 600 pounds rests on the beam at its middle point. How many wooden pins must be used to insure a factor of safety of 5 against horizontal shear? Neglect weight of beam.

Solution.—The vertical shear on each half of the beam is 300 pounds.

From equation 333A, we have

$$s_s = \frac{3}{2}\left(\frac{S_v}{bD}\right)$$

Making proper substitutions, we obtain;

$$s_s = \frac{3}{2}\left(\frac{300}{6 \times 8}\right) = \frac{75}{8}$$

Total shear $= \frac{75}{8} \times 18 \times 12 \times 6.$

Ultimate value of wood in shear across grain = 3000 pounds.

Safe value with a factor of safety of 5 $= \frac{3000}{5} = 600$ pounds.

Area of one pin = .7854 sq. in.

No. of pins required $= \dfrac{\frac{75}{8} \times 18 \times 12 \times 6}{600 \times .7854} = 25.8$ or 26 pins.

TRUSSES.

1011. A truss is a triangular frame or system of triangular frames so arranged in a structure that the outside forces are applied only at the joints and consequently the members of the frame sustain only longitudinal stresses. In many trusses the pieces or members composing the trusses may be divided into two general groups; horizontal members which are called **chords,** and vertical or inclined members which are called **webs.** In roof trusses, the upper inclined members are called **chords.** Web members subjected to tensile stress are called **ties;** those subjected to compressive stress, **struts.**

For long spans, a system of trusses is very necessary. For example, an ordinary truss railway bridge with a span of 200 feet can sustain its own weight and that of a loaded freight train covering its entire span: but a square steel beam capable of sustaining its own weight and that of a freight train would have to be 64 inches on a side, and would weigh seven times as much as the truss bridge. It is thus seen that for long spans, a system of trusses is not only of great value in saving of metal but also is the only practicable method because of the enormous beams that would be required without the use of trusses.

1012. A very simple truss is shown in Fig. 1012A. The forces acting at the point B must be in equilibrium: therefore, the vertical component of the stress in AB and that in BD must each be equal to W. As shown in Fig. 1012B, the force along $A'B$ or BD' which will have a vertical component equal to W is given by the equation

$$\frac{BC'}{BA'} \text{ and } \frac{BC'}{BD'} = \sin \phi \qquad (1012A)$$

$$BA' \text{ and } BD' = \frac{BC'}{\sin \phi} = \frac{W}{\sin \phi} \qquad (1012B)$$

Each acts toward B and is compressive.

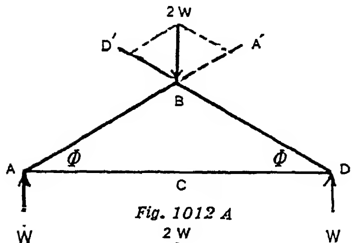


Fig. 1012 A

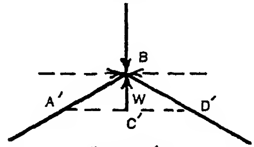


Fig. 1012 B

1013. Going to the point A (Fig. 1013A), we have the compressive force on $B'A$ equal to $\frac{W}{\sin \phi}$. Since the forces at A must be in equilibrium, we have

$$\frac{B'C'}{B'A} = \frac{B'C'}{\frac{W}{\sin \phi}} = \sin \phi \qquad (1013A)$$

$$B'C' = W \qquad (1013B)$$

Also

$$\frac{C'A}{B'A} = \frac{C'A}{\frac{W}{\sin \phi}} = \cos \phi \qquad (1013C)$$

$$C'A = \frac{W}{\tan \phi} \qquad (1013D)$$

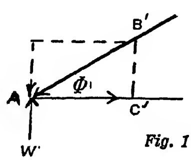


Fig. 1013 A

1014. The above problem may also be solved graphically by the methods of graphical statics. Briefly, this method consists in representing the intensities of forces by length of lines and in determining any unknown forces by the aid of a force polygon which is merely an elaboration of the force parallelogram with which the student of Mechanics is already familiar. The basic principle of the force parallelogram, as applied to graphical statics, lies in the fact that as there is equilibrium at each point, the force parallelogram or force polygon representing all the forces acting at the point must form a closed figure. For more in-

volved problems in graphical statics, the student is referred to Chap. XV of this text.

Applying graphical methods to Fig. 1012A, we find the stresses in AB and BD to be $A'B$ and BD' from the force parallelogram indicated. In this parallelogram, BD' was drawn through B extending BD. Through the upper end of the line representing by its length and location the force 2 W applied at B, a line was drawn parallel to AB. The intersection of this line with BD extended determined the intensities of the components of the force 2 W in the direction of AB and BD, which intensities must be equal and opposite to the stresses in AB and BD if there is to be equilibrium at the point B.

Similarly at the point A, we have, as the known forces acting at that point, the stress in AB and the vertical reaction W at the support. The only unknown force at this point is the stress in the member AD. From A, Fig. 1013A, lay off along AB to any scale a length representing the intensity of the stress in AB. Also lay off from the extremity of line thus determined a line parallel to the known reaction and equal to it in length using the same scale as for AB. The length from A to the point of intersection of AD with this line determines the intensity of the stress in AD.

THE BOOM DERRICK.

1015. The boom derrick (Fig. 1015A) is a machine of triangular frame for lifting weights and distributing them over a horizontal area. The **mast** BB' is of steel or wood, fixed vertically, but rotating about its vertical axis. The **boom** AH is of steel or wood, with its lower end piv-

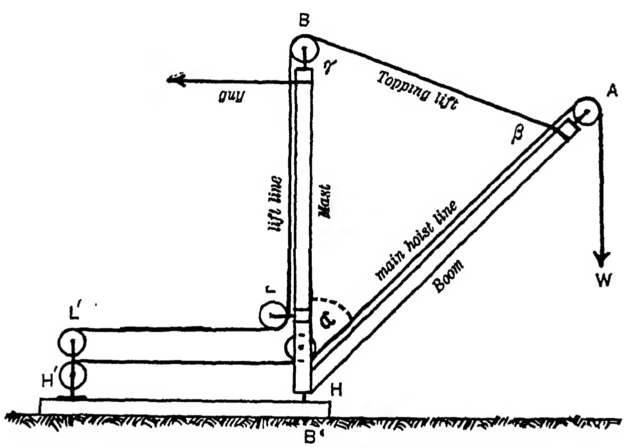


Fig. 1015 A
Boom Derrick

oted at H and its upper end free to move up or down as the length AB is increased or decreased. The **topping lift** is the member of the triangular frame which changes in length. It is the portion AB of a rope running from A to B and then down the mast where it is attached at L or runs to one of the engine drums at L'. The **main hoist** is a rope which runs from W to A, down the boom to H, thence to the engine drum at H'. The guys, usually four in number, are attached to outside supports for the purpose of holding the mast vertical.

Calculation of the stresses in the boom derrick is rendered more complicated by the existence of two stresses along the same line, as for example, a compression in the boom and a tension in the main hoist line, both acting along the line HA. However, there can be no change in the rule that each point or part or combination of parts must be in equilibrium; hence, there is no change in the system of calculating stresses.

Stresses in Boom Derrick.

1016. **Outer End of Boom.**—Let it be assumed that the hoist line from the weight stops at A and the topping lift at B. In this case the solution is not complicated by the existence of two forces acting along the same line. Resolving AW into its components in the direction of AB and AH, we have (Fig. 1016A)

$$AD : W :: \sin \gamma : \sin \beta \tag{1016A}$$

From the triangle ABH we have

$$AH : BH :: \sin \gamma : \sin \beta \tag{1016B}$$

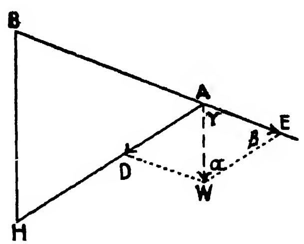


Fig. 1016 A

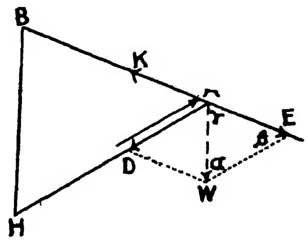


Fig. 1016 B

Thence

$$AD : W :: AH : BH \tag{1016C}$$

$$AD = W\frac{AH}{BH} \tag{1016D}$$

Taking the point A as a "free body," we see that to maintain equilibrium the compression in the boom must equal AD and act in direction DA (Fig. 1016B).

Since AH and BH are fixed for the derrick, the compression in AD is a constant factor of the weight and is independent of the angles of the boom and the mast.

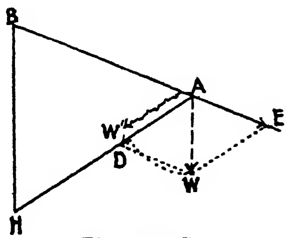


Fig. 1016 C

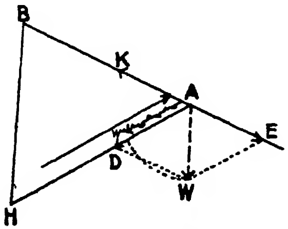


Fig. 1016 D

If the hoist line does not stop at A but goes on to H, we have now two forces along the line AH, one being the component of AW, and the other being an additional force equal to W. Therefore, to maintain equilibrium, the compression in the boom must counteract these two forces. Hence

$$\text{Total Compression in the Boom} = W + W\frac{AH}{BH} \qquad (1016E)$$

1017. **Topping Lift.**—In Fig. 1016A, showing AW resolved into its components in the direction of the topping lift and in the direction of the boom, we have

$$AE : W :: \sin\alpha : \sin\beta \qquad (1017A)$$

From the triangle ABH (Fig. 1015A) we have

$$BA : BH :: \sin\alpha : \sin\beta \qquad (1017B)$$

Thence

$$AE : W :: BA : BH \qquad (1017C)$$

Therefore to maintain equilibrium by counteracting AE, we have

$$\text{Stress in Topping Lift} = AE = W\frac{BA}{BH} \qquad (1017D)$$

Since BH is fixed, the stress in the topping lift increases as BA increases and is a maximum when BA is a maximum which is, under the conditions assumed, when AH is horizontal.

1018. **Top of Mast.**—Let the point B be taken as a "free body," and let it be assumed that the lift line stops at B. Then (Fig. 1018A), assuming the guy to be horizontal,

$$BF = BE\cos\gamma \qquad (1018A)$$

Substituting for BE its value AE from equation 1017D, we know that, to maintain equilibrium, we must have

$$\text{Stress in Mast} = BF = W\frac{BA}{BH}\cos\gamma \qquad (1018B)$$

This stress varies with $BA \cos \gamma$, and changes from compression in the mast, to zero, to tension in the mast as A moves from a point below B to one above B.

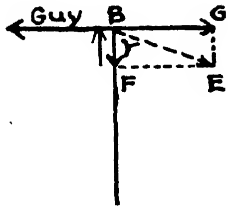


Fig. 1018 A

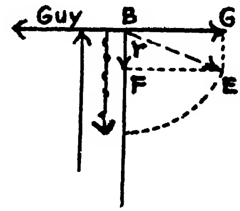


Fig. 1018 B

If the lift does not stop at B but goes on down the mast, we have (Fig. 1018B) an additional force equal to AE acting on the mast, so that, to maintain equilibrium, we must have

$$\text{Total Stress in Mast} = W\frac{BA}{BH} + W\frac{BA}{BH}\cos\gamma \qquad (1018C)$$

1019. **Guy.**—The stress in one or more guys will be due to BG (Figs. 1018A and B), the horizontal component of the topping lift. For a horizontal guy, we have

$$BG = BE \sin\gamma \qquad (1019A)$$

Substituting for BE its value AE from equation 1017D, we know that, to maintain equilibrium, we must have,

$$\text{Stress in Guy} = BG = W\frac{BA}{BH}\sin\gamma \qquad (1019B)$$

But from Fig. 1015A, we know that

$$BA : AH :: \sin\alpha : \sin\gamma \qquad (1019C)$$

Substituting in equation 1019B, we obtain

$$\text{Stress in Guy} = BG = W\frac{AH}{BH}\sin\alpha \qquad (1019D)$$

The only variable is $\sin\alpha$; therefore the stress in the guy varies with the angle α and is a maximum when $\alpha = 90°$, that is when the boom is horizontal.

If the guy is not horizontal, but makes an angle ϕ with the horizontal, then its horizontal component must equal BG, and the actual stress in the guy will be

$$W\frac{AH}{BH}\frac{\sin\alpha}{\cos\phi} \qquad (1019E)$$

1020. **Problem.**—The mast and the boom of a boom derrick are of equal length. The guy makes an angle of 45° with the horizontal. The main hoist line is made fast at the outer end of the boom. The topping lift is made fast at the outer end of the

boom and at the top of the mast. The load is 100 pounds. With the boom horizontal, find graphically the stress in the mast.

Solution.—Referring to Fig. 1015A, assume AH to be horizontal. Finding graphically the components of AW in the direction of the topping lift and of the boom, we have

$$\text{Component along topping lift} = 141$$
$$\text{Component along boom} = 100$$

Next, finding the components of the stress in the topping lift in the direction of the guy and the mast, we obtain

Component along the guy = Component along the topping lift (since both make an angle of 45° with the vertical) = 141.
Component along the mast (graphically or by calculation) = 200.

The stress in the mast is therefore 200 pounds. This stress is compression. By inspection it is also evident that it is compression, as the direction of the applied forces at B will cause compression in the mast.

1021. **Base.**—If the hoist line and the topping lift stop at A and B, respectively, we have for the reaction at the support the resultant of the stresses in the boom and the mast, as given in equations 1016D and 1018B.

If the lines stop at H and L, the reaction is the resultant of the compressive stresses as given in equations 1016E and 1018C minus the tensile forces equal to W and AE as carried by the lines along the boom and the mast to H.

If the lines go to drums at H' and L', as is the case in large derricks, the reaction is the resultant of the forces already stated plus the two tensile forces equal to W and AE as carried by the lines from H and L to the drums.

1022. **Problem.**—A boom derrick is made up as follows:—The mast is 24′ long; the boom is 32′ long and is attached to the mast at the base of the mast. The guy makes an angle of 30° with the horizontal; the main hoist line runs along the boom to its junction with the mast, where it is fastened; the topping lift is fastened at the outer end of the boom and at the bottom of the mast. The boom is horizontal. The load is 12 tons. Find the reaction at the foot of the mast.

Solution.—To solve this problem it is usually desirable to work from the point of application of the load, finding the stresses in each member of the structure and thus eventually obtaining the stress in the mast and the reaction at the base. It may also be solved merely for the reaction by considering the derrick as a structure in equilibrium under three external forces which, being in equilibrium, must be either parallel or concurrent. In this case they cannot be parallel as will be seen. They must be concurrent. These forces are the applied load, the stress in the guy, and the reaction at the foot of the mast.

The direction of the action line of the guy stress, the action line and intensity of the applied load, and the point of application of the reaction at the foot of the mast are known. Extend the two known action lines until they intersect. The action line of the reaction extended passes through this point of intersection because the system is concurrent. A parallelogram of forces constructed at this point of intersection on the known applied load gives the components of this known force in the direction of the

guy and the reaction. Performing the indicated operations graphically the reaction at the base of mast is found to have a numerical value of 26.6 tons.

Solving by the usual method and first finding the components of the load in the direction of the boom and topping lift, we have:

Component in direction of topping lift = 20 tons.
Component in direction of boom = 16 tons.
Total stress in the topping lift = 20 tons.
Total stress in the boom = 16 + 12 = 28 tons.

Next finding the components of the topping lift stress in the guy and along the mast, we have:

Vertical component of stress in the mast (due to stress in topping lift) = 12 tons.
Horizontal component of stress in guy (due to stress in topping lift) = 16 tons.

Stress in the Guy $= \dfrac{16}{\cos 30°}$.

Component along mast of stress in guy $= \dfrac{16 \times \sin 30°}{\cos 30°} = 9.2$ tons.

Stress along lift line = 20 tons.
Total stress in mast = 12 + 9.2 + 20 = 41.2 tons.

The forces acting at the bottom of the mast are then as follows:—

Vertical forces: 41.2 tons compression in mast, and 20 tons tension in topping lift rope, or a resultant of 21.2 tons compression.

Horizontal forces: 28 tons compression in boom, and 12 tons tension in the main fall rope, or a resultant horizontal force of 16 compression.

The reaction is equal to and opposed to the resultant of these two forces or

$$\text{Reaction} = \sqrt{21.2^2 + 16^2} = 26.6 \text{ tons.}$$

For more detailed information see:

Structural Details of Elements of Design in Timber Framing, Jacoby.
Civil Engineer's Pocket Book, Trautwine.

PROBLEMS.

P. 1001. (Fig. P. 1001.) The cantilever is made up of two pieces bolted one on top of the other, each 1.5″ deep by 3″ wide. How many ½″ wrought-iron bolts are necessary to safely resist the horizontal shear in *AB*? How many in *BC*?

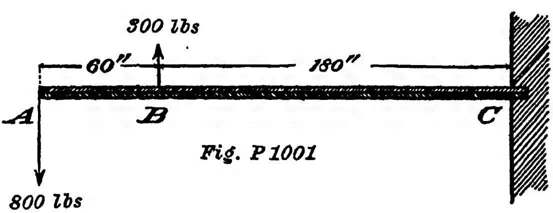


Fig. P 1001

P. 1002. A wooden beam, 18′ long, 8″ deep, and 6″ wide, supported at the ends, is made up of two pieces 4″ thick and 6″ wide, placed one on the other and held together by wooden pins one inch in diameter. A weight of 500 pounds rests on the beam at its middle point. How many wooden pins must be used to insure a factor of safety of 6? Neglect weight of beam.

P. 1003. A wooden beam, 18′ long, is made of two pieces 12″ by 12″, placed one upon the other, and rests upon end supports. The beam carries a uniformly distributed load of 200 pounds per lineal foot. If the two pieces are bolted together what is the total shear on the bolts? Neglect weight of beam.

P. 1004. A beam 10′ long and 6″ by 6″ in cross-section, resting on end supports, is made up of three 2″ by 6″ wooden planks superimposed. Three bolts, one at each end and one in the middle of the beam, hold the planks together. Neglect weight of beam.

(*a*) If the beam is subjected to a uniformly distributed load of 200 pounds per lineal foot what is the shearing stress on each bolt?

(*b*) If the bolt at one end should fail, what would be the shear on each of the other two bolts?

P. 1005. A wooden beam is composed of two pieces, placed one on the other and is 16′ long, 12″ deep, and 8″ wide, and rests on end supports. The top piece is 8″ wide and 8″ deep; the bottom piece, 8″ wide and 4″ deep. The two pieces act as a solid beam because of seven pieces of wood 2″ by 2″ by 8″ with grain horizontal being notched into both pieces at intervals of 2′. Find the safe load upon the middle of the beam. Factor of safety equals 6.

P. 1006. The mast and boom of a boom derrick are of the same length, each being 20′ long. The boom may have any position between the vertical and the horizontal. Find graphically the maximum stress to which the mast may be subjected.

P. 1007. The boom and the mast of a boom derrick are each 30′ long. The main fall is made fast at the outer end of the boom, and the topping lift at the outer end of the boom and at the top of the mast. The guy is horizontal. The boom may have any position between the vertical and the horizontal.

(*a*) Find analytically what stress the boom must be designed to carry.
(*b*) Find analytically what stress the mast must be designed to carry.

P. 1008. The mast and the boom of a boom derrick are of equal length. The guy makes an angle of 45° with the horizontal. The main fall is made fast at the outer end of the boom. The topping lift is made fast at the outer end of the boom and at the top of the mast. The load is 100 pounds. With the boom in a position making an angle of 60° with the mast, find analytically:

(*a*) the stress in the guy.
(*b*) the stress in the mast.

P. 1009. A boom derrick is made up as follows:—The mast is 30′ long; the boom is 32′ long and is attached to the mast 6′ from the ground; the guy makes an angle of 30° with the horizontal; the main fall runs along the boom to its junction with the mast, thence down the mast to the bottom of which it is fastened; the topping lift is fastened at the outer end of the boom and at the bottom of the mast. The boom may have any position from the horizontal to 10° from the vertical. Weight, 12 tons.

(*a*) Find graphically the maximum stress in the boom.
(*b*) " " " " " " " topping lift.
(*c*) " " " " " " " guy.
(*d*) " " " " longitudinal stress in the mast.
(*e*) " " " " shear in the mast.
(*f*) " " " " bending moment in the mast in (foot-pounds).
(*g*) " " " reaction at the foot of the mast when the boom is horizontal.

P. 1010. A boom derrick has a boom 30′ long attached to the mast at its base A. The mast is 40′ high. The main fall passes over a pulley at the other end, B, of the boom, down the boom and is attached to a cleat on the mast at A. The topping lift passes over a pulley at C (the top of the mast) and is attached to a cleat on the mast at A. The boom may have a position making any angle with the mast from 10° to 90°. The guy makes an angle of 45° with the mast.

(*a*) Find analytically the maximum compressive stress for which the boom should be designed in order to carry a load of 20 tons at B.

(*b*) Find analytically the maximum stress in the topping lift.

(*c*) Find analytically the stress for which the guy should be designed.

(*d*) Find analytically the stress for which the mast should be designed.

(*e*) Find graphically the value and direction of the reaction at the base A with the boom in the horizontal position.

P. 1011. A boom derrick has a mast 16′ high and a boom 16′ long, the boom being hinged to the mast 4′ from the bottom. The topping lift is attached to the outer end of the boom and the top of the mast. The main fall passes over the end of the boom, along the boom, over a pulley attached to the hinge between the mast and the boom and thence horizontally to the engine. The weight supported by the main fall is 12 tons. The boom may have any position from the horizontal to 10° with the vertical. The guy is horizontal. Find analytically: —

(*a*) The maximum stress in the boom.
(*b*) " " " " " topping lift.
(*c*) The maximum shear in the mast.
(*d*) " " stress in the guy.
(*e*) " " compression in the mast.
(*f*) " " bending moment in the mast in (foot tons).

P. 1012. (Fig. P. 1012.) The boom may have a position making any angle from 10° to 90° with the mast.

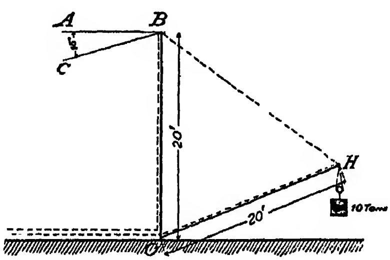


Fig. P 1012

(*a*) For what stress must the boom be designed?

(*b*) With the boom in the horizontal position find the reaction at the bottom of the mast.

CHAPTER XI.

MASONRY.

1101. Masonry is the art of erecting structures in natural and artificial stone. The term is also applied to the structures thus built. It is ordinarily classified as Stone Masonry, Brick Masonry, and Concrete Masonry.

1102. **Masonry structures** are divided into classes according to the kind of stresses they are to sustain. Their forms and dimensions are determined by the amounts and kinds of these stresses. They may be classed as follows:

a. Those which sustain only their own weight; as walls of inclosures.

b. Those which, besides their own weight, are required to support a vertical pressure arising from a weight placed upon them; as the walls of a building, piers of arches.

c. Those which, besides their own weight, are required to resist a lateral thrust; as a wall supporting an embankment, reservoir walls, etc.

d. Those which, sustaining a vertical pressure, are subjected to a transverse strain; as lintels, areas, etc.

e. Those which are required to transmit the pressure they directly receive to lateral points of support; as arches.

Walls, columns, and arches comprise most of the masonry structures in modern engineering: lintels and areas do not require much discussion.

1103. **Construction Rules.**—The following general rules should be followed in masonry work:

a. Build the masonry in a series of courses, which shall be perpendicular, or as nearly so as practicable, to the direction of the force it is to sustain.

b. Avoid the use of continuous joints parallel to the direction of the force.

c. Use the largest stones in the lower courses.

d. Lay the stratified stones so that the force shall be perpendicular to the planes of stratification.

e. Moisten all dry and porous stones before bedding them in mortar, and thoroughly cleanse from dust, etc., their lower surfaces and the bed of the course on which the stones are to be laid.

f. Reduce the space between the stones as much as possible, and completely fill the joints with mortar.

PRESSURES.

1104. The surface on which a structure rests is required to support the weight of the structure, and also any additional load or thrust imposed on the structure. The resultant of the weight of the structure and these other forces pierces the surface on which the structure rests at a point called the **center of pressure.**

1105. The pressure on each point of the surface is determined by the position of the center of pressure. In some structures, the center of pressure must be at such a point that all pressures on the surface are compressive. In other structures, this is not necessary. The surface is said to be **centrally loaded** when the resultant is perpendicular to the surface and pierces it at its central point. When this is not the case, it is said to be **eccentrically loaded.**

1106. **Distribution of Pressures.**—The theoretical distribution of pressure over the surface depends upon the following hypotheses:

a. The surface of cross-section does not change form under the action of the force.

b. All fibers offer equal resistances to tensile and compressive forces.

c. The unit stress developed in a fiber is proportional to its strain.

1107. Represent by Fig. 1107A a surface under pressure with center of pressure on an axis of symmetry.

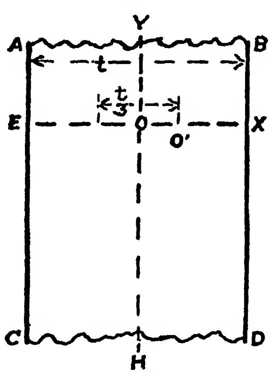


Fig. 1107 A

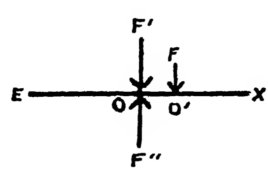


Fig. 1107 B

Let O = center of gravity of $ABCD$;

EX and HY = axes of symmetry;

F = component of the resultant force which is parallel to the axis of the column and perpendicular to the surface;

O' = center of pressure or point of application of F in the cross-section of $ABCD$;

s = unit longitudinal stress, due to F, at any point in the cross-section $ABCD$;
x_F and y_F = co-ordinates of point O', the center of pressure;
(then y_F = 0 for point on the axis of symmetry);
x and y = co-ordinates of any point of area $ABCD$;
(then y = 0 for point on the axis of symmetry);
r' = radius of gyration of area $ABCD$ about HY;
r'' = " " " " " EX;
t = width of the surface along the axis of symmetry EX.

Without changing the pressure, we may introduce at the center of gravity, O, two opposite forces, F' and F'', each equal to the force F (Fig. 1107B). This gives us a force F' at the center of gravity, and a couple FF'' with a lever arm x_F. The total stress equals the stress due to the compressive force F', plus that due to the couple, FF''.

The compressive stress due to F' is equally distributed over the whole area. Representing this unit stress by s', we have the same equation as in compression (222A)

$$s' = \frac{F'}{A} = \frac{F}{A} \tag{1107A}$$

for unit stress at any point on the area.

The couple FF'' has a tendency to rotate the area $ABCD$ about some line in its plane. Due to the couple above, this may be any line parallel to HY, since we know from Mechanics that the moment of a couple is the same for all such lines. But from hypothesis 1106c above, the couple will tend to rotate the area $ABCD$ around the line HY because it thus encounters less resistance. Also, from the same hypothesis, the unit stress at any point varies with its distance from the axis. Therefore, representing this unit stress by s'', we have the same equation as in bending (260A)

$$s'' = \frac{M_f y}{I} = \frac{M_f y}{Ar^2} \tag{1107B}$$

in which y is the distance from the axis to the fiber stressed.

The total stress is therefore given by the equation

$$s = s' + s'' = \frac{F}{A} + \frac{M_f y}{I} = \frac{F}{A} + \frac{M_f y}{Ar^2} \tag{1107C}$$

which is the general equation for unit pressure on a foundation.

To find the unit pressure at any point on the axis of symmetry (x, o) when the center of pressure is also on the axis of symmetry (taken at x_F, o), we substitute Fx_F for M_f, x for y, and r'^2 for r^2, and obtain

$$s = s' + s'' = \frac{F}{A} + \frac{F}{A}\left(\frac{x_F x}{r'^2}\right) \tag{1107D}$$

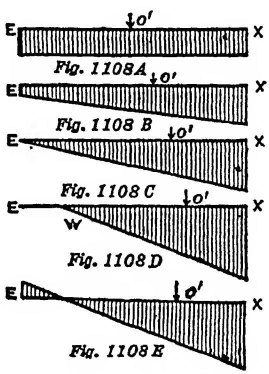
or,

$$s = \frac{F}{A}\left(1 + \frac{x_F x}{r'^2}\right) \tag{1107E}$$

For a rectangular surface, $r'^2 = \frac{t^2}{12}$. Substituting, we obtain

$$s = \frac{F}{A}\left(1 + \frac{x_F x}{\frac{t^2}{12}}\right) \tag{1107F}$$

1108. By assigning different values to x_F and to x, we can find the unit stress at any point along the X axis for any position of the center of pressure (co-ordinate x_F) along that axis. The resulting pressures



at all points for different positions of the center of pressure are shown in Figs. 1108A to E.

a. $x_F = 0$. The equation reduces to the form

$$s = \frac{F}{A} \tag{1108A}$$

This is its form for all values of x, positive and negative: that is, the stress over the area $ABCD$ is uniform (Fig. 1108A). Hence, *a central load always produces uniform stress in every cross-section.*

b. $x = 0$. The equation reduces to the form $s = \frac{F}{A}$ (1108B)

This is the form for all values of x_F, positive or negative. Hence, *the unit stress at the center of gravity will be the same for all positions of the center of pressure.*

c. x_F *varying from* 0 *to* $\frac{t}{2}$. Figs. B, C, D, and E show the corresponding values of stress for different points. To find the extreme value of x_F which

will give a compressive stress over the entire area, that is, give a stress of zero in a rectangular surface at the point where $x = -\frac{t}{2}$, we substitute this value in equation 1107F and have

$$0 = \frac{F}{A}\left(1 + \frac{(x_F)\left(-\frac{t}{2}\right)}{\frac{t^2}{12}}\right) \tag{1108C}$$

Solving,

$$x_F = \frac{t}{6} \tag{1108D}$$

Hence, *the stress over the entire surface is compressive* (Fig. 1108C) *if the center of pressure is on the axis of symmetry, and not more than* $\frac{t}{6}$ *from the center of gravity in the case of a rectangular surface.* This principle is used in the design of dams. In the general case, it is assumed that the joints take no tension; therefore, if the center of pressure is outside of the middle third (Fig. 1108D), a crack will form from E to W and water will penetrate and tend by upward water pressure to overturn the dam. If, however, the joints will take tension, as in reinforced concrete columns and in dams where reinforcing rods are used, the stress will be as shown in Fig. 1108E.

d. $x_F = \frac{t}{6}$, $x = \pm\frac{t}{2}$. When the pressure is at the edge of the middle third of a rectangular surface, we have from equation 1107F

$$s = \frac{F}{A}\left(1 + \frac{\frac{t}{6}\left(\pm\frac{t}{2}\right)}{\frac{t^2}{12}}\right) \tag{1108E}$$

For $x = -\frac{t}{2}$, we have $s = 0$.

$$\text{For } x = +\frac{t}{2}, \text{ we have } s = \frac{F}{A}(1 + 1) = \frac{2F}{A} \tag{1108F}$$

Hence, *when the center of pressure is at the edge of the middle third of a rectangular surface, the unit stress is zero at one extremity of the axis of symmetry and twice the mean stress at its other end.*

e. **Maximum Stress.**—The stress will be a maximum when the expression $\left(\frac{x_F x}{r'^2}\right)$ is a maximum. This will be the case when x_F and x have their maximum values.

In a rectangular surface, this will be when their values are $\frac{t}{2}$. Substituting in equation 1107F, we have

$$s = \frac{F}{A}\left(1 + \frac{\frac{t}{2} \times \frac{t}{2}}{\frac{t^2}{12}}\right) = 4\frac{F}{A} \qquad \text{(1108G)}$$

Hence, *the maximum stress in a rectangular surface is of the same character as and four times the value of the mean stress, and occurs at the extremity of the axis of symmetry when the center of pressure is at the same point.*

Minimum Stress.—The stress will be a minimum when the expression $\frac{x_F x}{r'^2}$ is a minimum. This will be the case when x_F and x have their maximum values but are of opposite signs. In a rectangular surface, this will be when $x_F = \frac{t}{2}$ and $x = -\frac{t}{2}$. Substituting in equation 1107F, we have

$$s = \frac{F}{A}\left(1 + \frac{\frac{t}{2} \times -\frac{t}{2}}{\frac{t^2}{12}}\right) = -2\frac{F}{A} \qquad \text{(1108H)}$$

Hence, *the minimum stress in a rectangular surface is of opposite character and twice the mean stress, and occurs at the extremity of the axis of symmetry when the center of pressure is at the other extremity.*

1109. **Problem.**—The cross-section of a masonry dam is a trapezoid with horizontal top and bottom and vertical back. It is 50′ high, 12′ wide on top, and 30′ wide on the bottom. The masonry weighs 150 pounds per cubic foot. Find the maximum pressure per square foot on the base when there is no water behind the dam.

Solution.—Consider the cross-section of the dam to be subdivided into a rectangle and a triangle. Then for a linear foot of the dam, the weights of the component parts are:

Rectangle $= 50 \times 12 \times 1 \times 150 = 90{,}000$ pounds.

Triangle $= 50 \times \frac{30 - 12}{2} \times 1 \times 150 = 67{,}500$ pounds.

Take moments about the heel of the dam to find the position of the resultant:

$$R \times t = +67{,}500 \times 18 + 90{,}000 \times 6$$

$$t = \frac{1{,}215{,}000 + 540{,}000}{157{,}500} = \frac{78}{7}$$

Then $\quad x_F = 15 - \frac{78}{7} = 3.86'$, which is within the middle third.

And $\quad s = \frac{F}{A}\left(1 + \frac{x_F x}{r'^2}\right) = \frac{157{,}500}{30}\left(1 + \frac{3.86 \times 15 \times 12}{30 \times 30}\right) = 9300$ pounds (Pressure at heel in pounds per square foot)

1110. If the center of pressure is not on an axis of symmetry of the surface, the resultant pressure may be resolved into two components whose centers of pressure are on these axes and also on a straight line passing through the center of pressure. The pressure is then found due to these components on the axes of symmetry, and the combined pressures give the pressure due to both. The discussion is omitted in this text.

WALLS.

1111. In a masonry wall the front side is called the **face;** the inside or opposite side, the **back;** the stones which form the front are called the **facing,** and those of the back, the **backing;** the portion between these forming the interior of the wall, the **filling.**

If a uniform slope is given to the face or back, this slope is termed the **batter.**

The section made by a vertical plane passed perpendicular to the face of the wall is called the **profile.**

Each horizontal layer of stone in the wall is called a **course;** the upper surface of the stone in each course, the **bed;** and the surfaces of contact of two adjacent stones, the **joints.**

When the stones of each layer are of equal thickness throughout, the term **regular coursing** is applied; if unequal, **random coursing.** The particular arrangement of the different stones of each course, or of contiguous courses, is called the **bond.**

Stone Walls.

1112. **Rubble.**—The stones used are of different sizes and shapes, prepared by knocking off all sharp, weak angles of the blocks with a

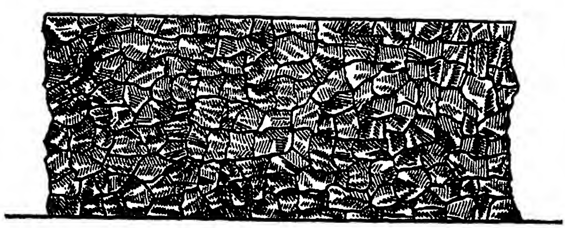


Fig. 1112 A
Rubble Wall

hammer. They are laid in the wall either dry or in mortar. If placed without reference to regular and continuous lines, the masonry is known as **uncoursed rubble,** or **common rubble** masonry.

In building a rubble wall (Fig. 1112A) the stones should be so placed that they will fit one upon the other, filling the interstices between the

larger stones by smaller ones; and the vertical joints should not be continuous.

The mean thickness of a rubble wall should be not less than one-sixth of its height; in the case of a dry stone wall, the thickness should never be less than two feet. It strengthens the wall very much to use frequently in every course, stones which pass entirely through the wall from the face to the back. These are called **throughs.**

1113. **Ashlar.**—The stones in this kind of masonry are prepared by having their beds and joints accurately squared and dressed. The

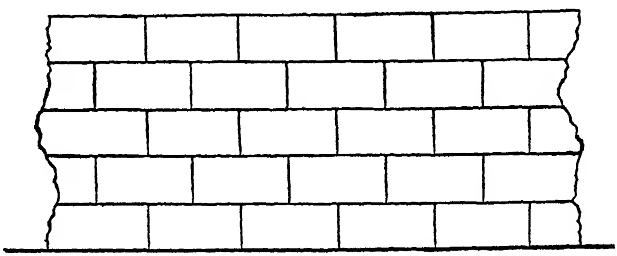


Fig. 1113 A Wall with facing of plane ashlar

sizes used in a wall depend upon the kind of wall to be built and the quarry sizes available. Ordinarily they are about one foot high, two or three feet long, and have a width from once to twice the height

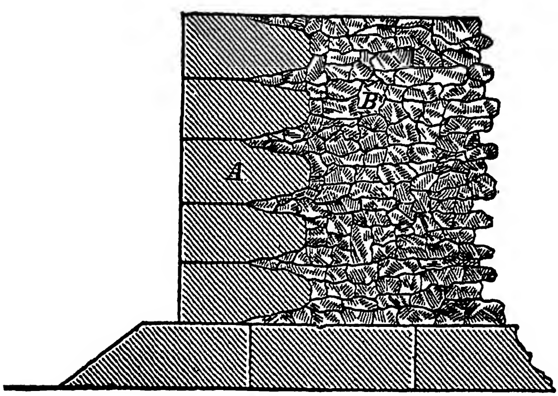


Fig. 1113 B **Section of wall with facing of ashlar and a backing of rubble**

(Fig. 1113A). They are used generally for the facing of a wall to give the front a regular and uniform appearance, and to produce a desired architectural effect.

Walls built with ashlar facing are backed with brick or rubble (Fig. 1113B). The question of economy will decide which is to be used. In the construction, **throughs** of ashlar should be used to bind the backing to the facing.

1114. **Cut-Stone.**—Where great strength is required in the wall, each stone is prepared by cutting it into a particular shape, so that it can be exactly fitted in the wall. In other words every stone is ashlar.

Cut-stone masonry, when carefully constructed, is more solid and stronger than any other class. The labor required in preparing the blocks makes it the most expensive.

Brick Walls.

1115. The least proper thickness of a brick wall is 8 inches, which is the thickness of walls of a low brick dwelling. When brick walls must be especially strong, it is best to build **throughs** into them at intervals of two and one half to three feet.

Concrete Walls.

1116. Concrete walls are usually monolithic (of one piece). However, long walls are often made in sections, with tongues and grooves at the joints. In building breakwaters, dikes, etc., the concrete is often made into large blocks which are fitted in place if the water is excluded, or thrown in at random if the water is not excluded.

Bonds.

1117. **English Bond.**—This consists in forming each course entirely of headers or of stretchers, as shown in Fig. 1117A.

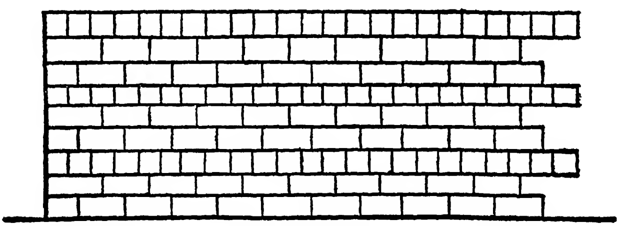


Fig. 1117 A
English Bond

Sometimes the courses of headers and stretchers occur alternately; sometimes only one of headers for three or four courses of stretchers. The effect of the stretchers is to tie the wall together lengthwise, and the headers crosswise. The proportionate number of courses of headers to those of stretchers depends upon the relative importance of the transverse and longitudinal strength of the wall.

1118. **Flemish Bond.**—This consists in laying headers and stretchers alternately in each course (Fig. 1118A).

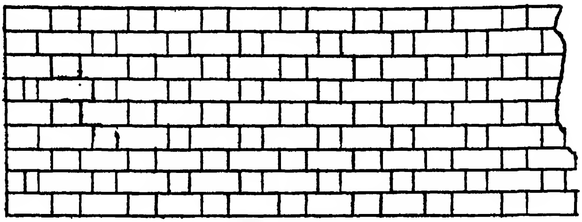


Fig. 1118 A
Flemish Bond

A wall with this bond presents a neater appearance than one built in English bond, and is, therefore, generally preferred for the fronts of buildings. It is not considered as strong as the English, as it has a less number of headers.

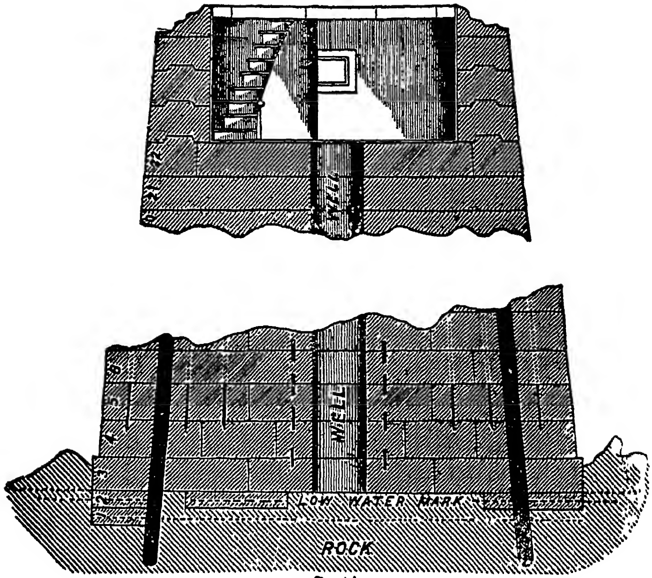
1119. **Special bonds** are often necessary, as in lighthouses where the tendency of the stones is to "jump out" immediately after receiving the blow of the wave. Fig. 1119A shows a lighthouse so constructed that the stones are dovetailed into each other and also fastened together with metal. Concrete is now generally used in such cases.

ARCHES.

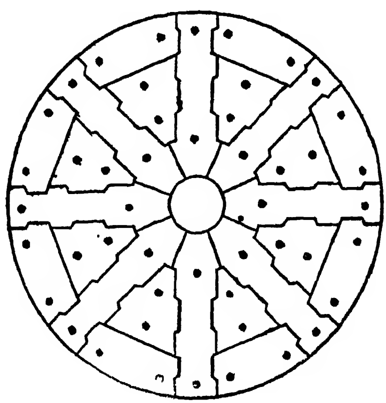
1120. **A stone masonry arch** is a combination of wedge-shaped blocks, called **voussoirs,** or **arch stones,** supporting each other by their mutual pressure, the combination being supported at the two ends (Fig. 1120A).

The supports against which the extreme voussoirs rest are almost invariably built of masonry, though in some cases the supports are of natural rocks. If a support holds up two successive arches it is called a **pier;** if it holds up an embankment, generally of earth, on one side, and an arch on the other, it is called an **abutment.** The inner (lower) surface of the arch is called the **soffit;** its outer (upper) surface, the **back.** The end surface of the arch is called the **face,** and sometimes the **head** of the arch. The surface of the top course of stones on the pier or abutment is called the **skewback;** it is generally sloping. The stones forming the skewback are called the **cushion-stones.** The highest course of an arch is composed of **keystones.**

The line in which the soffit of the arch intersects the pier or the abutment is called the **springing-line.** The chord, AB (Fig. 1120A), is termed the **span,** and the height, HC, of the keystone above this line is termed the **rise.** The **length** of the arch is that of the springing-line. The highest



Section



Plan

Fig. 1119 A

Plan and Section of Lighthouse
showing how stones are doweled and dove tailed

line of the soffit, that projected at C, is called the **crown.** The line in the plane of the springing-lines projected at H, symmetrically disposed with respect to the plan of the soffit on the plane, is the **axis** of the arch.

The **intrados** is the line of intersection of the soffit with a plane perpendicular to the axis; the **extrados** is the line of intersection of the back

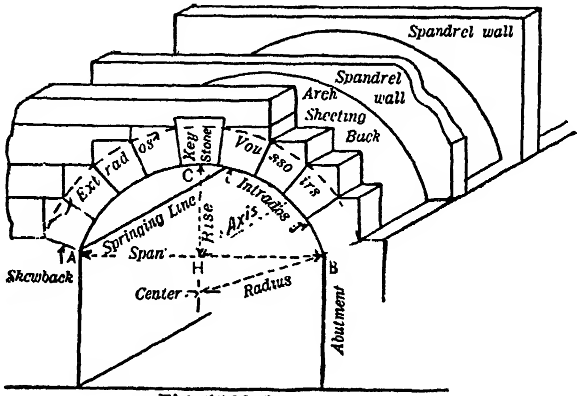


Fig. 1120 A
Details of Arch

with the same plane. The courses of stones parallel to the head of the arch are called **ring-courses;** those which run lengthwise of the arch are termed **string-courses.** The joints between the different ring-courses are called **heading-joints;** those between the different string-courses are termed **coursing-joints** or **bed-joints.**

A wall standing on an arch and parallel to the head is called a **spandrel-wall.** The filling between two spandrel-walls is called **spandrel-filling.**

1121. **Classification.**—Arches are classified according to the direction of the axis and the form of the intrados. A **right** arch is one whose axis

Fig. 1121 A
Segmental

Fig. 1121 B
Elliptical

Fig. 1121 C
Basket Handled Oval

Fig. 1121 D
Tudor

Arches

is perpendicular to the face. The arch is called **oblique** when the axis is oblique to the face; and **rampant** when the axis is oblique to the horizontal plane.

The intrados of a **full-center** arch is a semi-circle. **Segmental, elliptical, oval, tudor** arches have for intrados respectively a segment of a circle, a semi-ellipse, an oval with three or more centers, a pointed arch made up of two intersecting curves. (See Figs. 1121A to D.)

1122. The soffits of groined and cloistered arches are formed by the intersection of two cylindrical soffits having the same rise and intersecting axes. The **groined** arch (Fig. 1122A) is made by removing from each cylindrical arch those portions of itself which lie within the corresponding parts of the other arch; in this way the two soffits are so connected that

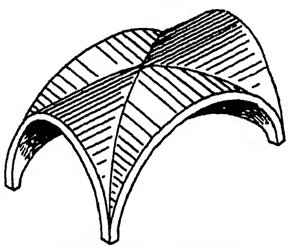


Fig. 1122 A Groined Arch

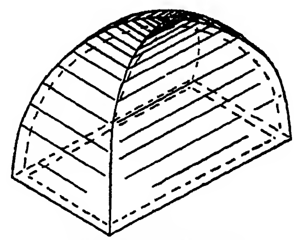


Fig 1122 B Cloistered Arch

the two arches open freely into each other. The **cloistered** arch (Fig. 1122B) is made by retaining in each cylindrical arch only those portions of itself which lie within the corresponding portions of the other arch; thus, a portion of the soffit of each arch is inclosed within the other, these portions forming a four-sided vaulted ceiling. This arch was much used in forming the ceilings of the cells of monasteries; from their object and use is derived the term cloistered.

A **dome** is an arch whose soffit is the surface of a hemisphere, the half of a spheroid, or other similar surface.

Arches, whose soffits are **warped surfaces**, are frequently used. The particular kind of warped surface will depend upon circumstances.

Pressure.

1123. An arch may fail by rotation, by sliding, or by crushing of the material. A failure by crushing may take place if the resultant of the forces acts on or near the outer or inner edge of a joint; but construction to avoid failure by rotation or sliding also provides against failure by crushing of materials of which arches are ordinarily constructed. Therefore, failure by crushing is not usually considered, and the design of an arch is based on its possible failure by rotation or by sliding.

1124. As shown in paragraph 1108, if the resultant is not within the middle third, a portion of the stress will be tensile or a part of the surface will have no pressure at all. A general rule is therefore adopted that the resultant must come within the middle third of the surface pressed.

In considering possible failure by sliding, the angle of friction of masonry on masonry is taken at 37°. As shown by Fig. 1124A, the action line of resultant pressure must make an angle of 37° or less with the normal to the surface.

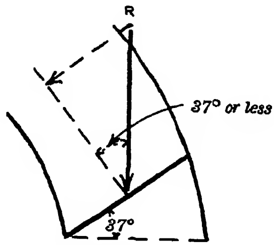


Fig. 1124 A

1125. Figs. 1125A to D show ordinary modes of rupture of arches. In Fig. 1125A, the resultant acts outside the middle third, causes outward rotation at CD and $C'D'$ with consequent failure also at EF, $E'F'$, and

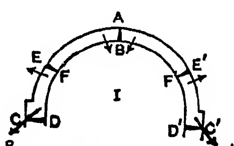


Fig. 1125 A

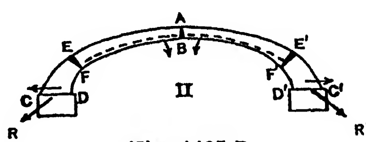


Fig. 1125 B

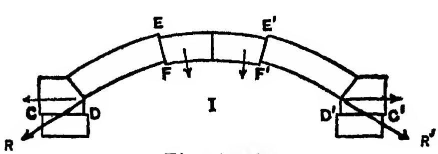


Fig. 1125 C

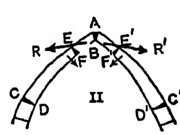


Fig. 1125 D

AB. In Figs. 1125B and C, the rupture is due to sliding, the resultant making an angle greater than 37° with the normal to the surface. In Fig. 1125D, the arch has a rise much more than half the span, and the resultant acts outside of the middle third of one of the joints near the crown, causing rupture, as at EF.

1126. **Theory of the Arch.**—Considering one half of an arch as a " free body " (Fig. 1126A), made of materials of infinite strength and without elasticity, there are three forces acting to keep it in equilibrium: (1) the weight of this half of the arch, $\frac{W}{2}$; (2) the thrust at the crown due

to the right half of the arch, F; (3) the reaction at the skewback, R. The intensity and action line of $\frac{W}{2}$ can be determined fairly closely from the dimensions of the arch and the specific gravity of the materials. Of

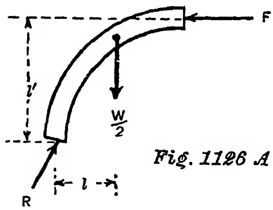


Fig. 1126 A

F and R, it is known that they should act within the middle third of their corresponding surfaces and must not make an angle greater than 37° with the normal to the surfaces.

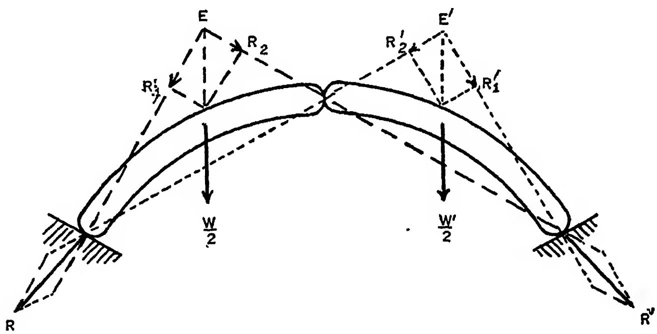


Fig. 1126 B

If we could construct an arch hinged as shown in Fig. 1126B, it would be possible to determine exactly the forces F and R. In the figure, the reactions due to $\frac{W}{2}$ would be equal and opposite to R_1 and R_2: those due to $\frac{W'}{2}$ would be equal and opposite to R_1' and R_2'. Their resultant reactions at each support are found by force polygons and drawn as shown, R and R'.

1127. If the arch is perfectly symmetrical and its corresponding elements are exactly equal in weight, the thrust, F, at the crown will be horizontal. As arches are thus symmetrically designed, the thrust at the crown may be taken as horizontal.

Remembering that the thrust at the crown and the reaction at the skewback must, for safety, lie within the middle third, and assuming for first computation that the thrust and reaction act at the center of their respective joints, we have the direction and point of application of the thrust at the crown, the point of application of the reaction, and full knowledge of the vertical force.

Taking moments around the point of application of the reaction, we have (Fig. 1126A):

$$\frac{W}{2} \times l = F \times l' \tag{1127A}$$

Hence

$$F = \frac{W}{2} \times \frac{l}{l'} \tag{1127B}$$

We are thus able to obtain full knowledge of the thrust at the crown and the reaction of any particular arch, under the above assumptions. We could also assume other locations for the points of application of the thrust and reaction and determine their intensities and directions.

1128. **Design.**—The assumption that the points of application of the thrust and reaction are at certain points, preferably within the middle third, is not necessarily correct. Fig. 1128A shows that, for an

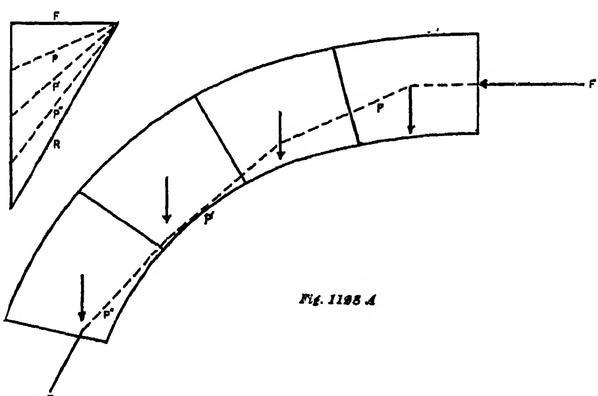


Fig. 1128 A

arch with shape as shown, and F and R calculated as in paragraph 1127, the reaction is not at the center or even within the middle third if we start with F at the center of the crown and obtain resultants and directions, p p' p'', with the weight of each voussoir. It is thus seen that equation 1127A is based on incorrect assumptions and should include a value of R times its lever arm with respect to the middle point of the skewback as the center of moments.

Fig. 1128A shows where the resultant at each joint pierces the joint, for that arch with weights as calculated. It is seen that three of these resultants pierce the joint outside the middle third; therefore, this arch is not satisfactory.

The design of an arch thus becomes a series of approximations. The shape of the arch or the weights of its elements, must be changed, and the pressure curve again drawn. Fig. 1128B shows a suitable arch for

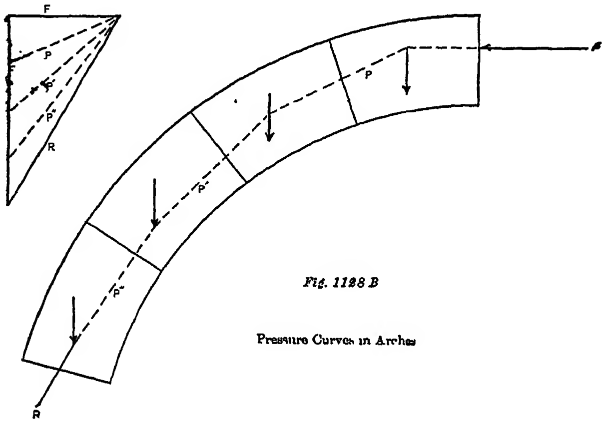


Fig. 1128 B

Pressure Curves in Arches

the same span as that shown in Fig. 1128A. In Fig. 1128B, the action lines at every joint are within the middle third, and at no joint make an angle greater than 37° with the normal to the joint.

Construction.

1129. **Centers.**—Since in a completed arch, each semi-arch is supported by the pressure of the other half transmitted through the keystone, it

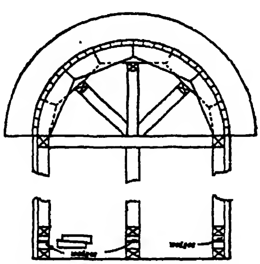


Fig. 1129 A Center for Arches

is necessary to support the arch during the construction until the keystone is inserted, or in concrete arches until the concrete has properly set. The frame employed for this purpose is called a center (Fig. 1129A).

It is composed of a cylindrical shell of boards upon which the masonry rests, and a series of wooden arched or trussed ribs which support the shell. The whole is called a form. For small arches such as window-caps, culverts, and sewers, the truss may be omitted; for very long spans, the shell is supported by a very strong truss. All centers must be so constructed with folding wedges or similar devices that they may be removed without shock to the completed arch.

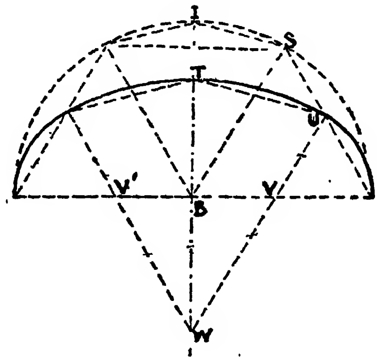


Fig. 1130 A

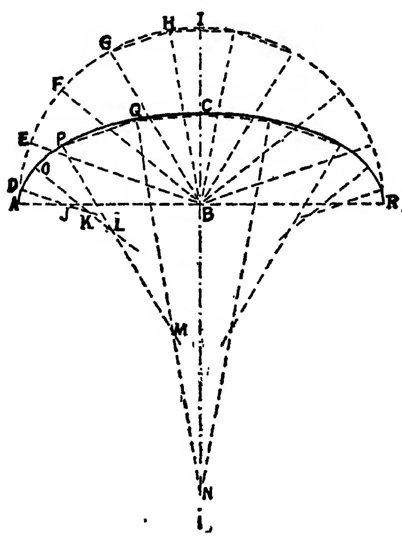


Fig. 1130 B

1130. **Intrados.**—The circular arch is the one most commonly constructed; next, are the oval arches with an odd number of centers.

Fig. 1130A shows the **basket-handled arch,** which is a three center oval and is considered the most graceful form of oval. It is employed in the

construction of ornamental arched bridges. To draw it, inscribe in a semi-circle three sides of a hexagon (Fig. 1130A). Assume the rise BT. Draw IS and parallel to it TU. Draw SB and parallel to it UW. Then V, W, and V' are the centers of the oval.

Fig. 1130B shows the method of drawing an oval of any odd number of centers. On the span AR describe the semi-circle AIR, and divide it into a number of parts corresponding to the desired number of centers. Draw radii to the points of division. Lay off BC equal to the desired rise. Assume the first center J, so that AJ is less than BC. Draw DJ parallel to EB. Assume center K, and draw OK parallel to FB. Assume center L, and draw PL parallel to GB. From P draw PQ parallel to the chord GH, and from C draw CQ parallel to the chord HI. From their intersection draw QM parallel to HB. Then will J, K, L, M, and N be the centers of the circular arcs forming the oval arch $ADPQC$.

1131. **Bond.**—The arch is constructed in the same way as a masonry wall. The joints parallel to the axis are continuous; but the voussoirs break joints, as in walls, so as to have no continuous joints parallel to the face. If the arch is backed with rubble masonry, some of the stones are made long enough to be **throughs,** that is, to bond together the soffit and the backing.

1132. **Voussoirs.**—In important work with large stones, the shape and dimensions of the voussoirs are determined by geometrical drawings and numerical calculations. Brick used are of regular size and shape, it being no longer good practice to make special wedge-shaped brick. Concrete is laid in one monolith; or if this is not practicable, in adjacent monoliths held together by a tongue and groove. Steel reinforcement is very often used in modern arches.

ALLOWABLE STRESSES IN MASONRY.

1133. The table below shows the allowable stresses in the different classes of masonry:

Kind of Masonry	Allowable Pressure
Portland Cement Concrete	250 pounds per square inch
Rubble Masonry with Cement Mortar	150 " " " "
Sandstone	250 " " " "
Limestone	400 " " " "
Granite	600 " " " "
Brick with Cement Mortar	150 " " " "

By means of this table, it is easy to determine the size of wall plate to transmit a given load, and also the probability of a failure of the structure by the crushing of its material due to excessive unit stress.

PRESERVATION.

1134. Masonry is frequently injured by the mortar being washed out of the joints, by unequal settling, and by the expansion and contraction of the material due to changes of temperature.

Pointing.—The washing out of the mortar from the joints may be prevented by means of pointing. This consists in cutting out the mortar at the edge of a joint to a depth of about an inch, brushing the opening clean, moistening it, and filling it with rich Portland cement mortar.

Unequal Settling.—A certain amount of settling always takes place in masonry, due to the shrinkage of the mortar and other causes, and the engineer must take every precaution to insure that this settling is equal throughout. This is especially difficult when parts sustain unequal loads, and are required to be firmly joined together. In such cases, unequal settling is shown by cracks and ruptures in the masonry.

To avoid this unequal settling, it is advisable to use the same thickness of mortar throughout, to pay particular attention to the bond and correct fitting of the courses, and to carry up all parts of the wall simultaneously: also, to allow lower courses to become well set before adding upper courses. A test load is often used to determine conditions of settling.

Effects of Frost.—Frost is one of the most powerful destructive agents against which the engineer has to guard in masonry construction. During severe winters in the northern parts of our country, it has been observed that the frost will penetrate the earth in contact with walls to a depth of ten feet; it therefore becomes a matter of the first importance to use every practicable means to drain all the ground in contact with masonry to whatever depth the foundations may be sunk below the surface. If this precaution be not taken, accidents of the most serious nature may happen to the foundations by the action of the frost. If water is apt to collect in any quantity in the earth around the foundations, it may be necessary to make small covered drains under them to convey it away, and to place a stratum of loose stone between the sides of the foundations and the surrounding earth to give the water a free downward passage.

For more detailed information see:

Masonry Structures, Spalding.
Design of Masonry Structures and Foundations, Williams.
Treatise on Arches, Howe.

PROBLEMS.

P. 1101. (Fig. P. 1101) represents a cantilever bracket, whose base $C'A'B'$ is firmly anchored. Find the stresses at A and B.

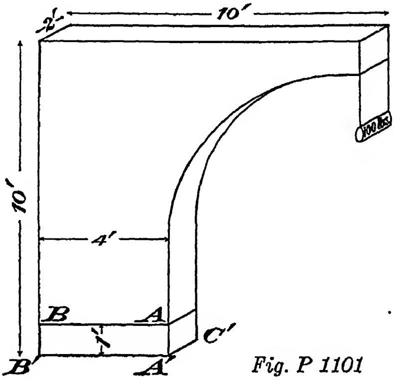


Fig. P 1101

P. 1102. The rectangular base of a shaft is 6′ by 12′, and the center of pressure is one foot from the center of figure, measured parallel to the longer side. The total pressure is 3000 pounds. Find the pressure at the corners.

P. 1103. The base of a wall is 8′ wide, the pressure at one edge is 30 pounds per square inch, and at the other edge is 8 pounds per square inch. What is the mean pressure, and where is the center of pressure?

P. 1104. The cross-section of a masonry dam is a trapezoid with horizontal top and bottom and vertical back. It is 60′ high, 12′ wide on top, and 39′ wide on the bottom. The masonry weighs 150 pounds per cubic foot. Find the maximum pressure per square foot on the base when there is no water behind the dam.

P. 1105. A rectangular wall of masonry is 20′ high, 40′ long, and 8′ thick; on one end of this wall is built a solid masonry tower, 8′ square and 39′ high above the top of the wall. The weight of the masonry is 150 pounds per cubic foot. What is the maximum pressure per square foot on the base of the structure?

P. 1106. A masonry column 80′ high, of square cross-section 10′ by 10′, is subjected to a horizontal wind pressure normal to one side, of 40 pounds per square foot of surface. Find the greatest unit pressure on the base. The masonry weighs 150 pounds per cubic foot.

P. 1107. (Fig. P. 1107.) $ABCD$ is the vertical end face of a stone block, one foot thick, whose top and bottom faces are horizontal rectangular. Find the unit pressures at A and B, with weight of masonry 135 pounds per cubic foot.

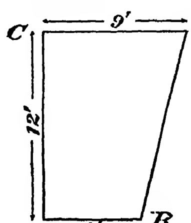


Fig. P 1107

P. 1108. A stone block, 10′ wide by 2′ high in vertical section, and weighing 135 pounds per cubic foot, overhangs by 3′ a masonry structure upon which the remaining 7′ are supported. Construct the line whose ordinate will give the pressure in pounds per square inch at any point, locate the center of pressure, and find the unit pressure at the edge of the structure under the block.

P. 1109. In problem 1108, if the masonry structure is only 5′ wide, so that the stone block overhangs by 3′ on one side, and by 2′ on the other side, construct the line whose ordinate will give the pressure in pounds per square inch at any point, and find the mean and maximum pressures.

P. 1110. A full center arch, of 30′ span, and 2′ thickness of ring, has a weight of 500 tons uniformly distributed horizontally over the arch ring. Find analytically the maximum and minimum thrusts at the crown due to this weight, assuming that the points of rupture are at the skewbacks and at the crown.

P. 1111. A segmental arch of 120° and 15′ radius has an arch ring 2′ in thickness and supports a weight of 400 tons uniformly distributed horizontally over the arch ring. The points of rupture are at the crown and springing lines. (*a*) Find graphically the maximum and minimum crown thrusts, assuming that the center of pressure must be within the middle third of the joints.

(*b*) Check by analytical solution.

P. 1112. A full center arch has a span of 30′ and an arch ring 3′ thick. Concentrated loads of 5 tons, 5 tons, 6 tons, 6 tons, 7 tons, 8 tons, 8 tons, and 9 tons act vertically through points 2′, 4′, 6′, 8′, 10′, 12′, 14′ and 16′ respectively measured in a horizontal direction from the center. Assuming that the crown thrust is applied at the upper extremity of the middle third of the crown joint and equals 25 tons, find the center of pressure on the joint 45° from the crown.

P. 1113. In the preceding problem (1112), construct the curve of pressure.

P. 1114. (Fig. P. 1114.) The figure represents half of a segmental arch, with the joints between voussoirs as indicated. The spandrel load, including the weight of the

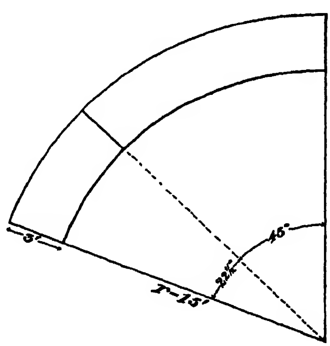


Fig. P 1114

voussoirs, is assumed to be 1 ton per foot uniformly distributed horizontally. Assume that the crown thrust is 12 tons and that its point of application is at the upper extremity of the middle third of the crown joint.

(*a*) Determine graphically the center of pressure at each joint of the arch.

(*b*) Determine graphically whether or not the arch is safe against sliding along the voussoir joints.

P. 1115. An arched rib of 24′ span has its intrados one half of a regular octagon and the arch is composed of four voussoirs, made of timber 3′ by 3′ in cross-section. The joints between the voussoirs are radial. The voussoirs are prevented from sliding at the joints by short dowel pins. A load of 2000 pounds per lineal foot is distributed so as to be uniform horizontally over the span. Under the theory of minimum crown thrust does the center of pressure intersect each voussoir joint within the middle third?

P. 1116. A full center arch of 15′ radius is made up of 6 voussoirs, each subtending an angle of 30°. The arch ring is 3′ thick and the voussoir joints are radial in direction. The top of the spandrel filling is horizontal and 6′ above the extrados of the arch at the crown. The weight of masonry on the arch ring is 144 pounds per cubic foot. The specific gravity of the spandrel filling is $\frac{2}{3}$ that of the arch ring. Determine the load on each voussoir, including its own weight, and determine a point of application of this load.

CHAPTER XII.

COLUMNS.

1201. A column is a strut whose length is such that it will perceptibly bend or buckle under a compressive force before it actually breaks.

This buckling under a compressive force depends upon the material: certain materials will buckle when the length is 5 times the least dimension of cross-section, while others will not bend until the length is some 15 or 20 times the least dimension of cross-section.

1202. If the ends of a column are fixed, the curve of the axial fiber will be tangent to the original axis at the ends, and will have two points of inflection, one half-way between each end and the middle point

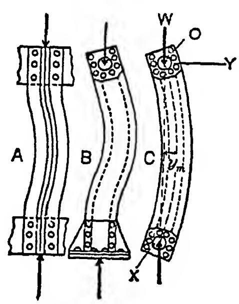


Fig. 1202 A

(Fig. 1202A). If one end of the column is **square,** and the other **round** or held by a **pin,** the curve of the axial fiber will be tangent to the original axis at the square end and there will be but one point of inflection, which will be about one-third of the height of the column from the square end (Fig. 1202B). If the column has two **round** or **pin-connected** ends, the curve of the axial fiber has no points of inflection (Fig. 1202C).

1203. From experiment and by deduction, it is found that a column with square ends will sustain greater stress before rupture than will a column with round (pin-connected) ends or one with one square end and one round end. However, in engineering practice, it is now customary to use formulas for columns with square ends. A large percentage of all columns are now fixed at the ends and riveted, and even in pin-con-

nected columns the ends are often so rusted as to become fixed in position; therefore, as the formulas for fixed ends have a large factor of safety, they are used in the general case where it is evident that they will be satisfactory.

COLUMN FORMULAS.

1204. No column formulas are exact. All are based on deduction and on experiment. In the deductions, it is assumed that the stress on the surface fiber consists of two stresses: (1) the unit stress due to compression and (2) the unit stress on the surface fiber which causes its buckling or bending. Expressed by an equation, we have the same equation as 335A, namely:

$$s_m = \frac{F}{A} + \frac{M_f y}{I} \tag{1204A}$$

in which s_m = maximum stress (in surface fiber) due to both compression and bending.

1205. **Rankine's Formula.**—This formula is of the following general form:

$$\text{Allowable load per square inch on column} = \frac{a_c}{1 + \frac{Cl^2}{r^2}} \tag{1205A}$$

in which a_c = allowable unit compressive stress of the material in the column.

l = length of the column in inches.
r = least radius of gyration of cross-section of the column.
C = a coefficient, determined by experiment.

This formula may be obtained from equation 1204A as follows:

$$s_m = \frac{F}{A} + \frac{M_f y}{I} \tag{1204A}$$

Substituting for M_f its value Fy_m at the dangerous section, in which y_m = maximum deflection of the column, we have

$$s_m = \frac{F}{A} + \frac{Fy_m y}{I} \tag{1205B}$$

Substituting for I its value Ar^2, in which r is the least radius of gyration of the cross-section about the neutral axis, we have

$$s_m = \frac{F}{A} + \frac{Fy_m y}{Ar^2} = \frac{F}{A}\left(1 + \frac{y_m y}{r^2}\right) \tag{1205C}$$

In this formula, y and r^2 are dependent upon the dimensions of the cross-section, which are readily obtained: therefore it is necessary only to obtain some value for y_m in suitable terms.

To obtain such a value for y_m, we assume that the force causing the y_m in this column is the same which causes an equal y_m in a beam fixed horizontally at both ends and loaded at the middle point. From the table, paragraph 319, we have

$$y_m = \frac{Fl^3}{192\ E_f I} \tag{1205D}$$

Substituting in equation 1205C, we have

$$s_m = \frac{F}{A}\left(1 + \frac{(Fl^3)}{(192\ E_f I)}\frac{y}{r^2}\right) \tag{1205E}$$

This is the desired equation, but it is usually expressed in terms not including y and I. To eliminate these, we use again the table in paragraph 319 and find in second column the expression for unit stress for bending alone to be:

$$s_{fm} = \frac{Fly}{8I}, \qquad \text{or} \qquad y = \frac{8\ s_{fm} I}{Fl} \qquad \text{1205F)}$$

Substituting this value of y in equation 1205E, we have

$$s_m = \frac{F}{A}\left(1 + \frac{s_{fm} l^2}{24\ E_f r^2}\right) \tag{1205G}$$

Replacing $\frac{s_{fm}}{24\ E_f}$ by C, we have

$$s_m = \frac{F}{A}\left(1 + \frac{Cl^2}{r^2}\right) \tag{1205H}$$

Or

$$\frac{F}{A} = \frac{s_m}{1 + \frac{Cl^2}{r^2}} \tag{1205I}$$

In this equation s_m is the unit stress in the surface fiber. Giving this its allowable value, a_c, we have $\frac{F}{A}$ equal to the allowable unit stress on the column. The equation then becomes

$$\text{Allowable Unit Stress on Column} = \frac{a_c}{1 + \frac{Cl^2}{r^2}} \tag{1205J}$$

The suppositions made in this deduction are only approximately true for columns, as may be seen by comparing the values of C obtained by

experiment with those obtained by substituting the ultimate strength of any material for s_{fm}, and its modulus of longitudinal elasticity for E_f, in the expression $C = \frac{s_{fm}}{24 E_f}$.

Multiplying the allowable unit stress on the column by the area of cross-section, we have

$$\text{Total Allowable Stress on Column} = A\left(\frac{a_c}{1 + \frac{Cl^2}{r^2}}\right) \qquad (1205K)$$

1206. The following table gives the values of a and C as used in applying the formula to different materials:

Material	a_c	C
Yellow Pine	1,000	$\frac{1}{3,000}$
Oak	900	$\frac{1}{3,000}$
White Pine	700	$\frac{1}{3,000}$
Cast Iron	10,000	$\frac{1}{5,000}$
Medium Steel	12,500	$\frac{1}{36,000}$
Soft Steel	11,250	$\frac{1}{36,000}$
Wrought Iron	10,000	$\frac{1}{36,000}$

From the above table, it is seen that Rankine's formula for medium structural steel becomes

$$\text{Allowable load per square inch on column} = \frac{12,500}{1 + \frac{l^2}{36,000\, r^2}} \qquad (1206A)$$

$$\text{Total Allowable Load on Column} = A\left(\frac{12,500}{1 + \frac{l^2}{36,000\, r^2}}\right) \qquad (1206B)$$

1207. **Problem.**—A hollow round cast-iron column is to carry safely a load of 60,000 pounds. Its thickness is $\frac{1}{2}''$, length $12'$. Find its external diameter. Use Rankine's formula.

Solution.—Rankine's formula for cast-iron columns is:

$$F = A\left(\frac{10,000}{1 + \frac{l^2}{5,000\, r^2}}\right)$$

A for a hollow circle $= \pi\left(\frac{D^2}{4} - \frac{D_1^2}{4}\right) = .7854\ (D^2 - D_1^2)$ where D is the external diameter and D_1 is the internal diameter. In this particular case $D_1 = D - 1$ and $A = .7854\ (D^2 - D^2 + 2\ D - 1) = .7854\ (2\ D - 1)$

r for a hollow circle $= \frac{\sqrt{D^2 + D_1^2}}{4}$ or in this case $r = \frac{\sqrt{D^2 + D^2 - 2\ D + 1}}{4}$

and
$$r^2 = \frac{2\ D^2 - 2\ D + 1}{16}$$

Making the proper substitutions in the formula above, we have

$$60{,}000 = .7854\ (2\ D - 1)\left(\frac{10{,}000}{1 + \dfrac{144^2}{5{,}000\left(\dfrac{2\ D^2 - 2\ D + 1}{16}\right)}}\right)$$

The reduction of this expression to a simpler form in which it can be handled more easily is rather tedious and the intermediate operations are omitted. Clearing of fractions, combining like terms, and dividing through by the coefficient of D^3, we have:

$$D^3 - 5.41\ D^2 + 4.77\ D - 128.4 = 0$$

The easiest method of solving an expression of this sort is to substitute trial values for d until one is determined which satisfies the equation. The correct value is first bracketed by assuming two values quite far apart and then more closely approximated by trying values lying between the bracketing values.

Substituting for D in the above expression a value of 7, we have:

$$343 - 265 + 33.4 - 128.4 = -17.0$$

Therefore 7 is not the correct value of D since the expression does not reduce to zero.

Trying $D = 7.5$, we have:

$$422 - 304.3 + 35.8 - 128.4 = +25.1$$

D must lie between 7 and 7.5 since one value gives a positive value to the expression and the other gives a negative value.

Trying $D = 7.2$, we have:

$$373.2 - 280.2 + 34.3 - 128.4 = -1.1$$

The correct value of d should apparently be a little more than 7.2 as the value 7.2 so nearly reduces the expression to zero. The value of D will therefore be taken as 7.22″.

1208. **A.R.E.A. Formulas.**—The American Railway Engineering Association has adopted the following formulas in which D = least dimension of cross-section, and r = least radius of gyration.

$$\text{Allowable Unit Stress on Column} = 16{,}000 - \frac{70\ l}{r} \text{ (structural steel)}$$
$$= 1{,}300\left(1 - \frac{l}{60\ D}\right) \text{ (longleaf pine)}$$
$$= 1{,}200\left(1 - \frac{l}{60\ D}\right) \text{ (Douglas fir)}$$
$$= 1{,}100\left(1 - \frac{l}{60\ D}\right) \text{ (spruce)}$$
$$= 1{,}000\left(1 - \frac{l}{60\ D}\right) \text{ (white pine)}$$

These are formulas of the first degree (right line formulas) and are shown by experiment to give fairly accurate results.

1209. **A.B.C. Formulas.**—The American Bridge Company has adopted certain "Construction. Specifications," which are not really formulas but which give allowable unit stresses on structural steel columns for certain values of $\frac{l}{r}$. These are as follows:

$\frac{l}{r}$	Allowable Unit Stress on Column	$\frac{l}{r}$	Allowable Unit Stress on Column
0 to 60	13,000	130	6,500
70	12,000	140	6,000
80	11,000	150	5,500
90	10,000	160	5,000
100	9,000	170	4,500
110	8,000	180	4,000
120	7,000	190	3,500

By multiplying the area of cross-section by the allowable unit stress on columns as given in the table, interpolating between values where necessary, the total allowable stress on the column is obtained.

1210. **Problem.**—Find the allowable load on a column 15′ long, with r of 5.10″, and area of cross-section equal to 10.17 square inches, using the Construction Specifications of the American Bridge Co.

Solution.—In the column mentioned, length was 15′ or $l = 180''$.

The least radius is given as 5.58″.

Therefore $\frac{l}{r} = \frac{180}{5.10} = 35.3$.

Referring to the specifications above we see that when $\frac{l}{r}$ is between 0 and 60 the allowable unit stress is 13,000 pounds.

The total allowable load is $2 \times 10.17 \times 13{,}000 = 264{,}420$ pounds.

1211. **Radius of Gyration.** — In the above formulas, the allowable unit stress on the column depends upon the value of $\frac{l}{r}$ or $\frac{l}{D}$, the allowable unit stress decreasing as the values of $\frac{l}{r}$ or $\frac{l}{D}$ increase. As the required length, l, of a column cannot ordinarily be changed in the plan for a structure, it is necessary to increase the allowable unit stress by selecting or constructing a column of such a cross-section that its **least radius of gyration** or least dimension of cross-section will be as large as practicable. No other value than the least r or the least d need be considered. For example, if a column with a cross-section of 8″ × 4″ can support Q pounds

per square inch, it can support no more **per square inch** if its cross-section is changed to 16″ × 4″, as the least r and d are not changed: but if its cross-section is changed to 8″ × 8″, it will support much more per square inch because its least r and least d have been increased.

1212. **Problem.**—A builder requires a wooden column 8′ long to support a load of 20,000 pounds. He has available timbers in the following sizes: 4″ × 6″, 6″ × 6″, 4″ × 10″, all of Douglas Fir. Using the proper A.R.E.A. formula, determine which is the most economical size to use.

Solution.—A.R.E.A. formula for Douglas Fir is:

Allowable unit stress $= 1200\left(1 - \frac{l}{60\,D}\right)$.

Considering the timbers mentioned, we have:

For 4″ × 6″ timber, allowable unit stress $= 1200\left(1 - \frac{8 \times 12}{60 \times 4}\right) = 720$ pounds per square inch.

For 6″ × 6″ timber, allowable unit stress $= 1200\left(1 - \frac{8 \times 12}{60 \times 6}\right) = 880$ pounds per square inch.

For 4″ × 10″ timber, allowable unit stress $= 1200\left(1 - \frac{8 \times 12}{60 \times 4}\right) = 720$ pounds per square inch.

Multiplying these values by the corresponding areas, we obtain:

For 4″ × 6″ timber, safe load	$= 4 \times 6 \times 720$	$= 17{,}280$	pounds
" 6″ × 6″ " " "	$= 6 \times 6 \times 880$	$= 31{,}680$	"
" 4″ × 10″ " " "	$= 4 \times 10 \times 720$	$= 28{,}800$	"

The 6″ × 6″ and 4″ × 10″ will safely carry the load. But the most economica column to use will be the 6″ × 6″ since it has a lesser area and therefore lesser cost than the 4″ × 10″.

1213. The engineering handbooks have tables showing the values of r for different cross-sections, as squares, rectangles, circles, and structural forms. By selecting the least r or the least d from the handbooks, it is possible to substitute in the formulas and determine the allowable unit stress for any column. Then, by multiplying this allowable unit stress by the area of cross-section (also in the handbooks) the total allowable stress is obtained.

1214. For ordinary structural shapes, the handbooks give tables showing the total allowable stress, and this can be taken direct from the tables without any calculations at all. These tables are prepared by calculations with certain formulas. If unusual shapes are used or if it is desired to use other formulas, special calculations must be made.

KINDS OF COLUMNS.

1215. **Built-up Columns.**—In order that there be no waste of material, it is desirable that a column be equally strong to resist bending

in all directions, as is the case in a column of circular cross-section, either solid or hollow.

A **built-up column** is made up of two or more longitudinal pieces, so fastened together as to act as a single column. The aim in designing built-up columns is to design a cross-section so that the least r shall be the same in all directions, thus making the column equally strong to resist bending in all directions. As a rule only the two main r's can be made equal. If a column is made up of two rectangular pieces (Fig. 1216A) it will fulfill the required conditions as nearly as practicable if the unit stresses on the edges AB and AC are equal. They will be equal to each other if the moment of inertia or radius of gyration of the combined section about GH is equal to that about EF.

1216. Let it be required to find the distance between the inner faces of the rectangles AB and CD when the moments of inertia of the combined cross-section about EF and GH are equal to each other.

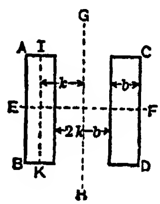


Fig. 1216 A

Let A = area of each rectangle
I' = moment of inertia of the rectangle AB about EF;
r' = radius of gyration " " " " "
i' = moment of inertia " " " " IK
r'' = radius of gyration " " " " "
I'' = moment of inertia " " " " GH

The moment of inertia of the combined section about EF is $2\,I'$, which expressed in terms of r gives

$$2\,I' = 2\,Ar'^2 \qquad (1216A)$$

The moment of inertia of the combined section about GH may be determined from the principle of mechanics that the moment of inertia of any mass with reference to any axis is equal to the moment of inertia about a parallel axis through its center of gravity, plus the product of the mass into the square of the distance between the two axes. Hence we have the moment of inertia of the rectangle AB about the axis GH through the center of gravity of the combined section, expressed in terms of the radius of gyration.

$$I'' = i' + Ak^2 = Ar''^2 + Ak^2, \qquad (1216B)$$

in which k = distance between axes IK and GH, or between the center of gravity of the area of the rectangle AB and the center of gravity of the combined section.

For the combined section we have

$$2\,I'' = 2\,Ar''^2 + 2\,Ak^2 \qquad (1216C)$$

By hypothesis, the distance between IK and GH is such that $2\,I'' = 2\,I'$, hence

$$2\,Ar'^2 = 2\,Ar''^2 + 2\,Ak^2, \qquad (1216D)$$

$$r'^2 = r''^2 + k^2, \quad \text{or} \quad k = \sqrt{r'^2 - r''^2} \qquad (1216E)$$

from which we can determine the value of k, when r' and r'' are known. The distance between the inner faces of the two rectangular pieces is $2\,k - b$, in which b is the breadth of the cross-section AB.

If r'' is very small with respect to r', its value in the second member may be omitted, and we then have

$$k = r' \qquad (1216F)$$

This value of k is slightly greater than its true value.

In practice it is not unusual to make $k = r'$ in built-up metal columns.

1217. It is not enough to space the two members at the proper distance apart; it is necessary to unite them so that they will act as one column; otherwise each member will act simply as one column with a least r equal to the least r of a member acting alone, and the load

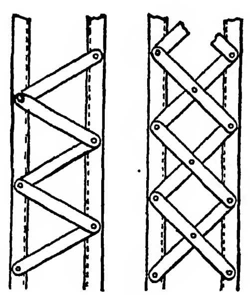


Fig. 1217 A
Latticing used on
Steel Columns

carried by both members will be twice that carried by one member. To unite the members, lacing or latticing is used, that is, strips of material connecting the two members as shown in Fig. 1217A. The effect of the latticing is to divide each member into a series of short columns. To determine accurately the length of these short columns (distance

between connections of lacing or latticing pieces), we decide that

$$\frac{l}{r} \text{ of short columns must } = \frac{l}{r} \text{ of the entire column.} \qquad (1217A)$$

This gives the same allowable unit stress in the short column as in the entire column. In this equation (1217A) all of the quantities are known except the l of the short column; therefore, it can easily be determined.

1218. **Problem.**—A built-up column is composed of two channels 15″ 35 pound, placed back to back and single latticed. They are spaced 10″ back to back. The column is 15′ long. Find the safe load on the column using the A.R.E.A. formula.

Solution.—The first step is to find the least radius of gyration of the cross-section of the column, which is r in the formula above.

r for the channels assumed is 5.58 about the axis perpendicular to the web (see Handbook).

The moment of inertia about the axis parallel to the web must be computed. The computation is as indicated below:

I for one channel about its own axis parallel to the web is 8.4 from Handbook.

k, the distance between axis of channel and axis of cross-section, is $5 + .79$.

Area of cross-section of one channel is 10.23 square inches.

$Ak^2 = 10.23 \times 5.79^2 = 343.$

The total moment of inertia for the cross-section is twice that for one channel, or $2\ (8.4 + 343) = 702.8.$

$$r^2 = \frac{702.8}{2 \times 10.23} = 34.35$$

$r = \sqrt{34.35} = 5.865$ which is greater than the r about the other axis. Therefore it is necessary to substitute in the formula $r = 5.58$.

Allowable unit stress $= 16{,}000 - \dfrac{70 \times 180}{5.58} = 16{,}000 - 2655 = 13{,}745$ pounds per square inch.

The total allowable load on column $= 13{,}745 \times (2 \times 10.23) = 281{,}120$ pounds.

1219. **Structural Forms.**—Figs. 1219A to D show some of the forms of cross-sections of columns made up from rolled steel forms. The plate

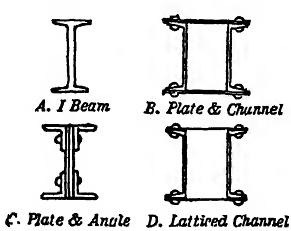


Fig. 1219
Columns

and channel column has the advantage over the others of a greater r for an equal area of cross-section; it has the disadvantage that it cannot be painted on the inside.

1220. **Problem.**—An upper chord member of a bridge truss is 20′ long and is under a compressive stress of 220,000 pounds. (*a*) Select a column composed of latticed channels which will take this load. (*b*) Select a column made up of plates and angles.

Solution.—(*a*) Referring to the handbook we find that two channels 12″ 30 pound, single latticed, 20′ long will take a load of 229,000 pounds. This section is sufficiently strong and would be used unless in the process of designing the bridge it was found necessary to use some other size of channel for convenience in making riveted connections. We will assume that considerations of design require that 10″ channels be used. Referring again to the handbook we find that the lightest column which will take the load among the 10″ channels is one composed of 2 channels 10″ 15.3 pound and 2 plates 12″ × $\frac{3}{8}$″.

(*b*) Under plate and angle columns opposite the length of 20′ we find that the lightest column is one composed of 4 angles 6″ × 4″ × $\frac{1}{2}$″ attached to a web plate 10″ × $\frac{3}{8}$″ which will take a load of 226,000 pounds.

1221. **Hollow Cast-iron Columns.**—These are manufactured with cross-sections either round or rectangular. They have the advantage of a large r for the area of cross-section: they have the disadvantage of being of cast metal which makes them more expensive than the structural forms, and the metal is not so regular as steel, not so uniform, and not of equal physical characteristics.

1222. **Wooden Columns.**—These are usually round or square. Special wooden columns may be made with hollow interiors; or they may be made of solid pieces fastened together as in built-up beams, this being advisable when good solid timbers cannot be obtained, and being necessary when timbers of sufficient size cannot be obtained.

1223. **Reinforced Concrete Columns.**—Figs. 1223A, B, C, show methods of construction of reinforced concrete columns. In practice they are made so short that buckling need not be expected; but the steel

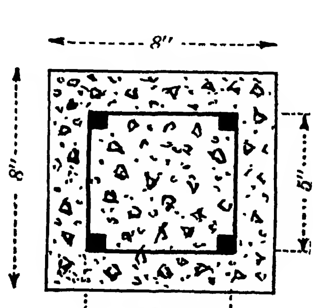


Fig. 1223 A
Vertical reinforcement

is nevertheless placed near the outer edge with a view to preventing buckling by taking up the tensile stress. The usual formulas for rein-

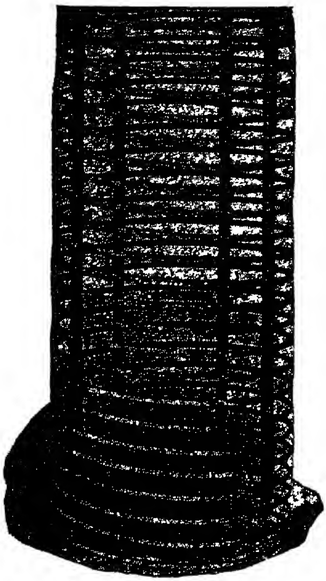


Fig. 1223 B
Hooped reinforcement.

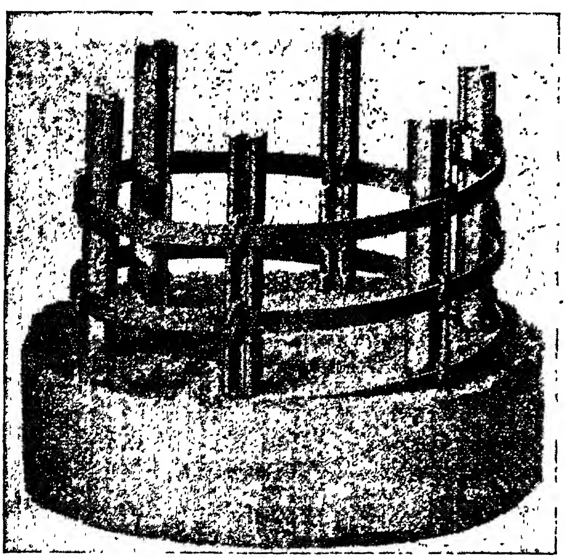


Fig. 1223 C
Spiral hooped column reinforcement

forced concrete columns apply only to those short columns whose length is not more than 15 times their smaller dimension of cross-section; the allowable unit stresses are based on the allowable compressive stresses, which are taken as 650 pounds for concrete and 16,000 pounds for steel. For longer reinforced concrete columns, there are special and complicated formulas which are not considered in this text.

PROBLEMS.

P. 1201. What is the safe load for a hollow cast-iron column 12′ long, inside diameter 7″, outside diameter 8″? Use Rankine's formula.

P. 1202. Determine the safe load on a hollow cast-iron column 12″ square, and three-quarters of an inch thick, and 18′ long.

P. 1203. Select an I-beam column from the Handbook, 14′ long, which will sustain a load of 160,000 pounds, safely.

P. 1204. What is the safe load of a solid round wrought-iron pillar 18′ long and 5″ in diameter?

P. 1205. What is the safe load that may be borne by a round white pine column with fixed ends, whose diameter is 7″ and length is 14′? How does this compare with the results obtained from the Handbook?

P. 1206. A wooden cypress beam 16′ long and 6″ × 9″ in rectangular cross-section is to be used as a column. What will be its safe load by the A.R.E.A. formula?

P. 1207. What is the proper distance between the inner sides of two 12″ × 2″ oak planks so that they may be latticed together and form a column of maximum strength?

P. 1208. It is desired to build a column of two pieces of lumber, each of which is 12″ × 3″ in cross-section. Find accurately how far apart the pieces should be placed.

P. 1209. How far apart should three planks, each 12″ × 1″, be placed in order to make a built-up column of maximum strength.

P. 1210. In the following two cases, determine spacing between backs of two 8″-11.5 pound channels so as to produce equal resistance to buckling in the two directions: 1st, webs inside; 2nd, webs outside.

P. 1211. What safe load will a hollow cast-iron column with fixed ends support if it is 14′ long, outside diameter 10″ and inside diameter 8″? Compare the load it could support for a length less than eight feet with the result obtained.

P. 1212. Determine the load on a hollow, round, cast-iron column, external diameter 12″, thickness 1″, and length 14′ for a factor of safety of 8.

P. 1213. A cylindrical wrought-iron column is 10′ long, 6.4″ outside diameter, 6″ inside diameter, and carries a load of 50,000 pounds. Find its factor of safety.

P. 1214. Determine the size of a square wooden column 30′ long to safely sustain a load of 20 tons.

P. 1215. A wrought-iron piston rod has a diameter of 2″ and a length of 4′. Considering it as a column, what is the allowable diameter of the piston, if the steam pressure is 60 pounds per square inch, for a factor of safety of 10?

P. 1216. Select a column of channels, 18′ long, to carry a load of 250,000 pounds, from those given in Handbook. The one selected should be the lightest one that will do the work.

P. 1217. Compute the allowable load on the column selected in P. 1216 using the A.R.E.A. formula.

P. 1218. A column is composed of a web plate $12'' \times \frac{1}{2}''$, 4 angles $6'' \times 6'' \times \frac{1}{2}''$, and a plate $14'' \times \frac{1}{2}''$ riveted to two of the angles and perpendicular to the web plate. It is 14′ long. Find the allowable load on the column by the American Bridge Company specifications.

P. 1219. A column is composed of a T-shaped rolled form. The T cross-section is 12″ deep over-all and the top of the T is 10″ wide. The thickness of the leg and the top of the T is 2″. Find the allowable load on this column using the A.R.E.A. formula. Metal is structural steel.

P. 1220. Determine the safe load of a hollow cast-iron column, 15″ square, 3″ thick, and 15′ long.

P. 1221. Determine the minimum size of round hollow cast-iron column, 18′ long to support a load of 220,000 pounds. Use same specifications as in P. 1220.

CHAPTER XIII.

FOUNDATIONS.

1301. **General Types of Foundations.**—A foundation is the name given to the part of a structure which distributes the load to the **soil.** On its proper design and construction depends the safety of the structure. There are three general types of foundations: footings, spread footings, and piles. A footing is simply a widening of the structure at its base until the required area is obtained to properly distribute the load. The spread footing increases the area by placing beneath the base a grillage of steel I-beams imbedded in concrete (Fig. 1317A) or a mass of reinforced concrete. A pile foundation consists of a footing resting on piles which serve either to distribute the load in depth by the friction between the piles and the soil or by the column action of the piles in resting on a solid stratum to transfer the load to that stratum.

EXPLORATION AND TESTING OF SOILS.

Exploration of Sub-Surface.

1302. The first consideration in the design of any foundation is the soil upon which the structure is to be built. It is imperative to obtain accurate information regarding the local conditions beneath the surface of the ground or below the bed of the stream so that the proper type of sub-structure can be designed. This necessitates the making of excavations, or borings, or the driving of test piles.

1303. A common method of preliminary examination of the soil is to drive sounding rods (iron rods or pipes 1″ or $1\frac{1}{2}$″ in diameter) by hand into the earth. The resistance encountered by these rods gives a general idea of the type of soil and its bearing capacity. For shallow foundations a test pit may be opened, a little deeper than the foundation is to be placed, so that the conditions for a few feet below will be known. The bearing capacity of the soil can then be tested.

1304. Where a test pit is not advisable, due to the necessity of excessive excavation or the presence of water, borings are made. Hand borings may be made with an earth auger in which the necessary depth is obtained by the use of a sufficient length of joined pipe sections. When proper depth is obtained, the auger is withdrawn and the soil contained in the pipe sections is studied.

Exploration of deeper or more compact strata can be made by wash borings. A hollow shaft is driven: water is then forced down the hollow shaft, out through an orifice in the bit, and in rising to the surface brings samples of the earth encountered. Wash borings are often unsatisfactory due to the water changing the natural condition of the material penetrated. Where rock is encountered, or where it is desired to take out dry samples of the sub-strata, core borings are made. For this work a drill with an annular bit whose cutting surfaces are black diamonds, chilled steel shot, or tempered steel teeth, is used to cut a core or cylinder which is broken off and raised to the surface. This method gives an accurate cross-section of the earth passed through, but is more expensive than other methods.

1305. In the deep waterways survey made in 1900 for the barge canal through New York State, 1910 feet of rock were drilled at a cost of $3.13 per foot of depth; wash borings were made on the same survey for from 12¢ to 85¢ per foot of depth.

Bearing Power of Soils.

1306. For ordinary structures, the knowledge of the sub-soil, as determined by one or more of the methods of examination described above, is sufficient to determine its bearing capacity. However, where large and important structures are to be built, it is necessary to test the bearing capacity of the soil at the depth at which it is planned to place the foundation, unless the presence of solid rock at a practical depth has been ascertained. Where rock is not found, testing is accomplished by sinking an open well, leveling off the foot of the well and placing a definite load on a known area, daily observations being taken to determine settlement. The Building Department of the Borough of Manhattan, City of New York, requires the area tested to be at least four (4) square feet, and does not accept the load under consideration as safe unless it causes no appreciable settlement in two days, and unless a 50 per cent greater load causes no appreciable settlement in the last four of six days.

1307. The following table gives the approximate safe bearing capacity of various types of soils:

Soft clay and wet sand	1 ton per square foot
Ordinary clay and dry sand mixed with clay..	2 " " " "
Dry sand and dry clay	3 " " " "
Hard clay and firm coarse sand............	4 " " " "
Firm coarse sand and gravel	5 " " " "
Rock, according to thickness and hardness....................	10 to 200 " " " "

1308. The safe bearing capacity is derived from the ultimate bearing capacity by dividing by a factor of safety whose value depends upon the uniformity of the soil. If the stratum upon which the building is to rest overlies a softer stratum, it is also necessary to determine the extent and thickness of the firm stratum. In Chicago where the surface soil overlies a very soft soil, the building regulations allow a load of 2 tons per square foot on sand and $1\frac{3}{4}$ tons on clay, when the bed is at least 15′ thick; for thinner strata, the loads must be reduced.

The amount of water contained in a soil greatly influences the bearing capacity. Thus, quicksand if confined and dried can be used as a foundation bed, its supporting power varying inversely with the amount of contained water. Clay is greatly changed by water: hard dry clay mixed with gravel often makes an excellent soil on which to place a foundation.

GENERAL CONSIDERATIONS GOVERNING FOUNDATIONS.

1309. A foundation may be considered satisfactory when the vertical movement or settling of the structure is uniform and hardly appreciable, and there is no horizontal movement of the structure on its base.

If the soil under every part of the structure has the same bearing power, the settling will be uniform if the unit pressure is the same at every point of the base. This uniformity is secured by designing the base of every section of the structure so that the resultant pressure shall pierce the base as closely as practicable to the center of the figure, and by enlarging the base of each foundation wall and pier until with a proper value of allowable unit bearing pressure the actual unit bearing pressure has a constant ratio to the bearing capacity of the soil. If the soil under the structure is not uniform throughout, the unit pressure on the base at any point must be proportioned to the unit bearing power of the soil at the same point; this is effected by varying the dimensions of the footing. The greatest unit pressure must be where the bearing power is greatest, and the converse. If a part of the structure rests upon rock and the remainder upon compressible soil, it will be practically impossible to so adjust the pressure and resistances as to prevent unequal settling. The structure should, if possible, rest wholly on the rock or wholly on the compressible soil; if this is not possible, a cushion of sand may be placed on the rock.

1310. If the center of pressure of any section of the base does not coincide with the center of figure, the pressure at that part of the base will not be uniformly distributed; and if the soil is uniform, the structure will tend to rotate about some line in its base. In the outer walls of

buildings a slight tendency to rotate inwards is not objectionable, since such inward rotation will bind the parts of the building together.

The factor of safety of a foundation is the ratio of the unit bearing power to the unit pressure; it should be as large as practicable, as appreciable settling is to be avoided.

1311. The soil underneath a structure must be protected from all disturbing influences. On land the foundation walls of buildings should extend below the limits of frost, and in rivers the foundations of piers should be deep enough to be protected from the undermining action of the current. The extreme depth of the frost line depends upon the climate; in New York City the building regulations require the base of the foundation walls to be at least four feet below the surface of the soil.

1312. Failure in a foundation may occur by horizontal as well as vertical movement. Where the horizontal component of the resultant pressure on the base is greater than the friction produced by its vertical component, the structure will move horizontally unless the foundation is designed to counteract this horizontal thrust.

To avoid excessive excavation when the surface of the soil is much inclined, the base for the foundation wall may be a series of horizontal steps instead of a single inclined surface, or it may be made level by placing concrete where necessary.

FOUNDATIONS ON LAND.

1313. If solid rock is found near the surface of the ground, it is only necessary to remove all loose and decayed material under the piers and foundation walls and then fill the crevices and level the bed with a layer of concrete. The surface of this concrete becomes the bed of the foundation proper. If the rock is intersected by wide crevices, these may be bridged by masonry arches or by strong iron beams imbedded in concrete; if the surface is inclined, steps may be cut in the solid rock for the bed of the foundation.

Firm Soil with Light Loads.

1314. Where the load upon the foundation is light and the soil near the surface is loam, sand, gravel, or clay free from water, a trench is dug to a depth below the frost line and its bottom leveled to receive the footings of the foundation walls and piers.

The footing (Fig. 1314A) is a mass of masonry or concrete by means of which the base of the wall or pier is widened so as to decrease the unit pressure upon the soil and in some cases at the same time move the

center of pressure to avoid eccentric loading. If the center of pressure does not pass through the center of the foundation, the latter must be designed with reference to this eccentric loading, as explained in Chapter XI on Masonry.

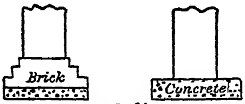


Fig. 1314 A
Types of Footings

A footing is necessary whenever the safe bearing power of the soil is less than the unit pressure on the base of the wall or pier. A deep footing is usually stepped, since with less masonry it can be made as strong as a footing without steps.

The vertical thickness of the footing may be deduced by assuming that the projecting part is a cantilever acted upon by a unit upward pressure equal to the unit downward pressure upon the entire base produced by the weight of the wall and its load.

The width of the footing in feet at any cross-section is determined by dividing the total pressure per running foot of the base of the wall by the safe bearing power of the soil per square foot. The safe compressive stress of the masonry must also be considered; this for brick masonry is about 175 pounds per square inch, and for concrete it varies from 250 pounds per square inch for a 1 : 3 : 6 mixture to 450 pounds per square inch for a 1 : 2 : 4 mixture.

According to standard building regulations, the thickness of a concrete or stone footing should be at least one foot. Each stone must extend the full width of the wall. If the footing is stepped, each step of a concrete or stone footing is made at least one foot thick. The steps of a brick footing are either one or two brick thick; if one brick thick, the steps are $1\frac{1}{2}''$ wide; if two brick thick, the steps are $3''$ wide. A brick footing should always rest on a bed of concrete.

1315. **Problem.**—The foundation wall of a building carries a load of 20,000 pounds per linear foot. It is 14″ wide. The building is to rest on firm gravel. Design a footing of plain concrete. Assume $a_f = 40$, $a_s = 36$.

Solution.—Assume the footing to be of the rectangular type shown in Fig. **1314A**, marked "concrete."

From paragraph 1307 we find that gravel will support 5 tons per square foot, hence

$$\frac{20{,}000}{5 \times 2000} = 2 \text{ square feet.}$$

Therefore, since the load used is for a linear foot of wall, the footing will have to be at least 24″ wide — assume 26″ — adding 2″ for safety.

The problem then reduces to one of determining the necessary thickness to resist

bending and shear on the projecting part of the footing which acts as a cantilever. This portion projects 6″ beyond the edge of the wall on each side.

Considering one linear foot of the foundation the maximum bending moment in the cantilever will be

$$\frac{20{,}000}{26} \times 6 \times 3 = 13{,}846 \text{ inch pounds.}$$

Using equation 267A

$$a_f = \frac{My}{I} \qquad (267A)$$

transposing and making the proper substitutions, we have

$$\frac{I}{y} = \frac{13{,}846}{40} = 346.2$$

but $\frac{I}{y}$ for a rectangle is $\frac{bD^2}{6}$, hence

$$\frac{bD^2}{6} = 346.2$$

$$D^2 = \frac{346.2 \times 6}{12} = 173$$

$$D = 13.2 \quad \text{or} \quad 14''.$$

Testing $d = 14''$, as found above for shear, the area in shear will be $12 \times 14 = 168$ square inches.

The total allowable shear will be $36 \times 168 = 6048$ pounds

The actual shear will be

$$\frac{20{,}000}{26} \times 6 = 4615 \text{ pounds.}$$

The depth of 14″ is therefore safe in shear.

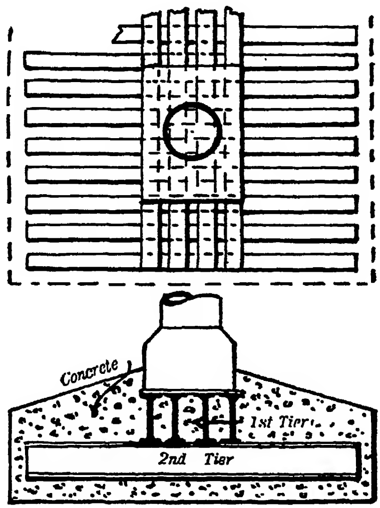
The footing will then consist of a slab of concrete running the length of the wall 14″ deep and 26″ wide.

Firm Soils with Heavy Loads.

1316. As the load on the foundation increases, the type of footing discussed above becomes so large as to be uneconomical, due to the cost of a large mass of masonry and the vertical space occupied by it. This condition has led to the development of two types of spread footings: the I-beam grillage, and the reinforced concrete footing.

1317. **The I-beam grillage** consists of two or more tiers of I-beams imbedded in concrete. Each tier of I-beams is at right angles to the tier below it. To prevent rusting, it is necessary to surround each beam completely with concrete. To permit the placing of this concrete the beams are separated from each other by separators of cast iron or pipe. The opening between the flanges of the beams should be not less than three inches nor greater than one and a half times the width of the flange of the beams. (See Fig. 1317A.) The whole footing is encased in concrete, care being taken to fill the spaces between the beams. Four inches is sufficient covering for the steel except on the bottom where

there should be not less than twelve inches. In designing the footing the concrete is considered as taking no stress, the load being carried from tier to tier of the beams and to the soil through beam action.



Steel "I" Beam Grillage
Single Column
Fig. 1317 A

This type of foundation was first developed in Chicago about 1878 and is today in wide use throughout the country. In the Woolworth Building, New York City, the loads are carried through I-beam grillages to reinforced concrete piers resting on bed rock 115 feet below the street.

1318. **Problem.**—Design a grillage footing for a steel column, using two tiers of I-beams. Base of column, 40″ × 40″. Load on the column, 800,000 pounds. Foundation soil is dry clay.

Solution.—(1) Dry clay has an allowable unit pressure of 6000 pounds per square foot, hence

$$\frac{800{,}000}{6000} = 133.3 \text{ square feet}$$

is the required area in bearing. Hence use a 12′ × 12′ foundation.

(2) Maximum bending moment — upper tier. The load on the upper tier of I-beams will be a uniformly distributed load along the under side of $\frac{800{,}000}{12}$ pounds per linear foot acting up, while on the upper side the central 40″ or 3.3′ will be uniformly loaded with $\frac{800{,}000}{3.3}$ pounds per linear foot acting down.

The maximum bending moment will then be

$$\frac{800{,}000}{12} \times \frac{12}{2} \times \frac{12}{4} - \frac{800{,}000}{3.3} \times \frac{3.3}{2} \times \frac{3.3}{4} = \frac{800{,}000\,(12 - 3.3)}{8}$$

$$= 870{,}000 \text{ foot pounds}$$

(*a*) Assume 4 I-beams; use the bending equation, $a_f = \dfrac{M_f}{\frac{I}{y}}$ in which $a_f = 16{,}000$ pounds per square inch.

The section modulus of one beam will then be

$$\frac{870{,}000 \times 12}{16{,}000 \times 4} = 163$$

Hence use 4 — 24″ 79.9 pound I-beams.

(*b*) Test for clear space or separation between the beams discussed in paragraph 1317.

$$7.00 \times 4 = 28'' \text{ (7.00 = approximate width of flange)}$$

hence $40.00 - 28.28 = 11.72''$ which divided by 3, since there are 3 spaces, gives 3.9″. This is sufficient.

(3) Second tier. (*a*) Try 16 I-beams.

The section modulus of one beam will then be $\dfrac{870{,}000 \times 12}{16{,}000 \times 16} = 40.8.$

Assume then 16 — 12″ 40.8 pound I-beams which will be satisfactory to carry the load.

Test for clear space, as above.

$$5.25 \times 16 = 84''$$
$$144 - 84 = 60''$$
$$\frac{60}{15} = 4.0''$$

Hence clear space is sufficient.

(*b*) Try 12 I-beams

The section modulus of one beam will be $\dfrac{870{,}000 \times 12}{16{,}000 \times 12} = 54.3$

Assume 15″ 42.9 pounds which will be satisfactory. Test for clear space.

$$5.5 \times 12 = 66''$$
$$144 - 66 = 78''$$
$$\frac{78}{11} = 7.09'' \text{ clear space.}$$

The maximum allowable clear space will be $\frac{3}{2} \times 5.5 = 8.25''$.

Hence 6.36″ will be satisfactory.

The grillage will then be made up as follows: 12 — 15″ 42.9 pound I-beams for the lower tier and 4 — 24″ 79.9 pound I-beams for the upper tier. Concrete to be placed 12″ below the flange of the bottom tier and 4″ on the sides.

1319. The **reinforced concrete footing** has the advantages of being less expensive than the I-beam grillage and of lending itself to construction in any shape. It is either of the slab or slab and beam type, although the shapes and method of reinforcing vary greatly. A comparison of plain and reinforced concrete footings can be made by reference to Figs. 1319A and 1319B.

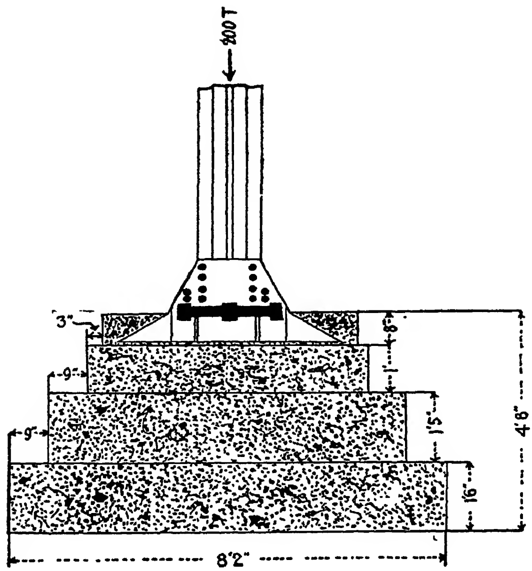


Fig. 1319 A

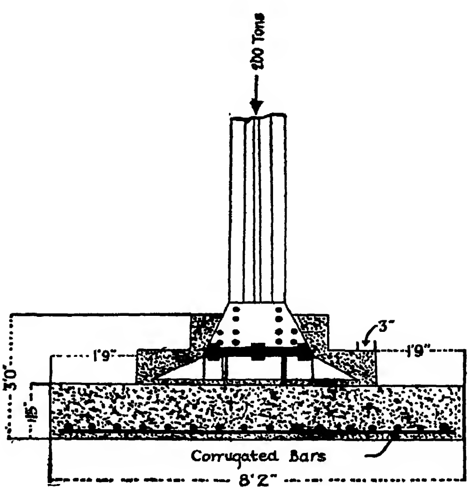


Fig. 1319 B

1320. Comparison of Cost of Single Footings.

(*a*) Plain concrete footing:

Excavation, 11.5 cu. yds. at 50¢ per yd.	$ 5.75
Concrete, 205 cu. ft. at 20¢ per cu. ft.	41.00
	$46.75

(*b*) Corrugated bar reinforced concrete footing:

Excavation, 7.5 cu. yds. at 50¢ per cu. yd.	$ 3.75
Concrete, 102 cu. ft. at 20¢ per cu. ft.	20.40
Corrugated bars, 383 pounds at 2.5¢ per pound	9.57
Extra column length, 85 pounds at 3.5¢ per pound	2.98
	$36.70

This comparison shows that even in the single piers a distinct saving is made by the reinforced concrete design. The percentage of saving increases with the size of the footing. Another advantage of the reinforced concrete construction lies in the greatly decreased volume occupied as compared with unreinforced concrete.

Soft Soils.

1321. The bearing power of the soft soils most nearly approaching in consistency to the firm soils may be increased sufficiently to bear the weight of an ordinary structure by increasing the area over which the weight is distributed. This may be done by making the footing wider, or by placing under the footing of the wall, a deep cushion of compressed sand much wider than the footing itself.

If the soil is marshy and is continually wet, a grillage made of several layers of beams in close contact may be employed instead of the sand. Timber is often used for this purpose since it will last indefinitely if kept continuously wet. Temporary weights such as the abutments of small military bridges are often supported by mattresses of poles and brush tied together by wire.

If the soil is too soft to admit of such constructions, the trench under each foundation wall must be excavated until a layer of firm soil is reached, or the foundation must be strengthened by the use of piles. The soft soil should be removed and the foundation laid on the firm soil beneath whenever this can be done without great cost.

Piles.

1322. A pile is a support which is driven or forced into the soil without previous excavation. Piles are classified in several ways: by their use, as bearing piles or sheet piles; by their material, as timber or concrete piles; by their slope, as batter piles; and by some special feature, as disk or screw piles. Piling is ordinarily of either timber or concrete,

although cast and wrought iron are employed in places where wooden piles would not be durable or where the pile acts as a column without lateral support and where great strength is required. A bearing pile is one

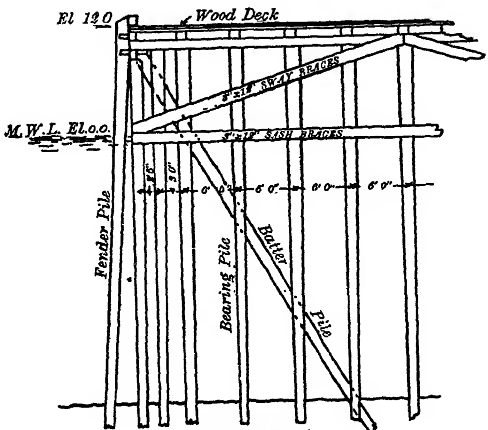


Fig. 1322 A
Wood Pile Pier

which resists a vertical force, a batter pile one which resists an inclined force (Fig. 1322A). Sheet piling is the name given to a special form of piling driven to form a wall resisting the lateral pressure of wet earth or water. In its simplest form it is a plank driven vertically into the ground.

1323. The timbers most often used for piles are: pine, oak, Douglas Fir, redwood, and cypress, although any wood which will stand driving is satisfactory for temporary construction. Timber piles are prepared from the lower portion of the tree trunk by removing all branches and bark and closely trimming all knots.

1324. A pile supports its load in one of two ways, depending on the soil conditions where it is driven. If the driven pile does not reach firm or hard soil it supports its load by frictional resistance developed on that part of the surface of the pile which is in the ground. The load in this case is transmitted to the soil in widening areas forming a conoid of pressure whose principal dimension is vertical. When the end of the pile rests on a hard stratum it acts like a column, more or less supported, and should be designed as such.

Pile Drivers.

1325. Piles are placed in position by the successive blows of a heavy hammer or by aid of a water jet. The machine used in driving piles is called a pile driver (Fig. 1325A). In its simplest form it consists of two

parallel vertical members called leads which guide the hammer during operation and which at the top hold the sheaves for the hammer cable. The leads are held in position by a truss arranged as shown in the figure. The height of the tower thus formed depends on the length of the pile

Fig. 1325-A
Field Pile Driver with Steam Hammer

which it is expected to use. This frame work is supported either on the deck of a barge, for use on water, or on a platform set on rollers or wheels, for use on land. This platform supports the boiler and hoisting engine, the latter being of the two drum* friction grip type. Either timber or steel may be used in the construction. With this rig are used two general types of hammers, the drop hammer and the steam hammer.

1326. The **drop hammer** is a solid iron casting with grooves to hold it between the leads of the pile driver, and with a pin at the top to which is attached a cable from the hoisting engine. The hammers used in the United States to drive timber piles vary in weight from 2000 to more than 4000 pounds. The hammer is raised to the top of the leads and a pile hoisted into place. Iron or wooden bars resting in hooks on the leads may be used to keep the pile in place during driving. The brake

* One for hoisting hammer, the other for hoisting pile.

is then removed from the cable drum of the engine and the hammer falls, driving the pile into the soil. The hammer is thus raised and dropped until the pile is driven far enough. It has been learned from experience that 15′ is the maximum height of fall that can be used without injury to the pile. Over-driving, that is, continued driving after the pile is properly seated, or driving with too great a hammer fall, causes the pile to shear, buckle, or broom, so that its bearing properties are greatly impaired.

1327. **The steam hammer** consists of a heavy iron frame encasing a movable piston, which is raised and forced downward by steam pressure. The piles are driven by the blows of the piston, which under the action of steam falls through a distance varying from a few inches to two feet according to the size of the hammer. The frame is constructed so as to slide in the leads, and carries at its lower end an anvil shaped to receive the head of the pile, this anvil receiving the blows from the piston. The weight of the frame helps to drive the pile. The steam hammer is much superior to the drop hammer in the following respects: (a) its blow is lighter, and therefore it does not broom the pile so much; (b) it drives more rapidly, since it strikes from 100 to 300 blows per minute and this does not allow time between blows for the soil to close in on the pile. Commercial types range in weight from about 175 pounds, used to drive light sheet piling, to over 8000 pounds, used to drive the heavier types of timber and concrete piling. The McKiernan-Terry No. 9 steam hammer (Fig. 1327A), has a total weight of 8100 pounds, with a piston weighing 1250 pounds, which under 80 pounds per square inch steam pressure strikes 200 blows per minute, each blow exerting a downward force of 7100 pounds.

1328. **The Water Jet.**—If the soil is sand, silt, or mud without boulders or other obstructions, a pile may be forced into the ground by placing a load on its top and then forcing a stream of water to its lower end. The water softens and stirs up the material so that the pile sinks under the load placed upon it. In sinking a wooden pile the stream passes through a tube which is attached to the pile by means of staples. When the pile is in place the tube is withdrawn. Reinforced concrete piles and other piles which would be shattered by the hammer may be driven in this manner.

The water jet tube is usually made up of one or more straight pipes 2″ or $2\frac{1}{2}$″ in diameter and carries a nozzle (usually about 1″ in diameter) at its lower end. The top of the tube is connected by flexible couplings to the discharge of a pump delivering from 50 to 250 gallons of water per minute according to the fineness and compactness of the soil. Water pressures of from 60 to 200 pounds per square inch are used. The water

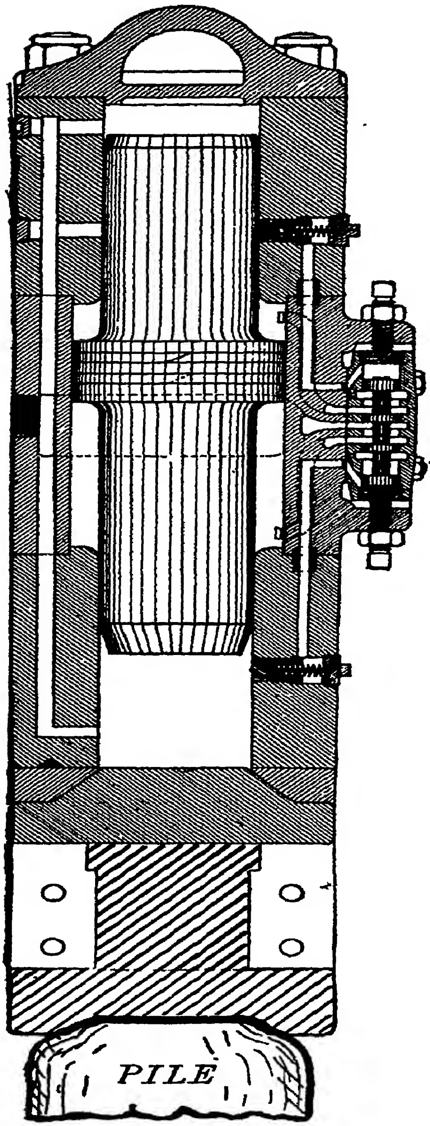


Fig. 1327 A
McKiernan-Terry
No 9 Steam Hammer

not only softens the soil at the tip of the pile, but by coming to the surface around it reduces the frictional resistance on the sides of the pile, which sinks under the weight. This method of placing piles is extensively used with concrete piles, and with other piles in sand or fine compact silt. Its greatest advantage is that the pile is placed without injury to it.

Load on Piles.

1329. The resistance of a driven pile to further movement could be accurately determined at any time were the intensity, direction, and law of distribution of the earth pressure upon its surface and bottom known. As these cannot be definitely ascertained, all formulas for the safe load upon a pile must be theoretical, based upon hypothesis, or simply empirical, based upon observation.

1330. **Theoretical Formula for Drop Hammers.**—Captain Sanders, Corps of Engineers, deduced a formula based on the hypothesis that the work of the pile's resistance developed by the last blow of the hammer is equal to the theoretical kinetic energy of the hammer at the instant of striking, or

$$R'd = \frac{MV^2}{2} = Wh, \qquad \text{or} \qquad R' = \frac{Wh}{d} \tag{1330A}$$

in which R' = ultimate resistance of pile, or its ultimate load, in pounds;
M = mass of hammer;
V = striking velocity of hammer in feet per second;
W = weight of hammer, in pounds;
h = height of fall of hammer, in feet;
D = penetration of pile at last blow, in feet.

Recognizing that the value of R' in this formula was too great because some of the kinetic energy of the hammer was expended in overcoming the friction of the hammer in the guides, in deforming the pile and the hammer, in producing heat, etc., he proposed as the safe load of a pile, one-eighth of R'. His allowable load was therefore

$$R'' = \frac{Wh}{8D} \tag{1330B}$$

To provide for the losses due to friction, etc., in the original formula it has also been proposed to increase the denominator of the theoretical formula by a constant determined from experiment. If the constant $\frac{1}{12}$ is adopted, the formula (1330A) becomes

$$R' = \frac{Wh}{D + \frac{1}{12}} \tag{1330C}$$

If h is left in feet, and d is expressed in inches, this becomes

$$R' = \frac{Wh}{\frac{D}{12} + \frac{1}{12}} = \frac{12\,Wh}{D+1} \tag{1330D}$$

The most complete theoretical formula that has been deduced is that of Mr. E. P. Goodrich, which in its general form consists of twenty-five terms and several coefficients. It has been found by experiment that many of these terms can be evaluated without exceeding an allowable error. When this is done the formula reduces to

$$R'' = \frac{10\,Wh}{3\,D} \tag{1330E}$$

where d is in inches.

1331. **Empirical Formulas.**—The most generally used formula in American practice is

$$R'' = \frac{2\,Wh}{D+1} \tag{1331A}$$

in which D is average penetration under last few blows in inches, h in feet, R'' and W in pounds.

This is known as the Engineering News formula. It was derived by Mr. A. M. Wellington from experimental data. This formula is the general standard now used. It will be noted that it can be obtained from the last form of Captain Sanders' formula by introducing a factor of safety of 6. The figure 1 appearing in the denominator takes care of the initial resistance of the pile on the last blow, which is taken to be equal to 1″ of penetration.

For use with steam hammers, this formula is modified to allow for the difference in initial resistance by substituting 0.1 for 1 in the denominator. Recent tests would seem to indicate that this constant should be 0.3. As more data are accumulated, this constant will be more accurately expressed. The most satisfactory way of determining the ultimate and safe loads is to drive several piles on the site selected, and some time after they are driven load them until they begin to sink. This will give the ultimate bearing power, from which the safe bearing power may be obtained by the introduction of a factor of safety.

Pile Foundations.

1332. When the weight of the structure is to be borne by the piles, the piles are driven along the bottom of the foundation trench in rows so arranged that no two piles are less than $2\frac{1}{2}'$ or $3'$ apart. The tops are sawed off in a horizontal plane and are covered with a grillage or with a layer of concrete.

1333. The grillage is made by capping each row of piles with a heavy longitudinal beam, which is fastened to the piles by mortise and tenon joints or by drift bolts. A plank platform may be spiked to the capping; or one or more layers of beams at right angles to each other and in close contact may be laid between the capping and the platform, each layer being fastened to the layers above and beneath. When a concrete bed is employed, the earth about the tops of the piles is replaced by a bed of concrete which holds the piles in place and distributes the weight over the entire area as well as over the piles.

1334. If the soil to a considerable depth is very soft, the piles are braced laterally by throwing loose stone in the spaces between them before the grillage is constructed. This is usually done when the piles are driven into a firm stratum which is covered by a thick layer of soft material.

If intended simply to compress the soil, piles from 8′ to 15′ long, 8″ to 12″ in diameter, are driven close together over the bottom of the trench or over the area of the foundation. The weight of the structure rests on both the compressed soil and the piles.

1335. The diameter of a bearing pile at the butt should be from 12″ to 18″. The length of a pile depends on the load and the character of the soil, and may be as great as 100′. Bearing piles should be straight and have tips of at least 7″ or 8″ in diameter. Piles should be cut off below the lowest ground water level, to insure their preservation.

Sheet Piling.

1336. Sheet piling may be timber, steel, or reinforced concrete. The best and most common commercial form of timber sheet piling is the Wakefield (Fig. 1336A). It consists of three planks bolted and spiked together forming a groove on one edge and a tongue on the other. Dressed lumber is used for the central plank so that a tight fit is obtained between each tongue and groove. The bottom of each pile is bevelled toward the longer center line of cross-section to aid alignment, and on one edge, toward the shorter center line of cross-section to force each pile tightly into the wall. The sheet piling is driven like ordinary round piling. As the excavation takes place, the piling is braced to resist lateral pressure due to water or soft soil.

1337. Steel sheet piling (Fig. 1337A) has been developed as an improvement on the use originally of structural steel shapes so arranged as to form a water-tight wall when driven as piling. One of the earliest forms was made up of alternate piles composed of standard I-beams, and of two standard channels bolted together with separators, to form a

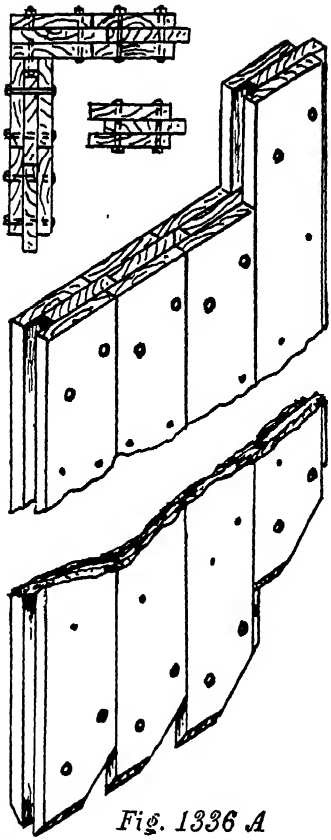
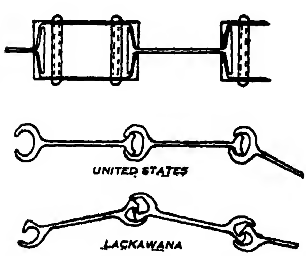


Fig. 1336 A

Wakefield Timber Sheet Piling



Steel Sheet-Piling

Fig. 1337 A

recess between the flanges of the channels, which received the flange of the I-beam. The success of steel sheet piling, due to its greater strength, its durability, and its lesser resistance to driving, soon led to the development of special shapes, all of the piles being alike, and interlocking when driven. In structural steel handbooks will be found information on these patented shapes, which are usually known by trade names. Steel sheet piling is more easily driven than timber sheet piling, requires less bracing due to its greater strength, and can be used to greater depths. Although more expensive in original cost, it is more economical than wood since it is easier to withdraw without injury, and hence can be used more than once.

1338. Reinforced concrete sheet piling is principally used where it is to be left in place to form part of a structure. This often occurs in piers and wharves and in some types of causeway construction where the concrete sheet piling forms a permanent wall of support. It is very durable and, although a little harder to handle in driving, is coming into general use. Each pile is designed as a flat slab reinforced concrete beam.

1339. **Life of Timber Piles.**—Timber which is kept constantly wet will last indefinitely in fresh water. One example is Attila's bridge over the Danube, the piles of which were recently discovered in good condition. If, however, the timber is subjected to alternate wet and dry periods, it decays within a few years. In the warm salt waters of the South Atlantic, Gulf, and Pacific States, timber piles are very short lived due to marine borers. The borers, or worms, the most destructive of which are the teredo and the limnoria, eat into the piling and soon honeycomb it, destroying its strength. To combat the attacks of these marine animals, timber piles in these waters are either impregnated with creosote oil or mechanically protected by metal or earthenware sheathing.

Concrete Piles.

1340. Due to the short life of timber piling under various conditions, and the increasing difficulty in obtaining long and heavy piles, concrete piles are being used extensively. There are two general classes of concrete piles: (1) the pre-molded, where the pile is cast, cured, and then driven like a timber pile; and (2) the cast-in-place, where the concrete is cast and sets in position.

Pre-molded piles are always steel reinforced. Several forms of this type have been patented, while other forms in general use are unpatented. In design and reinforcing, pre-molded piles are very similar to ordinary reinforced concrete columns, the longitudinal rods being placed just far

enough inside the concrete to prevent corrosion and being strengthened laterally by hoops or spirals of small cross-section.

Practically all forms of the cast-in-place type are patented. The most generally used is the Raymond pile. This pile is constructed by driving a sheet steel casing into the soil by aid of a collapsible core, which, when sufficient penetration is obtained, is removed so that the casing may be filled with concrete. Cast-in-place piles are ordinarily made without reinforcing.

Screw and Disk Piles.

1341. For piers, wharves, lighthouses, etc., hollow cast or wrought-iron piles 8″ to 12″ in diameter are sometimes used, having either a flat disk or a set of screw blades at the bottom. The disk of the disk pile is usually 3′ or 4′ in diameter with one or more holes for water jetting.

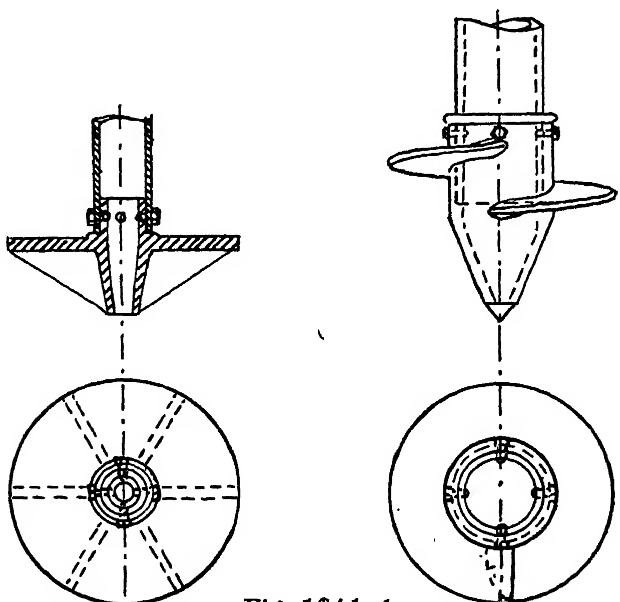


Fig. 1341 A

Disk Pile Screw Pile

The pile is usually made in sections which are fastened together as it is driven. Screw piles are usually made with a wrought-iron shaft and with cast-iron blades 3′ to 4′ in diameter. (Fig. 1341A.) Screw piles are usually sunk by men working with capstan bars. Disk piles are sunk by a water jet.

Screw piles may be forced through soils in which the water jet cannot be employed, and are used on shoals along the coast where the water is so rough that the ordinary pile driver cannot be employed. Like the disk pile, the screw pile has a large bearing area to resist not only the downward pressure but also the lifting power of the waves. The resistance of the screw and disk piles is due principally to the resistance of the soil under the disk or screw. This is increased, however, by the resistance along the sides, which varies from 200 to 800 pounds per square foot of this area, depending upon the firmness of the soil.

FOUNDATIONS UNDER WATER.

1342. The foundations of bridges (piers, and abutments), wharves, dock walls, breakwaters, etc., are constructed on sites covered with water. The difficulty of construction depends upon the depth of the water, the velocity of the current, the variation of depth of the water, wave action, and the care which must be taken to prevent settlement.

There are two general methods of executing the work: in the first, the foundation is laid without removing the water from the site; in the second, the water is removed from the site so that the work may be executed practically in open air. Whatever the method of execution, if the structure is an important one, the soil must be previously examined by borings or soundings.

WITHOUT DRAINING THE WATER FROM THE SITE.

Random Rock.

1343. The simplest method of constructing foundations under water is to throw in layers of stone until the stone reaches the surface of the water. The top of the rip-rap is then covered with a level bed of concrete which becomes the base of the superstructure. This method is employed in the construction of breakwaters, jetties, etc. The foundation is rendered compact by the action of the waves, and slight settlement is not objectionable. On exposed sites very large blocks of concrete are preferred to natural stones.

Cribs.

1344. If the soil is firm, the foundations for light structures may be made by sinking common cribs. These are constructed of round or squared logs halved at the ends and firmly fastened together with bolts or iron clamps. The crib is divided into a number of pockets by strong cross-walls. Some of the pockets are floored near the bottom and others are left open. The crib is floated to the site, held in place by piles or anchors, and sunk in place by filling the closed pockets with

quarry stone. When the crib reaches the bottom, all the pockets, both open and closed, are filled with stone and the crib is thus securely anchored. The stone in the open pockets settles under those parts of the crib which do not touch the bottom. Cribs require less stone than random rock foundations. If cribs are to be used for foundations of masonry structures, the crib work should be terminated below the low-water line; for piers of temporary bridges the crib extends above the water level.

Rock in Shallow Water.

1345. If the bottom is of rock in shallow water and the current is not strong, forms may be placed and the foundations made of concrete. Since concrete dropped through water is injured by the mortar being washed from the stones, the following methods have been employed to get it into place with as little injury as possible:

(1) The concrete is lowered to the bottom in open-mesh bags which are carefully laid by divers;

(2) it is lowered in water-tight boxes with doors in the bottom which are opened by means of a cord when the box reaches the bottom;

(3) it is passed through tubes which reach from the deck of a scow to the bottom. The concrete is placed and rammed in layers.

Concrete laid in water is not as uniform or compact as when laid in air. For this reason the usual practice when possible is to lay a few feet to seal the bottoms of the forms, which are practically water-tight; then to pump out the water and complete the concrete work in the air.

Piles under Water.

1346. If the river bed is of earth, and the water is not too deep, some form of pile foundation may be employed. When piles are used as a foundation for a masonry structure, they are cut off a short distance above the bed of the stream and capped either with a timber grillage or with concrete. Where a timber grillage is used, each cut-off must be at the same elevation so as to obtain an even bearing. When concrete is used to cap the piles, it is not necessary nor advisable to cut the piles at the same elevation, since it strengthens the concrete to have the tops of the piles in more than one horizontal plane.

Piling which extends some distance above the bed of the stream is strengthened laterally by bracing or by placing rip-rap around it. Disk and screw piles are made long enough to extend above the water and may be leveled by adjustable caps. They are stiffened laterally above water by struts and ties connecting each pile with at least three others.

Metal Shell Cylindrical Pier.

1347. This pier is a concrete pillar resting on a pile foundation and surrounded by a cylinder of wrought or cast iron. Its diameter varies from 4′ to 10′. To construct this pier, a cluster of piles covering as large an area as can be conveniently surrounded by the cylinder selected is driven well into the bed of the stream. The piles of the cluster are in

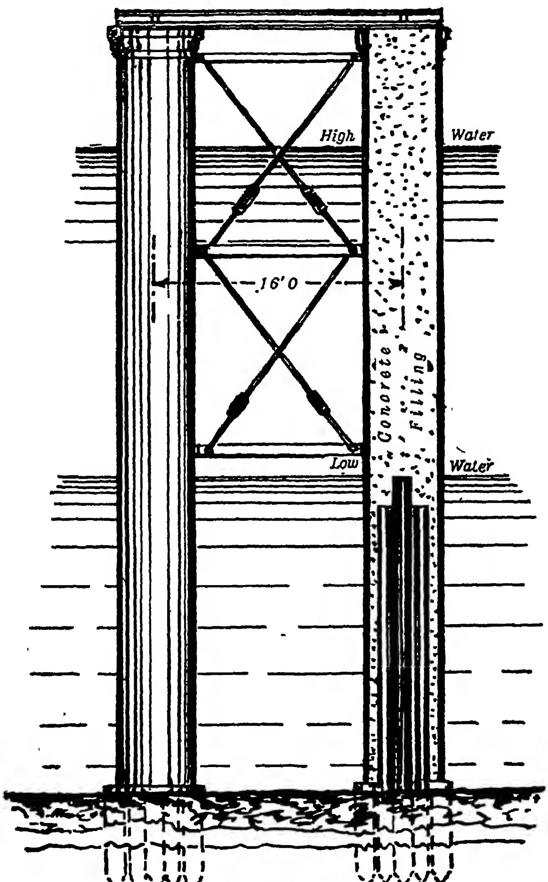


Fig. 1347 A
Metal Shell
Cylindrical Piers

close contact. The piles may be cut off just below the water line, although this is frequently omitted. The iron cylinder is lowered over the piles in sections 5′ or 10′ long which are bolted or otherwise fastened together. The cylinder rests on the bottom of the stream unless there

is a soft bottom which has not been entirely dredged away, in which case it sinks a short distance under its own weight. The cylinder is then filled with concrete with or without first pumping out the water. The piers are usually constructed in pairs or groups and united by braces (Fig. 1347A). These piers are used in the construction of highway and short-span railway bridges.

In firm soil and still water the cluster of piles may be omitted and the concrete made to rest on the soil itself; in soft soil the same method may be employed, but the diameter of the pier must be increased. However, if any horizontal forces are to be expected, the pier must be bonded to the soil. In soft soil, this is done with piles as described above: with rock bottoms, it is done by drilling holes in the rock and grouting in iron bars which project up into the concrete.

1348. The piers of Bridge No. 5 of the Norfolk and Western Railway over the Elizabeth River near Norfolk, Va., are of the type described

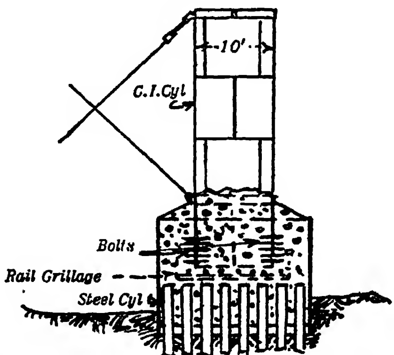


Fig. 1348 A

Cylinder Pier. N.&W.R.R.

above (Fig. 1348A). The cylinders are 20′ in diameter and 15′ 9″ high, made of $\frac{3}{8}''$ steel braced by $3\frac{1}{2}'' \times 3\frac{1}{2}''$ angles. They were sunk in the bed of the river by their own weight, dredged out, and 80 piles driven inside them and cut off 7′ above the bottom. The cylinder was then filled with concrete into which was imbedded a second and smaller cylinder carrying the bridge load.

EXCLUDING THE WATER FROM THE SITE.

Coffer-dams.

1349. Where the foundation is to rest at a depth of not more than about 30 feet below the water surface, a coffer-dam and pumping will usually be the most practical and economical means of excluding the

water. A coffer-dam is a temporary dam built around the site of the foundation so as to exclude the outside water from it either completely or sufficiently to keep the area dry by a practical amount of pumping. The inside water is pumped out and the substructure can then be built in air. It is found cheaper to pump some water which leaks through the coffer-dam than to make a perfectly tight dam.

1350. Coffer-dams may be divided into four general classes: earth, earth and sheet piling, sheet piling alone, and movable coffers.

1351. **Earth Coffer-dam.**—The earth coffer-dam is simply a temporary dam of earth placed around the site to be unwatered. It can be used only in shallow water where the current has a very low velocity. It is constructed of earth either brought from the shore or dredged from the bottom. Sand or gravel mixed with clay is the best material. The dam should be made about three feet higher than the water level and three or four feet wide on top. The water side should have the natural slope of the material while the inside slope should be more gradual.

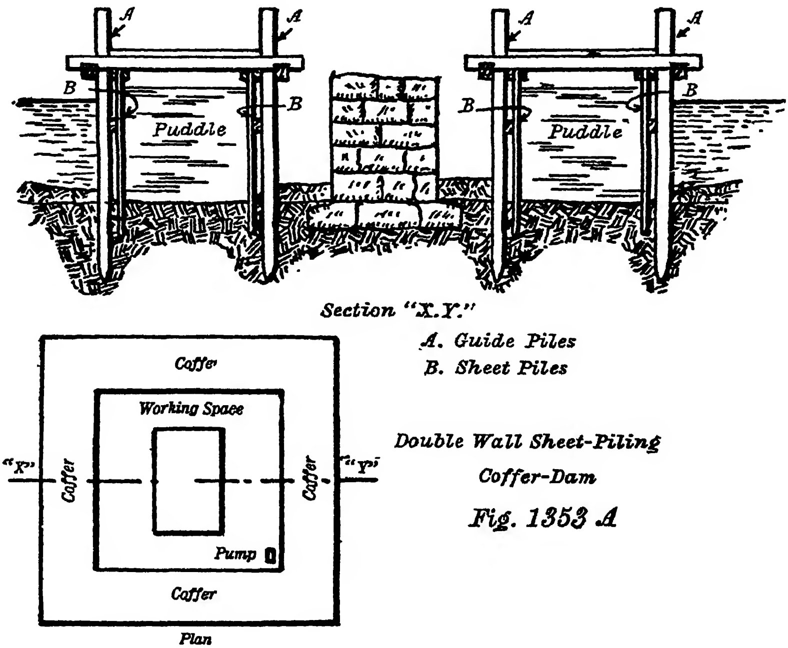
1352. **Earth and Sheet Piling Coffer-dam.**—In order to increase the water-tightness of an earth coffer-dam, plank or sheet piling is sometimes driven to form a core wall. This wall may consist simply of a single line of plank or sheet piling driven along the center of the dam or it may consist of two rows of sheet piling separated by a few feet. In the latter case, puddling is placed in the space between the two rows of sheet piling. The puddling should consist of material which is impervious to water and which will not be washed away if a leak is developed in the dam. A mixture of clay, sand, and gravel is the best material for the purpose; the finer material fills the interstices of the coarser, and the coarser material resists the movement of the water when a leak develops. Common loam, clay, and fine gravel are also employed as puddling material. The puddling is placed and tamped in such a manner that there shall be no distinct layers.

Experience has demonstrated that a thickness of two feet of good clay puddling is sufficient to prevent the percolation of the water through a coffer-dam of the usual height.

The sheet piles are first driven around the site; then the puddling and the embankments on the outer sides of the sheet piling are constructed at the same time.

1353. Coffer-dams are also built without the earth embankments when the embankments are not needed for stability, or when it is inadvisable to obstruct the stream with them. This type is shown in Fig. 1353A. Guide piles, marked *A* in the figure, are first driven around the site in two parallel rows connected by horizontal beams, called wales, bolted to the

piles. The wales are then connected by rods and cross-beams. The sheet piling marked B is then driven against the wales, and fastened to them. The puddle is then placed and, if floods are to be expected, the top is planked in to prevent the puddle from being washed out. The thickness of the dam is usually made about equal to the height for low



A. Guide Piles
B. Sheet Piles

Double Wall Sheet-Piling
Coffer-Dam
Fig. 1353 A

dams and equal to about one-half the height for high dams. The thickness of the sheet piles depends upon the distance between the wales. Piles must be driven sufficiently deep to prevent the puddling from escaping underneath them.

The guide piles and the weight of the dam must resist the tendency of the dam to overturn or slide due to the water pressure against the outside sheeting. This pressure will determine the length of the guide piles and the distance between them in each row; the latter is usually from four to eight feet.

The distance between the inner rows of piles and the foundation is at least three feet, if no inner excavation is necessary; if the soil must be excavated, this distance is increased by one and one-half times the depth of the excavation. When necessary, the stress in the piles may be relieved by inner braces which rest against the foundation itself or against adjacent sides of the dam. This type is often used when large areas are to be unwatered.

1354. Steel sheet piling may be used without support, several important works having been completed this way. This method was used in constructing the coffer-dam built in Havana Harbor by the Corps of Engineers around the U. S. S. *Maine*, Lackawanna steel sheet piling being used. This dam was elliptical in form, 219 feet by 398 feet along the axes, and was in water varying in depth from 29 to 37 feet. The ellipse was composed of cylinders each 50 feet in diameter which were filled with mud dredged from the harbor bottom. The cylindrical form was adopted because of its inherent strength against outside pressure; each cylinder was so designed as to be self-supporting, adjoining cylinders being connected by arcs of sheet piling to increase the strength and water-tightness.

1355. **Sheet Piling Coffer-dam.**—In this type of coffer-dam, the stability and water-tightness are dependent on a single wall of sheet piling driven around the site unwatered. They are generally called single-walled coffer-dams. The single-walled coffer-dam is less expensive than the double-walled and is better adapted to small areas. In narrow streams with swift currents it is very practical, since it interferes as little as possible with the flow of the water. Some form of support is required, which may be (*a*) internal bracing whereby the sides are braced against each other or against the structure as it is built; (*b*) cribs filled with stones; (*c*) piles acting as cantilever beams. Cribs, if used, are made on land, of round logs held together by drift bolts, then floated to the site and sunk by filling with stone. After the cribs are in place, the sheet piling is driven and made water-tight in case of leaks by banking clay on the outside or in some cases covering the sheet piling and the adjacent bed of the stream with canvas which is weighted with stone or other material to make it conform to the bottom. Where steel sheet piling is used, cement grout is placed in all joints to prevent leakage. The steel sheet piling, due to its greater strength, is now more generally used than timber.

When the bottom of the river is rock, the sheet piling cannot penetrate and is supported by cribs. Timber sheet piles are generally used in this case, the bottoms of the piles being mashed on the rock to give a close fit. When seams are found in a rock bottom they are grouted.

1356. **Movable Coffer-dam.**—When a coffer-dam is used to aid in placing a grillage which is to rest on soft rock or piles, it is usually constructed so that the walls can be easily detached and removed. This is known as a movable coffer-dam. (Fig. 1356A.) The grillage is made of two or more layers of heavy timbers placed in contact with each other and fastened together. The sides of the coffer-dam consist of frames to which planks are spiked. The sides are held in place by blocks spiked to the

bottom, and iron rods which at the bottom hook into iron rings fastened to the grillage beyond the sides and at the top pass through heavy cross-beams which are notched into the sides. When completed, it is thoroughly calked to make it water-tight.

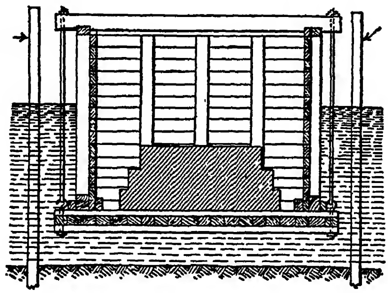


Fig. 1356 A

Movable Coffer

Each horizontal dimension is from four to six feet greater than the pier, and its height is several feet greater than the depth of the water. It is loaded with a few courses of masonry to give it steadiness and is then floated to the site which has in the meantime been prepared to receive it. It is accurately put into position, and is held in place by piles, anchors, or cribs while the masonry is being laid. When it has sunk nearly to the bottom, sufficient water is let in through valves so as to sink it in place. If it does not rest evenly on the bottom, the water is pumped out, the coffer-dam floated, and the foundation readjusted. When it rests satisfactorily on its foundation, the masonry is continued until it reaches the top. The iron rods are then removed and the sides taken away.

The grillage bottom may rest upon the soil itself if it is firm, especially in still water where there is no danger of undermining. The bed is prepared by dredging the soft material and then leveling it for the grillage.

1357. **Design and Construction.**—It is impracticable to design earth coffer-dams. The general method of design is to consider the hydrostatic pressure, and make allowance for changes in the water level and for floating material. When a coffer-dam fails, the cause is usually continued leakage and consequent washing out of the material of the coffer-dam; hence care must be exercised in construction to obtain the proper mixture of clay and gravel and to place it properly. Leaks should be located as they develop and checked before they create weakness in the coffer-dam.

When sheet piling is used, it is constructed to act as a simple beam

between supports and is designed to resist the pressure of the water and the earth filling. Experience has indicated many details of construction and design which cannot be theoretically calculated.

1358. **Dredging.**—When material from the bed of the stream is used in the construction of the coffer-dam, some form of dredging apparatus is used. The kind of apparatus depends upon the depth of the water and the character of the soil to be excavated.

The most common apparatus is the clam shell bucket attached to the hoisting cable of a crane. As its name suggests, it is a bucket made of two leaves each shaped like an ordinary clam shell. When open, their cutting faces are vertical; when closed, they form a semi-cylindrical bucket. A modification of this bucket, called the orange peel bucket, is hemispherical in form and opens out into four sections. These buckets are made of steel and when dropped open they penetrate the soil, due to their own weight. By a suitable device the buckets are closed while in the soil and the material is hoisted by a crane.

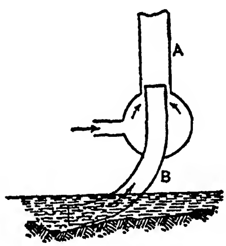


Fig. 1358 A

Sand Pump

The sand pump (Fig. 1358A) is often used in restricted areas. It operates on the principle of a reduction in pressure due to the production of a vacuum by the high velocity of a stream of water passing through an orifice; this causes the mud or sand to be carried with the water into a discharge pipe. The high pressure water is forced into a chamber at the base of the discharge-pipe, *A*, just below its junction with the suction-pipe, *B*. As the water escapes between the pipes it creates a partial vacuum in pipe *B*, sucks up the mud or sand and water and discharges it at the desired point.

Large centrifugal pumps are also used with what is called a suction dredge and the material is actually pumped, the pump having sufficient clearance to pass the gravel. Buckets fixed on endless chains or belts are also employed. The last three methods deliver large quantities of water with the soil, which prevents their use under some conditions.

1359. **Divers.**—Where sunken logs, boulders or other obstructions are encountered in driving sheet piling, or when leaks develop in a coffer-dam, it may be necessary to obtain the aid of a diver to clear up obstructions below the water surface.

The diver wears a strong air-tight suit surmounted with a metal helmet which rests on his shoulders. The helmet is connected by a flexible tube to a small hand air-compressor above water. To enable him to see, the helmet has a glass face, and to keep him in an upright position the soles of his boots are heavily weighted. The diver is raised and lowered by means of a rope which passes under his arms.

The pressure of the air in the helmet must be sufficient to withstand the pressure of the water on the flexible tube, or 15 pounds above the normal for every $34\frac{1}{2}$ feet of submergence. The extreme depth to which a diver has gone is about 207 feet.

Caissons.

1360. A caisson may be defined as a box which is placed in position and then filled with concrete or masonry to form a permanent part of the substructure. There are three general classes of caissons used in foundation work; the box caisson, which is open at the top and closed at the bottom; the open caisson, in which both top and bottom are open; and the pneumatic caisson, in which the bottom is open and the top closed, compressed air being used inside the box thus formed.

1361. **Box Caisson.**—Either wood or reinforced concrete is usually used in constructing box caissons. When made of wood, the bottom is a water-tight grillage of heavy timbers, and the vertical walls are of horizontal sheathing fastened to vertical studding. The height of the wooden caisson is such that when it is in position it will be entirely below low water. For this reason, removable coffer-dams are used to increase its height during placing. If a concrete box caisson is constructed, the expense is greater; but it is not necessary to have removable sides, since the caisson can be finished off above the water line as part of the structure without destroying its appearance. In water infested with marine borers, the concrete box caisson is particularly advantageous. The caissons are constructed on shore usually on skids to assist in launching; they must be reinforced to withstand both the exterior forces of sinking and the interior forces due to the filling.

1362. The most common use of the box caisson is where the foundation is to rest on piles or on soil which in its natural condition presents a firm and level bed. The method used in placing a foundation with a box caisson is to construct the caisson on the shore, adjacent to the site, and when completed to load it with enough of the foundation masonry

to make it stable in the water. It is then launched and moved to its position, where it is kept in place during sinking by guide piles, cribs or guy lines. The concrete or masonry is then placed and sinks the caisson by its weight. Due to the solid bottom of the caisson it is difficult to remove material from beneath it as it sinks, although this has been accomplished for short distances by the use of large numbers of water jets projecting through the bottom. If a small amount of soft material has to be removed to expose a solid stratum it is usually accomplished by dredging. It is therefore seen that the box caisson is limited in its use to those foundations where piling is used or a firm bearing can be obtained practically without excavation.

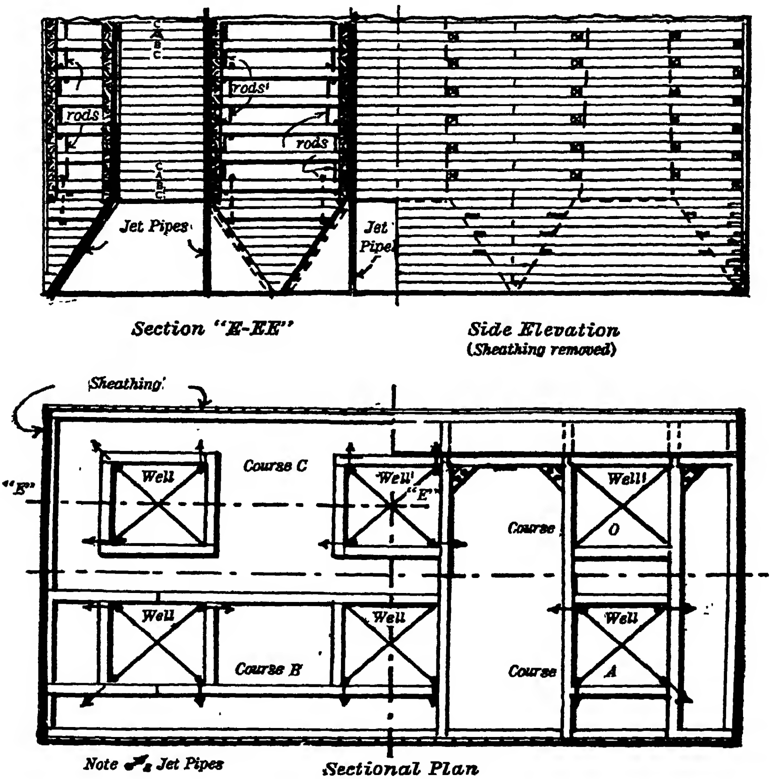
1363. **Open Caisson.**—Open caissons can be divided into three general groups or types; the single wall, the cylindrical, and the caisson with dredging wells.

The single wall open caisson consists of a framework of heavy timbers, steel, or concrete, either entirely or partly open at the top and bottom. It is employed in very soft material or where little sinking is required. The caisson is placed and loaded with heavy weights so that it will sink as the material is removed from within by dredging as described above. If the friction on the sides is too great, water jets placed along them are used to aid in the sinking. When in place, a layer of several feet of concrete is deposited through the water and allowed to set, the water is pumped out, and the remainder of the concrete or masonry placed in the dry, thus producing better concrete than if it were all placed through the water. The use of the single wall open caisson is limited by the fact that the sinking is produced by temporary weighting which makes it successful only in soft material or where the required depth is not great. It is standard practice to dredge as much of the soft material from the site as possible before sinking the caissons.

1364. **Cylindrical Open Caisson.**—Steel and reinforced concrete are used most frequently for cylinder caissons. Timber and cast iron, although used, are not generally satisfactory due to their lack of strength. The limit of practical use of cast iron is in caissons of about 5 feet in diameter. The cutting edge and the weights for sinking the single wall open caisson distinguish it from the cylindrical pier previously described. The single wall type is usually of small diameters and is used where the load to be carried is light but where considerable depth is necessary in order to obtain the proper bearing soil or to avoid scour. The sinking is accomplished by dredging out inside of the caisson and loading the top. This loading of the top of the caisson is expensive due to the cost of removing and replacing the weights as each new section is added to the caisson. For this reason it is customary in the large cylinders to use a

double wall so that part of the concrete for the foundation can be placed while sinking the caisson, thus either greatly reducing or eliminating the temporary loading. The double wall type is more often used than the single wall type due to this great advantage.

1365. **Open Caisson with Dredging Wells.**—It has been found that with large open caissons it is advantageous to have more than one dredging well so that the sinking can be better controlled especially



Open Caisson with Dredging Wells

Fig. 1365 A

when the hardness of the stratum is not uniform over the site. When built with more than one well, the caisson is usually larger. This type of caisson is used in placing the deepest foundations, there being no other method for depths of more than 110 feet.

The dredging wells are usually placed so that each corner of the caisson has a well to aid in regulating the penetration at that point (Fig. 1365A). The spaces between the wells form pockets which are filled

with concrete during the sinking, the completed structure resting on this filling as well as that placed in the dredging wells.

1366. **Advantages of the Open Caisson.**—The coffer-dam and pneumatic caisson methods of placing a foundation have two great advantages over the open caisson method: first, the work is more easily controlled; and, second, the foundation bed can be inspected before the masonry is placed. As has been stated, the usual limit of coffer-dam work is about 30 feet and it will be seen that the limit of the pneumatic caisson is about 110 feet while for greater depths the open caisson method is the only one available. It will often be cheaper however to use the open caisson instead of the pneumatic caisson for depths between 30 and 110 feet due to the fact that the work being entirely above ground, the labor cost is lower.

When the open caisson is used, it is customary to make it a little larger than necessary so as to provide against decreased effectiveness due to slight loss of control while sinking. The Poughkeepsie bridge piers were placed by this method, the largest caisson used being 60 $\times$ 100 feet. It rests on gravel 134 feet below high water.

1367. **The Pneumatic Caisson.**—The pneumatic caisson might be described as two box caissons placed bottom to bottom; the lower box opening downward, is the working chamber, and is usually about 6 or 8 feet high; the upper box, open at the top, is an actual box caisson. The pneumatic process consists in excluding the water from the lower box by means of compressed air. The workmen enter and leave the working chamber by doors in its roof which open into shafts running above the water level. Other shafts are used for lowering and removing materials and for the air pipes and electric wires. The shafts are of steel and about 4 feet in diameter; those for handling material may be smaller than those used by the workmen.

1368. To prevent the escape of the compressed air and to guard the health of the workmen, air-tight vestibules or air-locks are provided in each shaft. The air-locks open both into the outer air and into the shaft and are arranged so that the air pressure in them can be regulated. As the doors open towards the interior of the caisson, they are kept closed by the air pressure whenever there is a difference of pressure on the two sides of the wall in which they are made. An air-lock may be constructed at the top of the shaft, at the bottom, or at any intermediate position. The first is the best position for the workmen in case of accident, but the locks must be moved as the caisson is built up. The second is the most convenient, but also the most dangerous. An intermediate position is sometimes used by making the shaft discontinuous;

the upper part contains the air-lock at its base, the lower part is tangent to the upper and overlaps it to the height of the lock. A section of the shaft itself may be converted into an air-lock by inserting horizontal air-tight doors.

1369. Pressure in the air-lock is normal when the workmen enter from above, and is then gradually increased until it is equal to that in the working chamber below. In leaving the working chamber, it is necessary to reduce the pressure in the air-lock very gradually to prevent the caisson disease called "bends." Since the pressure has to be increased 15 pounds for every $34\frac{1}{2}$ feet head of water and since state laws generally set 50 pounds as the maximum pressure allowable, it is seen that about 110 feet is the limit for use of the pneumatic caisson.

1370. The working chamber is surrounded by wedge-shaped walls carrying at their bottom the cutting edge. This edge is blunt, usually

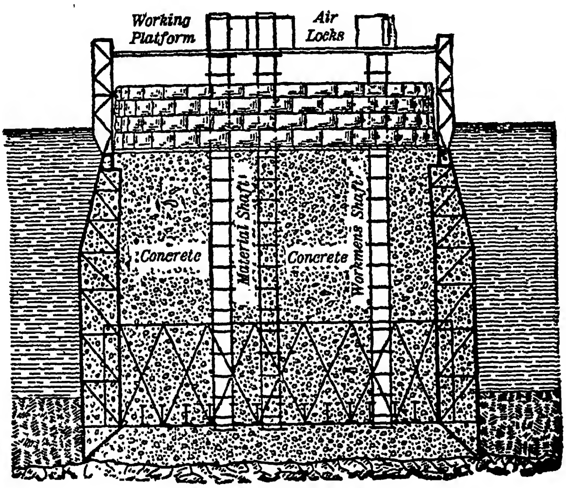


Fig. 1370 A

Tryical Pneumatic Caisson

4 to 8 inches wide, and made of steel. It must be designed to have sufficient bearing and yet be narrow enough to penetrate. Either timber or steel or both may be used in construction of the caisson. If of timber, the roof consists of several layers of 12 × 12 inch timbers laid in close contact and calked; if of steel, it is composed of trusses and girders connected by I-beams covered with plates. A movable coffer-dam (as previously described) may be placed on top of the caisson to increase its height as shown in Fig. 1370A. As soon as the masonry is completed above the water, the walls of the coffer-dam are removed.

1371. **Sinking the Caisson.**—The pneumatic caisson is partly constructed on shore, launched, partially loaded with masonry, floated into position and anchored. It is lowered to the bottom in much the same manner as the crib and cylinder caissons are sunk by the well process. When it comes to rest, compressed air is forced into the chamber, and the workmen descend into it and begin to remove the material. When sufficient material has been removed they either leave the chamber or take refuge in the shaft while the air is allowed to escape gradually. The weight of the caisson then causes it to sink a few feet. If more weight is needed, concrete may be added to that already placed on the roof of the working chamber. The difficulty is usually, however, the reverse, that is, to prevent the caisson from sinking too rapidly. This can be regulated by carefully excavating under the partitions or piers of the working chamber.

This process is continued until the caisson finally reaches the stratum upon which it is to rest. If this is gravel or approximately level rock, it is leveled with concrete and the chamber filled with concrete. The concrete is lowered through the small shafts. The lower doors of these shafts are closed and supported, and the material poured in; the upper doors are then closed, the pressure regulated, and the lower doors opened. Water is poured into the tubes before and after the concrete to prevent the premature setting of the concrete due to the heat of the compressed air.

1372. If the bed is an inclined stratum of rock, it may be blasted to a horizontal surface, blasted into steps, or if the inclination is not great, the foundation may be prevented from sliding by steel rods driven into the rock. The caisson is stopped as soon as one of its edges rests upon any part of the rock.

1373. The soil may be removed by placing it in buckets which are hoisted up one of the shafts or it may be removed by a sand or mud pump. The latter may be operated by compressed air from the working chamber.

1374. **Advantages of the Pneumatic Caisson.**—This method of placing a foundation gives the most positive control possible. The sinking of the caisson can be regulated so that it is kept vertical. Obstructions, such as large boulders or logs, can be removed and the foundation bed can be inspected by the engineer in charge, thereby assuring its proper preparation. The concrete filling for the working chamber can be placed in air, which results in stronger concrete than when it is placed in water.

FOUNDATIONS BELOW QUICKSAND.

1375. None of the methods heretofore described is practicable when the bed of the foundation lies below a thick stratum of quicksand or fine sand which runs easily. The Poetch method of overcoming this difficulty consists in substituting, for the wooden or masonry walls of the foundation shaft as heretofore described, walls of frozen quicksand.

To form the walls, a number of iron pipes, closed at the bottom, are sunk or driven through the quicksand to the desired depth. These pipes are placed at small and regular intervals along some polygon surrounding the site of the shaft. Smaller pipes, open at the bottom, are inserted in the first set, and both sets are connected with a refrigeration apparatus so that the freezing mixture flows into the smaller and returns by the larger pipes. A cylinder of frozen soil is thus formed about each pipe, and in time these cylinders interlock and form a wall within which the shaft may be safely excavated.

1376. Another method which has been utilized for overcoming the difficulties of working in quicksand is the forcing of cement grout into the sand by means of force-pumps. The cement in setting converts the sand into a solid mass.

For more detailed information see:

Foundations of Bridges and Buildings, Jacoby and Davis.
Details of Masonry Structures and Foundations, Williams.
Practical Treatise on Sub-Aqueous Foundations, Fowler.
Engineering and Building Foundations, Fowler.
A Practical Treatise on Foundations, Patton.

PROBLEMS.

P. 1301. A timber pile is driven with a drop hammer weighing 3000 pounds. The average penetration under the last three blows of the hammer with a fall of 12 feet was 3 inches. What is the safe bearing value of the pile?

P. 1302. The columns of a building are 18 feet center to center in both directions. The load on each column at the footing is 120 tons. The base of each column is 30 × 30 inches. The borings show the earth on the building site to be: fill, 10 feet; river mud and silt, 12 feet; then ordinary clay. The permanent water level is 11.5 feet down. If the columns are to be placed on pile foundations, a minimum allowable penetration of the ordinary clay would be 7 feet. Allowable load on timber piles to be taken at 15 tons, on concrete piles at 30 tons. Wooden piles in place cost $0.75 per linear foot, concrete piles $2.60 per linear foot. It will cost $3.25 a cubic yard for excavation. Concrete masonry in place will cost $16.50 per cubic yard. Under these conditions, show by cost data which type of foundation you would use.

P. 1303. Design a steel I-beam grillage for a steel column carrying 500,000 pounds with a base of 24″ × 24″. Use two tiers of I-beams. Soil is dry sand and clay mixed.

P. 1304. Determine whether it will be cheaper to put in a random rock foundation or to use a coffer-dam and construct a concrete footing for a bridge pier. Depth of water

is 15'. Bottom is solid rock and level. Random rock takes a slope of 2 horizontal to 1 vertical, and costs $4.00 per cubic yard in place. A double wall coffer-dam, 10' between walls, costs $20.00 per linear foot measured along the center line of coffer-dam. A working space 3' wide is to be left between the walls of coffer-dam and the wall of pier footing. Concrete in the footing would cost $3.00 per cubic yard in place. The footing, whether of rock or concrete, will be brought to the surface only and will be 12' × 12' at the surface.

P. 1305. A box caisson's outside dimensions are 30' × 20'. It is of reinforced concrete, walls and bottom 18" thick. Height of box is 25'. If reinforced concrete weighs 200 pounds per cubic foot and the water at the site of pier is 20' deep, how much, if any, weight will have to be added to sink the caisson in place?

P. 1306. Determine the cost of material for a double wall coffer-dam, inside dimensions 15' × 15', thickness of wall 12'. Concrete guide piles will be placed at intervals of 8' along both walls. United States sheet piling M105 will be used for the walls. The guide piles will be 40' long, the sheet piling 30' long. Cost of concrete pile is 30 cents per linear foot; of sheet piles, 5 cents per pound; of clay puddling used for filling between walls, $3.00 per cubic yard.

P. 1307. A pneumatic caisson is 30' in diameter at the base of the working chamber. The spoil is removed through the materials shaft in buckets, each of which holds 2 cubic feet. 20 buckets per minute can be handled. Assuming that all material is evacuated as soon as dug, how long will it take to lower the caisson one foot?

P. 1308. The pressure in the air-lock of a pneumatic caisson is atmospheric at the time workmen enter it on their way to the working chamber. Assuming that work is being carried on at a depth of 110' and that it is safe to vary the pressure in the air-lock at the rate of 2 pounds per square inch per minute, how much time is lost by each workman in the air-lock in a day?

P. 1309. The formula for safe load on a pile driven by a steam hammer is $R'' = \dfrac{2\,Wh}{d + .1}$. A timber pile is driven by a steam hammer weighing 2000 pounds with a fall of 6". The penetration in the last three blows was .1", .08", .06". What is the safe load on the pile?

P. 1310. A hollow steel form 30' long is sunk into position for use as a pile after it has been filled with concrete. Assume that the external pressure throughout the length of the pile is water pressure and that the pile is filled with concrete weighing 160 pounds per cubic foot. Diameter of pile 24". What must be thickness of steel wall?

P. 1311. A concrete chimney, 125' high rests on firm clay and is 48" in external diameter. Its thickness is 18". Concrete weighs 160 pounds per cubic foot. Design a grillage foundation of steel I-beams to support the chimney.

P. 1312. A masonry pier 40' long and 15' wide supports a maximum dead and live load of 1,200,000 pounds. The base of the pier is 20' below the top. Determine the area of the spread foundation so that the pier may rest safely on a bed of dry sand and clay mixed. Masonry weighs 150 pounds per cubic foot.

P. 1313. A box caisson of concrete is constructed on shore. Its dimensions are 20' × 20' × 10' measured externally. Its height is 10', and thickness is 18" throughout. It is to be launched by sliding it along a ramp of steel I-beams. The ramp consists of three parallel lines of single I-beams and no beam has an unsupported length of

more than 20′. The slope of the ramp is 30° with the horizontal. Determine the maximum bending moment on any one I-beam. Select the proper size I-beam to be used.

P. 1314. A sand bin to hold 1400 cubic feet of sand is constructed of concrete, 12″ thick, reinforced with steel. The bin consists of a cylinder 15′ high and 12′ in diameter with a hemispherical bottom; the whole bin is supported so that its lowest point is 10′ from the ground on four concrete pillars. How long must the pillars be? What is the total pressure at the base of each pillar, assuming that concrete weighs 160 pounds per cubic foot, sand 100 pounds per cubic foot, and that the columns or pillars are 24″ square? Design a plain concrete footing for one of these pillars to transmit the load to firm clay soil.

CHAPTER XIV.

METAL WORK.

1401. The greater portion of all metal used in engineering structures is the product of the rolling mills. Cast pieces are used when necessary, but rolled pieces, being cheaper and more satisfactory, are used whenever possible.

The product of these mills is in forms which can be produced by running the steel between rollers. The usual forms are **plates, eye-bars, channels, angles,** and **I-beams.** These are often combined and joined by **rivets** to form **plate girders, box girders, columns,** etc.

RIVETS.

1402. A rivet is a short cylinder of malleable metal, which is employed to fasten together the metal pieces in an engineering structure. The

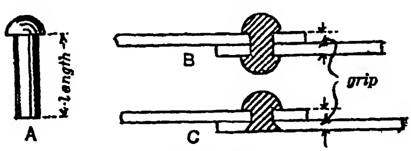


Fig. 1402.

rivet comes from the shop with a head on one end (Fig. 1402A); the other end is enlarged in the field by hammering, after being placed in the rivet hole, so that it will not pull out. In Fig. 1402B the headless end is shown enlarged to a **full head,** in Fig. 1402C to a **countersunk head.** The ordinary sizes of rivets vary from $\frac{3}{8}''$ to $1\frac{1}{4}''$ in diameter by differences of $\frac{1}{8}''$; intermediate sizes are also made. The $\frac{5}{8}''$, $\frac{3}{4}''$ and $\frac{7}{8}''$ rivets are the ones commonly used.

1403. To make a riveted joint, holes are punched or drilled in the pieces to be fastened together; the rivets are then pushed through the holes and secured in place by upsetting the headless ends by successive blows of a hammer or by pressure. Rivets over $\frac{1}{2}''$ in diameter must be heated to a red heat before using. The upsetting produces a second head, either hemispherical or conical. The holes for rivets are usually punched, as drilling is too expensive. In plates $\frac{5}{8}''$ or less, the holes are

punched to full size; in thicker plates the holes are punched $\frac{1}{8}''$ less than full size and then reamed to full size to remove the metal immediately about the hole which may have been injured in punching..

In very important work the holes in thin plates are also reamed, and those in plates 1″ or more are drilled. The diameter of the rivet hole is usually made $\frac{1}{16}''$ greater than that of the rivet; but in computations the net diameter of the rivet hole is assumed as $\frac{1}{8}''$ greater than the diameter of the rivet.

If the holes have been carefully placed, they will be accurately opposite each other when the plates are superposed, and the rivets will easily go through. In careless work, all the holes do not fit accurately, and a conical steel pin, called a **drift-pin**, is often used to bring the holes in line; but this "drifting" is prohibited in all specifications for high-grade work, as it weakens the material.

Riveted Joints.

1404. The kinds of riveted joints are shown in Fig. 1404. A and B are lap-joints, C, D, and E, butt-joints. Fish-plates, or cover-plates, are

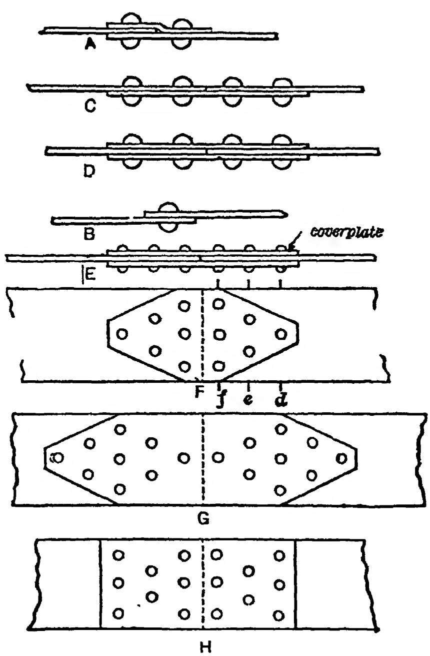


Fig. 1404

additional pieces riveted to both pieces to strengthen the joint as explained later.

A joint is single riveted when each piece is fastened by a single row of

rivets perpendicular to axis of piece (Figs. 1404A and B). It is double riveted when each piece is fastened by two rows of rivets (Figs. 1404C and D). It is chain riveted when it is fastened by more than two rows of rivets (Figs. 1404E, F, G and H). Rivets are staggered when they are placed in quincunx order (Figs. 1404F, G and H). The pitch or spacing of rivets is the distance between centers of consecutive rivets.

1405. **Failure.**—Riveted joints transmit either tension or compression, and are designed separately to carry each stress. A riveted tension joint (Fig. 1405A–A′) may be ruptured in any one of the following ways: viz., failure by:

a. Shearing of rivets, as shown in Fig. 1405B–B′;
b. Tension on plates, as shown in Fig. 1405C–C′ and D–D′;
c. Bearing on plates, as shown in Fig. 1405E–E′ and F–F′.

1406. *Shearing of Rivets.*—Each rivet has a certain allowable strength against shearing; this is equal to its area of cross-section multiplied by the allowable unit shearing strength of the metal. A study of the joint will show how many rivets must be sheared, so as to cause failure by shearing. If there are two cover plates, each rivet must be sheared twice, called double shear, thus making the total area to be sheared twice that in case of single shear.

In Fig. 1405B–B′, it is seen that four rivets must be sheared, in double shear. Therefore, taking A as the area of each rivet and 4 as the number of rivets, we have for this case

$$\text{Allowable Strength in Shear} = a_s \times A \times 2 \times 4$$

In the handbooks, tables show the shearing value of rivets of different sizes, and the strength of the joint can be easily determined.

1407. *Tension on Plates.*—Each rivet hole is taken as $\frac{1}{8}''$ larger in diameter than the rivet itself. Each of the rivet holes decreases the actual area in tension of the plates by the diameter of the hole times the thickness of the plate. A study of the joint will show how much net area of cross-section is taken from the plate, and by subtraction from the total cross-section it is possible to obtain the remaining available area. Multiplying this by the allowable unit tensile strength of the material, we obtain the tensile strength. In cases where there are two cover plates, each rivet takes out an area from each of the cover plates.

In Fig. 1405C–C′, the main plate itself is torn out, and as shown in the figure, its net cross-sectional area is decreased by the two rivet holes,

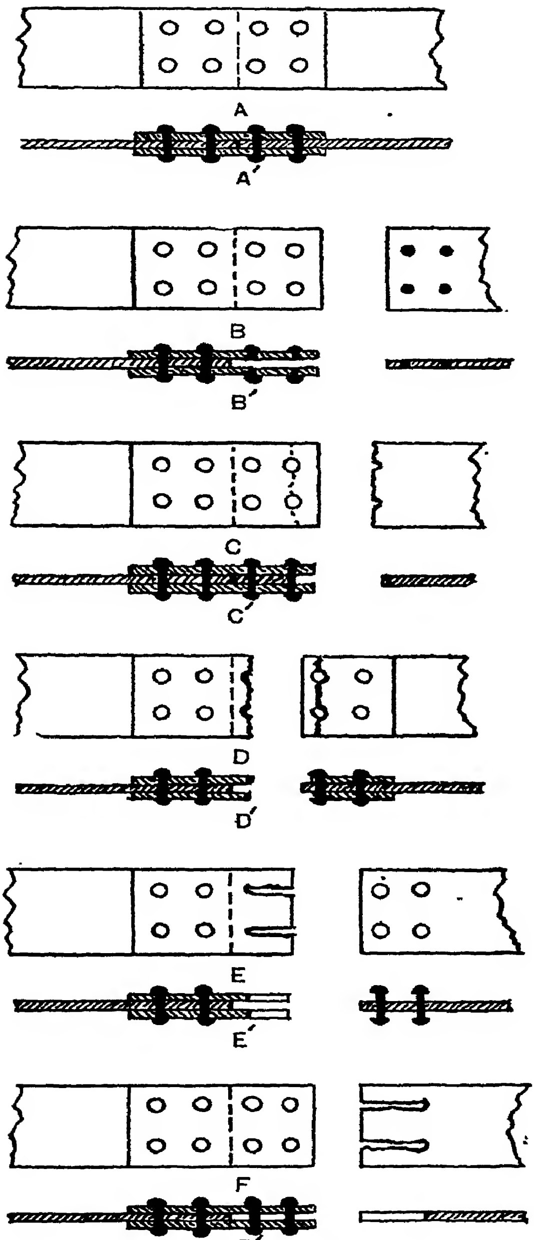


Fig. 1405

each of which causes a loss of area of ($d + \frac{1}{8}''$) times the thickness of the plate. In Fig. 1405D–D′, the two cover plates are torn, and as shown in the figure their net cross-sectional area is decreased by the two rivet holes, each of which causes a loss of area of ($d + \frac{1}{8}''$) times twice the thickness of one cover plate.

1408. *Bearing on Plates.*—If a plate is so thin that the rivets will tear through or crush or crumble it, the joint is said to fail by bearing on the plate. A study of the joint will show that an area equal to the thickness of the plate times the diameter of each rivet times the number of rivets must be torn out if the joint fails in bearing. Multiplying this by the allowable unit bearing strength of the material, we obtain the bearing strength of the joint. If there are two cover plates, each rivet tears or crushes a corresponding area in each of the cover plates.

In Fig. 1405E–E′, the joint fails by bearing of four rivets in the two cover plates. In Fig. 1405F–F′, the joint fails by bearing of four rivets in the main plate.

1409. *Pitch and Size of Rivets.*—In designing riveted joints, it is necessary to follow standard practice as regards the distance between centers and the size of the rivets. In the direction of the stress of compression members the distance between the centers of the rivets is not to exceed either 12″, or sixteen times the thickness of the thinnest plate; in a direction perpendicular to the stress it is not to exceed twenty-four times the thickness of the thinnest plate. The minimum distance between centers of rivets in any direction is three diameters of the rivets. The distance between the centers of the outside rivets and the edge of the plate is not less than $1\frac{1}{2}$ diameters nor more than eight times the thickness of the plate.

In handbooks, tables are given showing the sizes of rivets which should be used with different I-beams, channels, angles, and plates; also showing rivet spacing. The diameter of a rivet hole is never less than the thickness of the thickest plate which is to be fastened to it; in thin plates it is at least one-third to one-half greater.

1410. **Riveted Compression Joints.**—The only difference in design of a riveted compression joint as compared with one for tension lies in the fact that for compression no deduction is made for the rivet holes as is done for tension. In compression, the rivets are supposed to fill the holes completely and the whole cross-section of the plate is considered as taking compression. For bearing and shearing, there is no difference from the design of tension members. In ordinary riveted compression joints, the rivets themselves are designed to transmit the entire stress, and no allowance is made for the effect of the abutting surfaces in trans-

mitting it. This, however, is not the case in the joints of the compression chords of a bridge truss, as the surfaces in these joints are so accurately planed and fitted that the stress need not all be transmitted through the rivets.

1411. **Problem.**—Let it be required to design a riveted tension butt joint of structural steel which will safely transmit a tensile force of 70,000 pounds. Assume a_s = 9000 pounds; a_b = 14,000 pounds; a_t = 16,000 pounds; thickness of tension plate = $\frac{1}{2}''$. (See Fig. 1411A.)

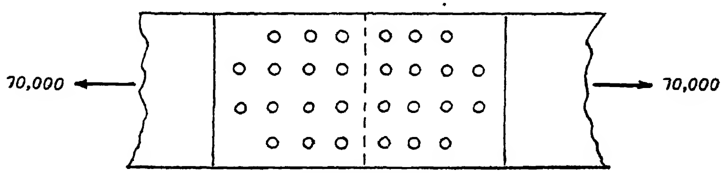


Fig. 1411 A

Solution.—In a butt joint there are usually two cover-plates; therefore, it is so assumed in this joint. The size of the rivets is taken as $\frac{3}{4}''$, which is one-half greater than the thickness of the plates.

Shearing.—The rivets are in double shear, and the handbooks give the double shear value of a $\frac{3}{4}''$ rivet as 7950 pounds; therefore 9 rivets will be needed.

Bearing.—The bearing value of a $\frac{3}{4}''$ rivet in a $\frac{1}{2}''$ plate is $\frac{3}{4} \times \frac{1}{2} \times 14{,}000$ pounds = 5250 pounds. (Also found in tables in handbooks.) Therefore the number of rivets must be 14. In comparing bearing and double shear values in the handbooks, it is at once apparent that the complete calculation for shearing was unnecessary in this case.

Tension.—As one square inch can transmit 16,000 pounds, there must be $\frac{70{,}000}{16{,}000}$ = $4\frac{3}{8}$ square inches of metal to resist tension which, remembering that the plate is $\frac{1}{2}''$ thick, gives a width of $8\frac{3}{4}''$ for the plate. If we assume the 14 rivets to be arranged four in each of the three inner rows, and two in the outside row, we first obtain the necessary width of the plate against tension in the row of two rivets. Each of these makes a gap in the plate of $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}''$. Therefore the width of the plate must be $8\frac{3}{4} + 2 \times \frac{7}{8} = 10\frac{1}{2}''$.

In the next row with 4 rivets there are two more gaps with loss of strength of $2 \times \frac{1}{2} \times \frac{7}{8} \times 16{,}000$, but there are gains by two rivets that must be pulled out by bearing with a value of 2×5250 pounds, double shear being 2×7950 pounds. It is seen that this section is weaker than the one with two rivets. Its available tensile area must be $\frac{70{,}000 - 10{,}500}{16{,}000} = 3\frac{3}{4}$ sq. in. Therefore its width must be $3\frac{3}{4} \times 2 + 4 \times \frac{7}{8} = 7\frac{1}{2} + 3\frac{1}{2} = 11''$. It is necessary to determine if this width is sufficient for proper rivet spacing. Following the practice as given in paragraph 1409, the rivets in each row must be placed at intervals of at least $2\frac{1}{4}''$ and an interval of $1\frac{1}{8}''$ must be left between the outside rivet and the edge of the plate. As this spacing requires a plate only 9″ wide, the main plates of $11\frac{1}{8}''$ are sufficiently wide.

Thickness of Cover Plates.—The weakest section of the cover plates is seen to be through the rows of four rivets on either side of the center of the joint, where the cover plates are reduced by four rivet holes, leaving a net width of cover-plate of $11 - 4 \times \frac{7}{8} = 11 - 3\frac{1}{2} = 7\frac{1}{2}''$.

The area of the cover plates must be $\frac{70,000}{16,000} = 4\frac{3}{8}$ square inches.

Therefore, their combined thickness must be $\frac{4\frac{3}{8}}{7\frac{1}{2}} = 0.58''$.

Each plate should therefore be $0.30''$ thick. The nearest standard plate to this is $\frac{5}{16}''$ thick.

As the combined thickness of the two cover plates is more than the thickness of each tie, it is unnecessary to test the cover plates for bearing, as their bearing power for each rivet must be greater than the bearing value for either tie.

PINS.

1412. **Pins** are forged steel cylinders (Fig. 1412A) used to join together two or more pieces of an engineering structure meeting at a

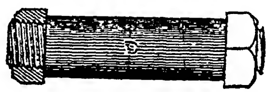


Fig. 1412 A

Pin

common vertex. They have either a head and a nut or two nuts at their extremities. The standard sizes of pins vary in diameter from 1″ to 12″.

Pin joints are not always used to join pieces of an engineering structure. Until recently pins were used in all important steel structural joints, as in bridges; but it is now common practice to rivet together the pieces at these joints.

1413. **Eye-bars.**—An eye-bar (Fig. 1413A) is a wrought-iron or steel bar of circular or rectangular section, having at one or both extremities

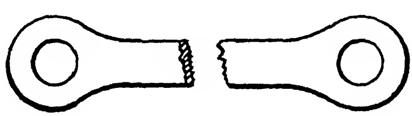


Fig. 1413 A

Eye-Bar

an enlarged, flattened head, with a circular hole or eye to receive the pin. Eye-bars carry tensile stresses. The pin hole of the eye-bar is about $\frac{1}{50}''$ larger than the diameter of the pin; and the sizes of the pin hole vary within limits to fit different pins. The form and thickness of the metal about the pin hole is so designed that the weakest cross-section in an eye-bar shall be in the straight portion of the bar; and in calculating the strength of standard eye-bars, it is necessary to consider only the

straight portion. Standard eye-bars vary in width from 2″ to 16″, and are made to fit pins varying in diameter from 1¾″ to 16″.

1414. **Columns** for pin joints are usually made in the form of two channels laced together. They carry compressive stress. The pin hole is made through the webs of the channels, and sometimes also through a reinforcing plate. The metal about the pin hole is often reinforced by additional plates riveted on to prevent the pin from crushing into the web. The necessary thickness is calculated as in the bearing of rivets; and this bearing value for different sizes of pins may be taken directly from the handbooks.

1415. **Design of Pins.**—Pins must be calculated for shearing, bending and bearing stresses, but one of the latter two will in most cases determine the size. When the size of pin has been determined from the bending stress, the thickness of the bars or web of the column should be investigated to see if there is sufficient bearing area, the bars being thickened or pin plates added if necessary. Therefore it is seen that the design of a pin consists in determining a pin of sufficient size to resist the bending stress, the shearing, the bearing in the eye-bars, and then reinforcing the columns as may be necessary so that they will be safe in bearing for the size of the pin already determined.

1416. The design of a pin against bending consists in determining the various forces acting on the pin joint, calculating the maximum bending moment, and determining the proper pin to resist this. First, it is to be noticed, as shown in Fig. 1417B, that the forces do not all act in one plane; therefore, they are all resolved into components of two coplanar systems at right angles to each other, usually vertical and horizontal. Then if

M_f = resultant moment at the section;
M_{fh} = moment at the section due to the horizontal components;
M_{fv} = moment at the section due to the vertical components;
S_s = resultant shear at the section;
S_h = shear at the section due to the horizontal components;
S_v = shear at the section due to the vertical components.

At every section of the beam we shall have

$$M_f = \sqrt{M_{fh}^2 + M_{fv}^2} \qquad (1416A)$$

and

$$S_s = \sqrt{S_h^2 + S_v^2} \qquad (1416B)$$

The dangerous section of the pin will be where M_f is a maximum. The pin may be tested for shearing strength at the section where S_s is a maximum, but this seldom governs.

The dangerous section of the pin may usually be seen by constructing and inspecting the curves of the bending moments for the vertical and horizontal components separately. The section of greatest shear may usually be seen by constructing and inspecting the lines of shear for the vertical and horizontal components separately.

The bearing strength of the ties and struts which abut on the pin should always be tested, as they generally determine the size of the pin. Usually, pins are selected large enough for bearing in the eye-bars, as eye-bars are not reinforced; but this is not done for bearing in the columns, as extra plates are easily put on the columns if necessary to increase the bearing value of the column.

1417. **Problem.**—Assume five forces as shown in Fig. 1417A, around the pin-joint O. Assume the forces OB, OD, OE, and OF are each transmitted by two members; the pin-joint is thus composed of members as shown in Figs. 1417B and C. Test the joint for safety.

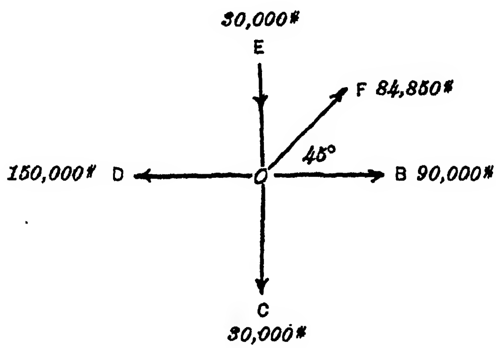


Fig. 1417 A

Solution.—Resolve OF into components in the vertical and horizontal plane. Then solving each set of forces separately, we have the curves of bending moment and shear for the horizontal forces as shown in Fig. 1417D, and for the vertical forces as shown in Fig. 1417E.

The maximum bending moment is at C and is

$$M_{fm} = \sqrt{54{,}375^2 + 13{,}125^2} = 55{,}936 \text{ inch pounds.}$$

The maximum shear is at any point between B and D and is

$$S_{sm} = 45{,}000 \text{ pounds.}$$

The size of the pin to resist the above bending moment and shear may be taken from the handbooks. By calculation, using the formula $a_f = \frac{M_f y}{I}$ for bending moment, and $a_s = \frac{F}{A}$ for shear, and assuming values of 20,000 pounds for a_f and 10,000 pounds for a_s, we obtain

Diameter for bending moment $= 3.06''$
Diameter for shear $= 2.4''$

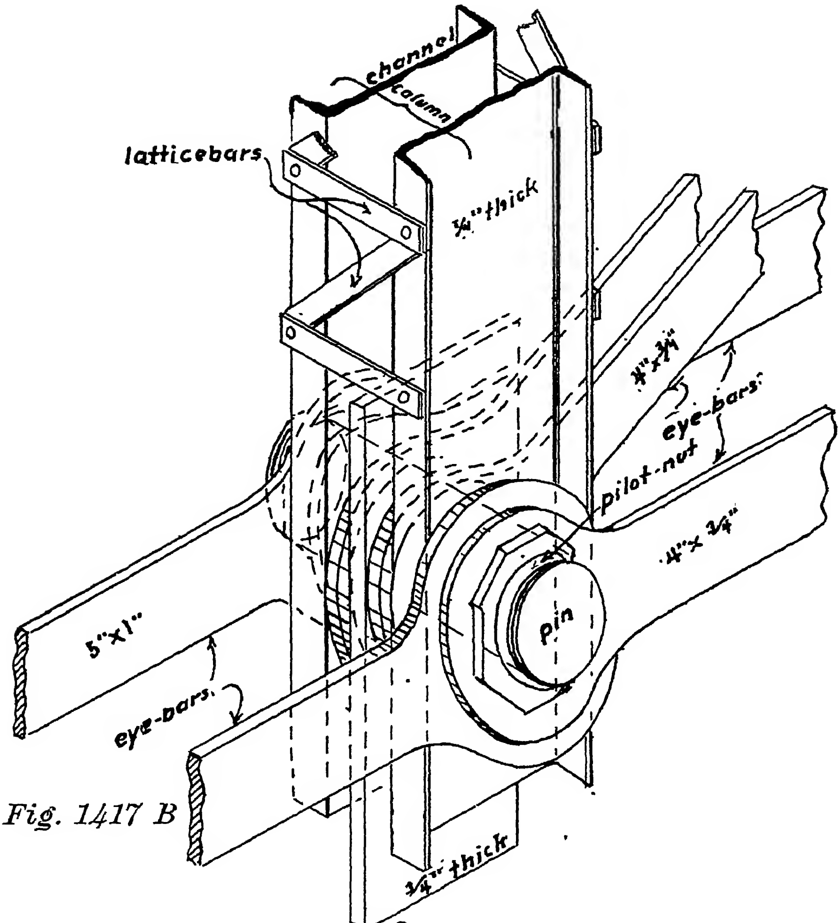


Fig. 1417 B

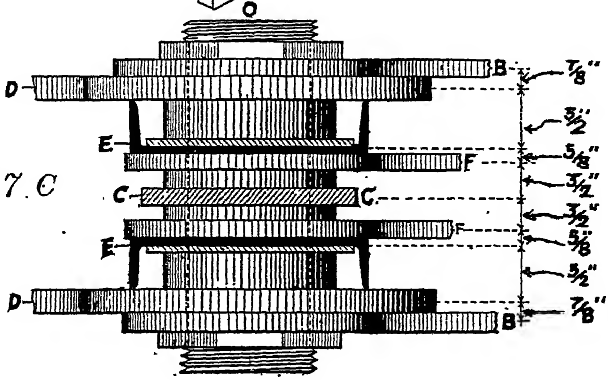


Fig. 1417 C

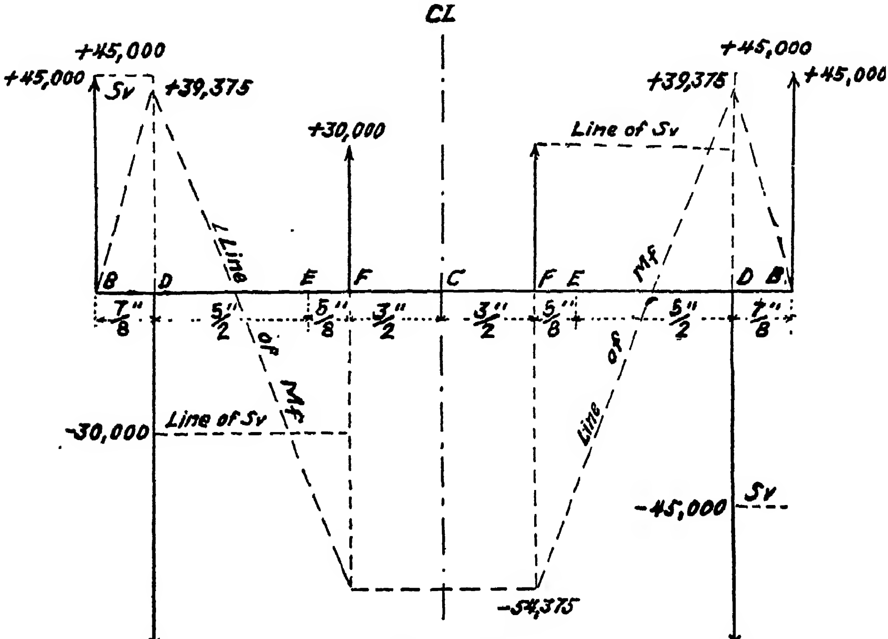


Fig. 1417 D

Curves of Horizontal Bending Moment and Shear

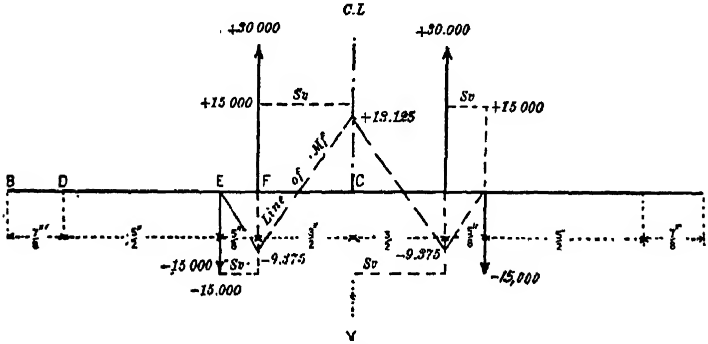


Fig. 1417 E

Curves of Vertical Bending Moment and Shear

However, the bearing of the 3.06″ pin is not sufficient. As seen in Figs. 1417A and C, the eye-bar OD transmitting 75,000 pounds requires the largest bearing area. It is 1″ thick. Therefore assuming a_b = 15,000 pounds, the diameter of the pin must be 5″, which determines the size of the pin. This could have been taken directly from the handbook.

The bearing of the column OE to transmit 15,000 pounds must be tested. Assuming a_b = 15,000 pounds, the necessary thickness must be .2″. But the web of the channel is .25″; therefore no reinforcing plate is needed. Nevertheless, it is usual to employ one, as shown in the figure where a $\frac{1}{2}$″ reinforcing plate is used.

1418. **Arrangement.**—In arranging the order of the pieces on a pin, it must be remembered that the smaller the maximum stress in any one of the pieces, the smaller will be the maximum shear and the maximum bending moment; hence if the calculated pin diameter is too large, it may be decreased by substituting two or more bars for each of the single bars carrying the greatest stress or by placing positive and negative bars in alternation and by placing the bar with the greatest negative stress adjacent to the bar with the greatest positive stress. The single bar in the center is usually the tension-bar having the smallest stress.

If the bars are separated by a considerable distance, washers are used between them; if no washers are used, an allowance of $\frac{1}{16}$″ is made for space between bars. Also, the flanges of the channels may be removed in the vicinity of the pin, if necessary in order to decrease the lever arm of the pieces outside the channels and thereby decrease the maximum bending moment. In the figure, the flanges have been left on, in order that the arrangement of the pieces may be clearly seen.

I-BEAMS AND PLATE GIRDERS.

I-beams.

1419. I-beams (Fig. 1419A) are solid metal structural steel beams with cross-section of form of the letter I. The upper and lower hori-

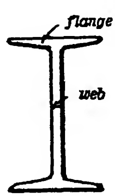


Fig. 1419 A
I-Beam

zontal branches of the I are called the flanges; the vertical connecting part is called the web. They are produced in rolling mills; and the commercial sizes are designated by depth in inches and weight in pounds

per foot. Thus, I-beams are designated as 20″ 90 pound, 20″ 81.4 pound, 18″ 60 pound, etc. The dimensions of web and flanges for a certain depth vary with the weight.

1420. The allowable load which may be placed on an I-beam may be determined by the formula in equation 263A

$$a_f = \frac{M_f y}{I} \tag{263A}$$

Substituting Ar^2 for I and $\frac{D}{2}$ for y, we have

$$a_f = \frac{M_f D}{2\,Ar^2} \tag{1420A}$$

The value of M_f can be determined from the loading of the beam and its length; the values of A, D, and r are given in the handbooks. It is to be noted that r is the radius of gyration about the neutral axis perpendicular to the web, as this is the r which affects the carrying power of the beam.

1421. **Problem.**—What weight may be safely distributed uniformly along an 8″ 18.4 pound I-beam, 14′ long, if a_f = 15,000 pounds?

Solution.—Using the formula, $a_f = \frac{M_f D}{2\,Ar^2}$

we have, a_f = 15,000 pounds

$$M_{fm} = \frac{(wl)}{2}\frac{(l)}{2} - \frac{(wl)}{2}\frac{(l)}{4} = \frac{wl^2}{8} = \frac{Wl}{8} = \frac{W}{8} \times 14 \times 12$$

$$D = 8$$
$$A = 5.34$$
$$r = 3.26$$

Substituting, we have

$$15{,}000 = \frac{\frac{W}{8} \times 14 \times 12 \times 8}{2 \times 5.34 \times 3.26 \times 3.26}$$

Solving, we obtain

$$W = 10{,}134 \text{ pounds,}$$

which includes the weight of the beam, 18.4 × 14 = 257.6 pounds.
Deducting weight of beam W = 10,134 − 257.6 = 9876.4 pounds.

Plate Girders.

1422. Plate girders (Fig. 1422A) are beams made of structural steel plates and angles, so riveted together that the resulting cross-section has the form of the letter I. The web is a metal plate with thickness usually from $\frac{3}{8}$″ to $\frac{1}{2}$″; and with depth varying from one-eighth to one-twelfth the span in heavily loaded girders, and from one-twelfth to one-twentieth the span in light ones.

Each flange consists of two angles riveted to the upper and lower edges of the web as shown. The area of each flange may be increased by one

or more flange plates riveted to the horizontal legs of the angles. The flange plates are in contact with the angles only and not with the edges of the web, which are always kept inside the faces of the horizontal legs of the angles.

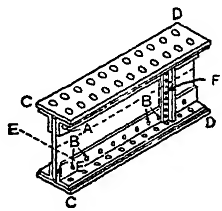


Fig. 1422 A

Plate Girder

At the supports and at points where the girder receives concentrated loads the web is strengthened against shear and buckling by **stiffeners,** which are angles or tees riveted in pairs, one on each side of the web, as shown by F in Fig. 1422A.

1423. The stresses in a plate girder are determined by one of the two following methods:

a. The girder is treated as a solid beam in the same way as an I-beam; that is, the longitudinal stresses are assumed to vary from the neutral axis to the surface, and the vertical shear to be uniformly distributed over the entire surface of the cross-section.

b. The flanges of the girders are considered as resisting the longitudinal stresses without the assistance of the web, and the web is considered as resisting the vertical shear without the assistance of the flanges. This method gives the greatest factor of safety and is the one usually employed.

The first method cannot well be employed in designing girders, as practically every dimension must be assumed and the whole must be calculated to see if the design will be satisfactory; but the method may easily be employed in determining the load which may be placed on a girder.

We use equation 263A:

$$a_f = \frac{M_f y}{I}$$

or

$$a_f = \frac{M_f D}{2\,Ar^2}$$

and make the proper substitutions.

In the second method the only important assumption necessary is the depth of the web. Calculations are then made to determine the areas of flanges, and the thickness of the web. The depth of the web is practically the depth of the girder; and good practice places it at $\frac{1}{8}$ to $\frac{1}{20}$ the span. In some cases, the position is such that the above limits are

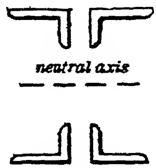


Fig 1423 A

not possible; and the depth must be taken at a certain dimension, in which case the method of design is simplified but the girder is not the most economical possible.

1424. **Design of Flanges to Resist All the Longitudinal Stresses.**—For design of flanges, the girder may be considered as shown in Fig. 1423A. We assume that the stresses are distributed uniformly over the area of the flanges, and that the center of stress is at the center of gravity of each flange. It is evident that the stress on any fiber increases with the distance of the fiber from the neutral axis; but in this case all fibers are well distant from the neutral axis, therefore the above assumption of equal stresses in all fibers is nearly correct.

With the above assumption, we use equation 263A in the form given in equation 1420A,

$$a_f = \frac{M_f D}{2\,Ar^2} \qquad (1424A)$$

in which,

D = distance between centers of gravity of flanges;
A = net area of cross-section of the two flanges;
r = radius of gyration of flange about neutral axis, and may be taken as equal to $\frac{d}{2}$.

Substituting, we have

$$a_f = \frac{M_f D}{2\,A\frac{D^2}{4}} = \frac{M_f}{A\frac{D}{2}} \qquad (1424B)$$

1425. With this formula, we can determine the area of either flange, $\frac{A}{2}$, at any section when we know the values of a_f, D, and M_f. The value of a_f

is obtained from standard practice, M_f is determined by calculations for bending moment of the beam being designed, and D must be assumed. Since in the largest angle used in making plate girders, the distance from its back to the center of gravity is only 2.41″ while the web may be 4′ or even more, it is seen that it will be sufficiently close for trial to assume d as from 0″ to 4″ less than the depth of the web. Equation 1424B can then be solved for $\frac{A}{2}$, the angles selected, and the exact value of d then determined.

1426. It is to be noted that $\frac{A}{2}$ is the true net area of the flange; therefore, in selecting angles, the area in the handbooks must be sufficient to give this net area of $\frac{A}{2}$ after deducting for the rivet holes in any cross-section. These rivet holes are for the rivets of flange to web, rivets of flange to flange plate if there is a flange plate, and rivets of stiffener to flange if there is a stiffener. If these rivet holes are not in the same plane, they are nevertheless so considered unless the oblique plane through them has a net area 30 per cent greater than the net area of the vertical section.

With a compression-flange, the gross area is made equal to that of the tension flange, although this is not really essential if the rivets fully fill their holes.

1427. **Design of Web to Resist All the Vertical Shear.**—The web of a plate girder is made up of one vertical plate or two or more side by side. If the girder is of great length, the web has a length of two or more plates, spliced end to end.

The shear is received by the flanges and transmitted to the web plates by means of stiffeners riveted to the flanges and to the web plates.

1428. The web must be strong enough to prevent failure by:

a. Shearing.—This is obviated by use of the shearing formula

$$a_s = \frac{F}{A}$$

and substituting for A, its value $D \times t$, of which D has been assumed and t is required. The value of D must be net depth after deducting for rivet holes by which the shear is transmitted to the web.

b. Bearing.—This is the same as already discussed under rivets. The number of the rivets in the stiffeners is known, and we can calculate the value of t of the web necessary to take by bearing (and double shear) the shearing force on the girder.

c. Buckling.—The stiffeners prevent buckling. They are small angles or tees placed in pairs on opposite sides of the web and riveted to the web and to each other. They are spaced generally not farther apart than the depth of the web plate, with a maximum limit of 5 feet. To determine the need of stiffeners, the web is considered as a column at points of support and wherever loads are concentrated. Stiffeners having been determined as necessary, their area of cross-section is likewise determined by considering a pair of stiffeners plus the web as one column. The formulas in use are empirical and based on experiment.

1429. **Box Girders.**—Box girders are composed of two or more rolled I-beams or plate girders placed side by side and connected by flange-plates which extend over and under all the beams or girders. They are employed when great strength is required.

1430. **Problem.**—Let it be required to design, with the aid of a handbook, a steel plate girder, 30′ long between supports which is to carry a load of 2000 pounds per linear foot including the weight of the girder. Assume the depth of the web to equal $\frac{1}{15}$ of the span and the allowable unit bearing stress to be 12,000 pounds, the rivets in single shear to be 6000 pounds and in double shear to be 12,000 pounds, unit flange stress to be 13,000 pounds, rivets to be $\frac{3}{4}''$, and rivet holes to be $\frac{7}{8}''$.

Solution.—*a. Design of Flanges.* Using equation 1424B, we have

$$a_f = \frac{M_f}{A\frac{D}{2}} \tag{1424B}$$

in which $a_f = 13{,}000$

$$M_f = 2000 \times 30 \times 30 \times \frac{12}{8} = 2{,}700{,}000 \text{ inch pounds}$$

$$D = 23.5'' \text{ (taken as a little less than the girder depth of 2′).}$$

Substituting and solving, we obtain

$$A = 17.68 \text{ square inches,} \qquad \text{or} \qquad \frac{A}{2} = 8.84 \text{ square inches.}$$

Try	two angles $5'' \times 3'' \times \frac{3}{8}''$....................		5.72 square inches gross area
	one flange-plate $12'' \times \frac{7}{16}''$....................		5.25 square inches gross area
	Total.................		10.97 square inches gross area
Deduct two $\frac{7}{8}''$ rivet holes connecting flange-plate and angles $2'' \times \frac{7}{8}'' \times (\frac{3}{8} + \frac{7}{16}'')$		1.42	
Deduct one $\frac{7}{8}''$ rivet hole connecting web plate and angles $1'' \times \frac{7}{8}'' \times \frac{6}{8}''$		.66	2.08
Net area....................................			8.89 square inches,

which is sufficient.*

b. Design of Web. For shearing, the web must have a

$$\text{Net area} = \frac{30{,}000}{12{,}000} = 2.5 \text{ square inches.}$$

* The flanges are not necessary near the ends of the plate girder, as the M_f is there so small that the area of the angles is sufficient. The discussion is omitted in this text.

For bearing, the web must have a thickness such that

$$\text{Thickness} \times \tfrac{3}{4} \times 12{,}000 \times \text{number of rivets} = \text{or} > 30{,}000.$$

Assuming thickness to be $\frac{3}{8}''$, which is suitable for a girder of this size, we find

$$\text{Number of rivets} = 9.$$

Testing to see if the net area of the web is still sufficient to resist shear after deducting the area of nine rivet holes, we find it to be sufficient. Testing to see if the nine rivets will fail by double shear before transmitting the 30,000 pounds, we find that they will not so fail.

Tests of the stiffeners and of the web itself as a column are omitted.

The girder (exclusive of the stiffeners and rivets) will then be made up as follows:

Four angles $5'' \times 3'' \times \frac{3}{8}'' \times 30'$	1176.0	pounds
One web $\frac{3}{8}'' \times 24'' \times 30'$	918.0	"
Two flange plates $\frac{7}{16}'' \times 12'' \times 30'$	1071.0	"
Total	3165.0	pounds

The weight of the girder is therefore about 5 per cent of the applied load.

For more detailed information see:

Design of Plate Girders, Moore.
Notes on Plate Girder Design, Hudson.
Structural Members and Connections, Hool and Kinne.
Structural Engineer's Handbook, Ketchum.

PROBLEMS.

P. 1401. Test by the first method the plate girder of paragraph 1430 to determine if safe.

P. 1402. Two plates are riveted together in a lap joint. How many $\frac{3}{4}''$ rivets are necessary to insure safety against shearing when the total stress to be transmitted is 50,000 pounds? $a_s = 6{,}000$ pounds.

P. 1403. Two $\frac{1}{2}''$ plates are riveted together in a lap joint with $\frac{3}{4}''$ rivets. How many rivets are needed to insure safety in bearing, if the total stress transmitted is 50,000 pounds? Allowable unit stress in bearing is 12,000 pounds per square inch.

P. 1404. Two plates, $\frac{3}{4}'' \times 12''$, are joined by a single covered butt joint; in each plate there are fifteen $\frac{3}{4}''$ rivets arranged in four parallel rows; four in the first three rows from the center of the joint, and three in the last row. What is the maximum tensile force that can be safely transmitted through the joint? The cover plate is $\frac{1}{2}''$ thick. Allowable unit stresses: shear, 6000; bearing, 12000; tension, 16000.

P. 1405. Two plates, $6'' \times \frac{5}{16}''$, are riveted together by a double covered butt joint, with six $\frac{3}{4}''$ rivets on each side. The cover plates are each $\frac{1}{4}''$ thick. Beginning at the center of the joint, three rows on each side contain 3, 2, and 1 rivets respectively. How will the joint become unsafe and at what tensile stress? Allowable unit stresses: shear, 8000; bearing, 14,000; tension, 16,000.

P. 1406. Two plates, $10'' \times \frac{3}{4}''$, are spliced by a double covered butt joint using 18 $\frac{7}{8}''$ rivets, in rows, beginning at the center of the joint, containing respectively 3, 3, 2, and 1 on each side. The cover plates are each $\frac{7}{16}''$ thick. How much will the joint safely transmit?

P. 1407. The upper chord of a Warren truss bridge carries a compressive stress of 200,000 pounds in one of its panel lengths. The member is composed of two 15″ 35

pounds channels double latticed together with their flanges inside. Design a riveted connection for a splice in this member.

P. 1408. A floor beam is connected to a plate girder by 2 angles $6 \times 6 \times \frac{3}{4}$ each of which is 36″ long. These angles transmit a total stress of 200,000 pounds to the girder. If possible design a connection that will be safe. Illustrate your design with a sketch showing rivet sizes used, the pitch and number used.

P. 1409. Find the safe tensile stress that may be put on two steel eye-bars $6'' \times 1\frac{3}{4}''$, on 5″ pins 22 feet apart.

P. 1410. Fig. P. 1410 represents a pin in a pin-connected joint.

(a) Determine the maximum shear in the pin and the section at which it occurs.

(b) Determine the diameter of the smallest steel pin that will be safe against shear alone.

(c) Determine the maximum bending moment in the pin and the section at which it occurs.

(d) Assuming an allowable fiber stress of 18,000 pounds per square inch, determine the diameter of the smallest steel pin that will withstand the bending moment.

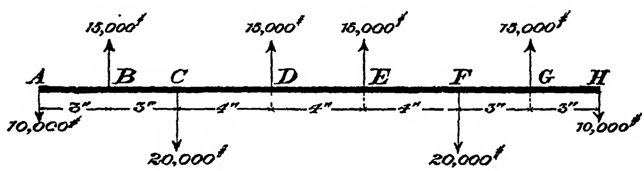


Fig. P 1410

P. 1411. A steel pin is supported by the web of two 7″ 12.25 pound channels spaced 4″ back to back. At the middle of the pin is a $5'' \times 1''$ eye-bar carrying 75,000 pounds tension. $a_f = 15000$; $a_b = 12000$.

(a) Choose a pin which will be safe for bending.

(b) Test this pin for bearing of the eye-bar. If not safe, choose a larger pin.

(c) Reinforce channel webs if necessary.

P. 1412. In Figs. P. 1412A and B, the forces A and B are each transmitted to the steel pin by two steel eye-bars; the eye-bars are $5'' \times 1''$. The force C is transmitted by two steel plates. $a_f = 15000$; $a_b = 12000$.

(a) Determine the maximum shear and maximum bending moment in the pin.

(b) Choose the smallest pin that will carry these stresses safely.

(c) Test this pin for bearing of the eye-bars. If not large enough, choose a larger pin.

(d) Determine the necessary thickness of the plates to carry the force C.

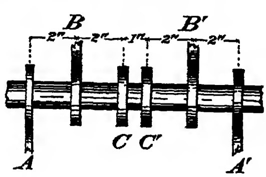


Fig. P 1412A

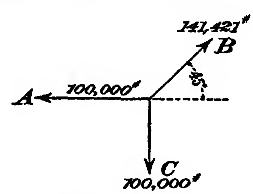


Fig. P 1412B

P. 1413. Fig. P. 1413A is the vertical projection of a pin A acted upon by the forces AB, AC and AD. Fig. P. 1413B is the horizontal projection of the same system showing the force AD equally divided between two eye-bars, $A'D'$ and $A'D''$, and the force

AB equally divided between the two eye-bars $A'B'$ and $A'B''$ and the force AC equally divided between the two channels C and C''. Center lines only of channels, eye-bars and pins are shown. The force CA = 110,000 pounds.

What is—

(a) Maximum horizontal shear?

(b) Maximum vertical shear?

(c) Maximum resultant shear?

(d) Maximum horizontal bending moment?

(e) Maximum vertical bending moment?

(f) Maximum resultant bending moment?

(g) What must be the diameter of the pin for safety in (1) bending moment, a_f = 15,000; (2) in shear, a_s = 12,000?

(h) With the larger pin found in (g), does the web of the channel in the column need reinforcing? Thickness of web = 0.24″.

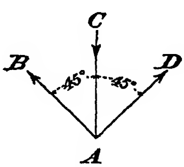


Fig. P 1413A

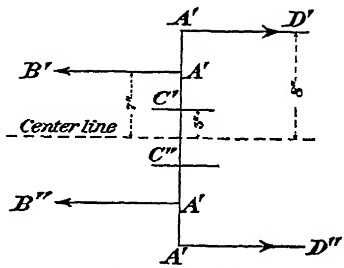


Fig. P 1413B

P. 1414. A steel bridge pin, 17 inches long, has horizontal forces, or components, of −10,000, +20,000, −30,000 and +20,000 pounds applied to it at distances of 4 inches, 6 inches, 7 inches and 8 inches respectively on each side of the middle point, and vertical forces of components of 10,000 pounds and −10,000 pounds, at distances of 4 inches and 5 inches respectively on each side of the middle point. The eye-bars that carry the forces of 14,142, 20,000 and 30,000 pounds are each one inch thick; two of the 10,000 pound vertical forces are carried on channels which have web thicknesses of $\frac{3}{4}''$. Design the pin to safely meet the conditions given.

P. 1415. A beam of 16 feet span, supported at the ends, is to be built of boards $1'' \times 12'' \times 16'$ placed horizontally, and is to be subjected to a concentrated load at its middle point of 10,000 pounds. (Neglect weight of beam.)

(a) If the planks are to be firmly spiked together, how many planks must be used?

(b) If the planks are to be superimposed without being spiked, how many planks must be used?

P. 1416. (a) What is the lightest standard I-beam that will safely carry 15,000 pounds uniformly distributed over a span of ten feet? a_f = 16,000 pounds.

(b) Will the deflection be greater or less than $\frac{1}{360}$ of the span?

(c) What is the section modulus of this I-beam?

P. 1417. A steel I-beam, 20 feet long, is to be fixed at the ends and to support a uniformly distributed load of 500 pounds per linear foot, exclusive of weight of beam. Select the beam. (a) When a_f = 16,000 pounds. (b) When a_f = 12,500 pounds.

P. 1418. By use of Handbook, select a steel I-beam which, when placed on end supports 16 feet apart, will safely bear a concentrated load of 25,000 pounds at center, exclusive of weight of beam. (a) When a_f = 16,000 pounds. (b) When a_f = 12,000 pounds.

P. 1419. An I-beam projects horizontally 10 feet from the wall and acts as a cantilever. The load, exclusive of the weight of the beam, is 100 pounds per linear foot uniformly distributed. Select the beam which will be safe. $a_f = 16{,}000$ pounds.

P. 1420. A plate girder 43 feet long, is made up as follows: web plate $36'' \times \frac{3}{8}''$, flange angles $6'' \times 6'' \times \frac{5}{8}''$, flange plates $14'' \times \frac{3}{4}''$. How much uniformly distributed load will it bear exclusive of its own weight?

P. 1421. A plate girder 50 feet long is made up as follows: web plate $48'' \times \frac{3}{8}''$, flange angles $6'' \times 6'' \times \frac{1}{2}''$. How much uniformly distributed load will it bear exclusive of its own weight?

P. 1422. A riveted beam girder 30 feet long is made up of one 20 inch 65.4 pound I-beam, with two cover plates $10'' \times \frac{5}{8}''$.

(*a*) What safe uniformly distributed load will it bear exclusive of its own weight?

(*b*) Should a plastered ceiling be put on this girder?

P. 1423. A built-up girder, 60 feet long, resting on end supports, is made up of the following pieces; web plate 5 feet deep by $\frac{1}{2}''$ thick; two $6'' \times 6'' \times \frac{3}{4}''$ angles in each flange; one $14'' \times \frac{3}{4}''$ flange plate in each flange. There is a clearance of $\frac{1}{8}''$ between the web plate and the flange plate; the area of each flange is weakened by one web and two flange rivet holes; all rivets are $\frac{7}{8}''$, the rivet holes being $3\frac{1}{8}''$ from center of holes to backs of angles.

(*a*) Find the true value of the effective depth.

(*b*) By the second method of solution (in which the flanges are assumed to bear the longitudinal stress and the web to bear the vertical shear), find the total uniformly distributed load that the girder will safely bear.

P. 1424. A built-up girder is 45 feet long and is made up of the following parts: the web plate is $42'' \times \frac{7}{16}''$; each flange consists of two $6'' \times 4'' \times \frac{3}{4}''$ angles (the short leg next to the web plate); and a flange plate $13' \times \frac{3}{4}''$. There is a clearance of $\frac{1}{4}''$ between the web plate and the flange plate. In addition there is a second flange or cover plate $13'' \times \frac{1}{2}''$, covering the central 24 feet of the flange. The angles are fastened to the web and flange plates by $\frac{7}{8}''$ rivets. The centers of all rivets are $2\frac{1}{2}''$ from the back of the angles. The flange area is weakened by one web and two flange rivet holes (one flange rivet on each side of the web). Unit stress in shear = 10,000 pounds; unit stress in flexure = 15,000 pounds; unit stress in bearing = 20,000 pounds.

(*a*) Find the value of the effective depth at end of girder.

(*b*) Assume the effective depth 41.7 inches. For what total uniformly distributed load is the flange section 10 feet from the end safe?

(*c*) Assume the effective depth 42.0 inches. For what total uniformly distributed load is center flange section safe?

P. 1425. Design a plate girder with specifications as follows:—span 20 feet; uniform load 4000 pounds per linear foot; depth of web $\frac{1}{10}$th of span; thickness of web $\frac{3}{8}$ inch. Allowable stress per square inch in simple shear = 12,000 pounds; allowable single rivet shear per square inch = 6000 pounds; allowable bearing rivet value per square inch = 12,000 pounds; allowable flange stress per square inch = 13,000 pounds.

(*a*) Determine maximum bending moment and maximum shear.

(*b*) Assume trial effective depth to be 23.5 inches and find trial flange area.

(*c*) Assume two angles $3\frac{1}{2}'' \times 3'' \times \frac{1}{2}''$ and one flange plate $10'' \times \frac{1}{2}''$ and determine net flange area and true value of effective depth, using $\frac{7}{8}''$ rivets. Assume clearance of $\frac{1}{8}''$ between web plate and flange plate and centers of rivets $1\frac{3}{4}''$ from back of angles (short leg of angle next to web). Are these results satisfactory?

P. 1426. A built-up girder is made of the following parts: — the web plate is $36'' \times \frac{3}{8}''$; each flange consists of two $6'' \times 6'' \times \frac{1}{2}''$ angles and a flange plate $14'' \times \frac{1}{2}''$ in cross-section. There is a clearance of $\frac{1}{8}''$ between the web plate and the flange plate. The angles are fastened to the web plate and the flange plate by $\frac{7}{8}''$ rivets. The centers of all rivets are $3''$ from the back of the angles. The flange area is weakened by one web and two flange rivet holes (one flange rivet on each side of web). Find the true value of the effective depth.

PART IV.

ENGINEERING STRUCTURES.

The general plan of a structure having been determined, the design of safe component parts, and thus of the entire structure, is simply a matter of calculation in accordance with the principles of the Mechanics of Engineering, taking into consideration the strength of building materials. However, the requirements of economy make it necessary to decrease the cost by lowering the factor of safety to a degree that would not be considered if money were no object. Therefore, although the theory of design of any engineering structure is mathematical, yet the final design and erection of the structure is not so simple. Safety and economy are often directly opposed; and sometimes economy wins too much, with resultant weakness and ultimate disaster.

This opposition does not cease after the design of the structure; inferior material and poor workmanship also reduce the expense and increase chances of disaster. However, economy must be considered; and the engineer in design and construction, while following the principles of Mechanics of Engineering and also requiring good material and workmanship, will use the least factor of safety which good practice has shown to be entirely safe in structures similar to the one being designed.

CHAPTER XV.

BUILDINGS.

1501. Buildings are generally arranged in classes by building laws and codes, the primary purpose of which laws and codes is to protect the lives and property of the inhabitants of the localities in which they are effective. The greatest menace to life and property is fire, the losses from which include not only the destruction or damage of the building and its contents, but also loss of life, and loss or disorganization of business until new buildings can be made ready. Fire losses in the United States amount yearly to more than a quarter of a billion dollars, and in addition a large amount is expended for fire prevention, fire departments, etc. The total yearly cost of fires in this country equals one half the cost of a year's new building construction.

1502. The City of Boston classification of buildings is as follows:

First-class buildings, to consist of fireproof material throughout, with floors constructed of iron, steel, or reinforced concrete beams, filled in between with terra cotta or other masonry arches or with concrete or reinforced concrete slabs; wood may be used only for under and upper floors, window and door frames, sashes, doors, interior finish, hand-rails for stairs, necessary sleepers bedded in the cement, and for isolated furrings bedded in mortar. There shall be no air space between the top of any floor arches and the floor boarding.

Second-class buildings, to comprise all buildings not of the first class, the external and party walls of which are of brick, stone, iron, steel, concrete, reinforced concrete, concrete blocks, or other equally substantial and fireproof material.

Third-class buildings, to comprise wooden frame buildings.

Composite buildings, to consist partly of second-class and partly of third-class construction; they may be built under the same restrictions as third-class buildings and need only comply with their requirements as to fire protection and exterior finish.

1503. The National Board of Fire Underwriters recommends the following classification:

a. **Fireproof construction,** comprising buildings of masonry, steel or reinforced concrete construction, built in accordance with approved methods hereinafter discussed.

b. **Non-fireproof construction,** comprising all buildings or structures having exterior masonry walls, with floors and other interior construction wholly or partly of wood. They are classified as:

(1) Ordinary construction: buildings having masonry walls, with floors and partitions of wooden joist and stud construction. The supporting posts and girders may be of wood, or of metal protected by an approved form of fire-resisting material.

(2) Slow-burning construction: buildings having heavy timber construction, with all hollow spaces and light timber construction within the walls or floors eliminated, and with masonry exterior walls.

c. **Frame construction,** comprising buildings having exterior walls or portions thereof of wood; also buildings with wooden framework veneered with brick, stone, terra cotta, or concrete, or covered with plaster, stucco, or sheet metal.

1504. Comparison of these two classifications shows that they are practically the same, and that fire resistance is the basis of classification either from the standpoint of the city in protecting the life and property of its inhabitants, or from an insurance standpoint alone.

Of recent years, consideration has been given to resistance against earthquakes. No great progress has been made. Experience indicates that buildings of steel frame construction offer greatest resistance to damage by earthquakes.

1505. Buildings are further classified, according to their occupancy or use, as public buildings, residential buildings, and business buildings, each of which is sub-divided into other classes which together specifically include all types of buildings which may legally be erected in any community. The purpose of this classification and sub-classification is to prescribe definitely and expressly the minimum requirements which each different type of building must meet. Thus, minimum limits are placed on depth of foundation, thickness of walls, heights, areas, allowable floor loads, etc.

FIRE-RESISTING MATERIALS.

1506. In the previous discussion of the Materials of Construction, they were considered principally with respect to their strength, durability, and ease of working. When any of these materials are considered for buildings, special consideration must also be given to their fire-resisting qualities.

1507. In so-called "fireproof" construction, none of the materials is actually fireproof. Materials of every kind will be rendered useless if

subjected to heat of great intensity for sufficient time. Furthermore, there are no entirely satisfactory fireproofing chemical preparations. Those on the market in the form of paints have limits of fire resistance, and have the added disadvantage of being composed of chemicals which often give off asphyxiating gases during a fire and are therefore dangerous. Fireproof construction resolves itself therefore into the selection of the most effective materials and the use of them in such a way as to best resist the effect of fire and the effect of water applied after the material has been subjected to intense heat.

1508. The principal materials used in fireproof buildings are steel, cast iron, brick, stone, gypsum blocks, hollow tile, and concrete. The most important advantages and disadvantages of these materials are given below.

Steel.—The principal disadvantage of steel is its inability to withstand high temperatures. At 500° F. it begins to lose its strength, at 1100° F. it will fail entirely. In fires, temperatures as high as 2500° F. have been reached; hence, steel if exposed will collapse and the building will be worthless.

However, steel is strong, is not very expensive, is available in standard shapes, and has therefore become practically indispensable in building construction. For protection against fire it is completely covered with concrete, $1\frac{1}{2}''$ to $3''$ usually, though this thickness is not enough to resist large fires.

Cast iron is not affected by fire as readily as steel, but it is very weak in tensile strength; in fact, it cannot be considered as taking any tension at all. It will not stand shock; and often it has defects of manufacture, such as blow-holes and unequal thicknesses of the metal in hollow columns due to shifting of the core during manufacture. It is often used in thick castings, which will not rust through and need not be painted. It is very largely used in ornamental shapes, columns, panels, etc.

The advantages of cast iron as compared with steel are very few; it is slightly cheaper, and has a higher compressive strength, but its many defects require a higher factor of safety, which practically offsets the advantages.

Brick is strong, inexpensive, and has good fire-resistant qualities; it is a poor conductor of heat, and has many joints to take up expansion and contraction. Its principal disadvantages are that it allows moisture and sound to pass through the wall, and is very heavy. In fires, the brick next to the fire expand and may cause ruinous warping, but as it is a poor conductor, the interior is not overheated. Brick is one of the best fire-resisting materials available; but its excessive weight, compared to other materials of the same class, prevents its being so extensively used for this purpose alone.

Stone.—Stone is widely distributed and is consequently generally available as a building material. However, it is expensive. As a fire-resisting material, it is very poor. Trap rock will stand high temperatures; but it is so hard that the cost of working it is excessive and as a building material it is practically limited to concrete aggregate. Granite will not resist fire; when hot it cracks, especially if a stream of water is turned on it. Nor will limestone resist fire; some building codes will not allow the use of limestone aggregate in concrete for fireproof building construction. Sandstone stands next to trap rock in its fire-resisting qualities, but this quality is not sufficient to warrant its use as a protective material. Marble (a limestone) is the poorest of all stones, except in appearance; it is the most expensive, the weakest, the poorest material for wear, and it crumbles to dust when burned. Stone, therefore, while it can stand great compressive forces and is very satisfactory for architectural effect, is little used in fireproof building construction except as a veneer or facing for the purpose of architectural embellishment.

Gypsum Blocks.—These consist of plaster of Paris mixed with straw or wood fiber and cast into blocks. Their principal disadvantages are their weakness and friability, because of which they are not used where strength is required. They are very light, and can be bored and cut to shape. Their coefficient of expansion is practically zero and they are good non-conductors of heat and sound. These qualities make them desirable in the construction of partitions and they are much used in this capacity. Because of their lightness, they are cast in large blocks, thus making construction rapid with little labor. They crack off on the outside but may be plastered up again.

Hollow Tile.—This is a most common material for fireproofing. It is made of clay pressed and burned at a high temperature. It is available in three grades, dense, semi-porous, and porous. The **dense tile** consists of hard-burned clay. It is strong and resists moisture; but is heavy, brittle, and likely to have cracks developed in its manufacture.

Porous tile is an excellent fire-resisting material. It stands fire better than the dense tile, is lighter, is a good non-conductor of heat, and stands a large amount of expansion. Where strength is needed, however, porous tile cannot be used. **Semi-porous tile** is the grade most commonly used. Its qualities are halfway between those of porous and dense tile.

Hollow tile is also called structural terra cotta; it should not be confused with architectural terra cotta which is principally a decorative medium, is very weak, and is not designed to carry any load.

Concrete.—Concrete is very widely used as a building material. Like brick, it is a poor conductor of heat; but also, like brick, the portion next to the fire expands and may cause failure by warping. As a fire-resisting

material, cinder concrete is most used. It is weak but light, and therefore meets such uses as require lightness but not much strength. It is used in floors and as a protection to steel columns. At least $1\frac{1}{2}''$ of concrete covering is required for resistance to even small fires.

FOUNDATIONS OF BUILDINGS.

Loads.

1509. The loads to be considered in the design of foundations of a building are:

a. The dead loads, or loads due to the actual weight of the completed structure, ready for occupancy.

b. The live loads, or loads due to the occupancy of the building and also to the weight of snow on the roof.

c. The wind loads, or the normal components of stresses on the structure produced by wind pressure.

1510. The dead load is simply the weight of the building. It can be accurately calculated by determining the weight of each element of the building and adding all these weights together. The portion of the dead load, and hence the weight, supported by each element of the foundation can be definitely determined, and each element can then be properly and accurately designed.

1511. The live load cannot be accurately determined; it is therefore assumed, and the elements of the building are then designed. Assumptions for the live load are made for each portion of the building, such as floors, lofts, roofs, as is explained later when each is discussed. The assumptions for each portion of the building must be sufficiently high, so that each will be designed to carry the maximum live load that may ever be placed on that portion. The sum of these partial live loads gives the maximum load that the building will ever carry; but in practice the total load on the foundation is never taken as the sum of the maximum loads on each portion of the building, as it is evident that all of these maximum loads will not occur at the same time.

1512. The wind load is generally calculated on the assumption that the wind will exert a uniform pressure, some 30 pounds per square foot, on the entire external area of one side of the building. This assumption will be modified in cases where deduction can be made for the protecting influence of adjoining buildings.

In designing a tall building to resist wind load, the building may be assumed to be a cantilever beam, set up vertically, with the point of support at the ground and with the wind load horizontal and normal to

the axis of the cantilever, which in this case is vertical. If the building is composed of a steel framework which carries all stresses, the cantilever becomes a truss, with as many points of support or attachments to the abutment as there are footings of the building, and with the wind load changed from a uniform load to a number of concentrated loads at each junction of floor girders with columns. Taken this way, the wind load produces shear in the building as a whole, the shear being taken up by the floor girders and carried by them from the windward side across to the leeward side, while the stresses of bending moment are sustained by the columns and carried by them to the foundations of the building.

To prevent deformation of the building, wind bracing is used. For this purpose, diagonals, knee braces, and the like have been used; but present practice prefers gusset plates (flat triangular steel plates) connecting floor girders and columns, as they do not project into the rooms or interfere with the windows. The stresses transmitted to the foundation under the above system of calculating wind loads are all vertical through the center of gravity of each footing. The footings on the windward side have reduced loads, while those on the leeward side have increased loads, due to the wind, these being in addition to and along the same lines as the live and dead loads of the building.

1513. **Loads for office buildings,** as prescribed by building codes of various American cities, usually are fixed at 150 pounds per square foot for the first floor and 75 pounds per square foot for the other floors. These are the maximum loads for which each floor system is designed. The loads transmitted through the columns to the foundations are not taken at the above maximum, as they will not be the maximum for all floors at the same time, but are reduced from full load on the roof and top floor by 5 per cent for each succeeding lower floor until a reduction of 50 per cent is reached, which reduction holds for all lower floors. Wind loads are usually considered 30 pounds per square foot for any one side of the building. Snow loads are added to the live load in those places where snow occurs.

Foundation Beds.

1514. Investigation of the foundation bed must be made. The investigation follows the usual methods as described in the chapter on Foundations, consisting of sounding with a steel rod, boring with earth augers or drills (with or without a water jet), boring with core drills, or digging open pits, according to the nature of the ground and character of the building.

1515. The allowable load on a foundation bed may be determined by a test load, but building codes are usually relied on, as they have been

compiled as the result of years of experience. As an example, the San Francisco allowable loads are as given below:

Soft clay	1 ton per sq. ft.
Dry sand mixed with clay	2 " " "
Firm dry loam, clay, or sand	3 " " "
Hard clay, firm coarse sand	4 " " "
Hard shale, unexposed	10 " " "
Rock	20 " " "

Footings.

1516. The general theory of foundations in Chapter XIII applies to the foundations for buildings. More detailed consideration brings out certain principles that apply especially to buildings:

a. The total dead load, live load, and vertical effect of the wind load, divided by the area of the footing, must not exceed the allowable load for the material of the foundation bed.

b. When the whole or parts of the foundation bed are compressible, the load and unit pressures must be so distributed that settlement will be equal.

c. The center of pressure on each supporting area should coincide with its center of gravity, so that the footing will not settle out of level. In special cases where this cannot be done, the center of pressure must fall within the middle third of the base.

1517. Only in rare cases is the entire area of ground underneath a building occupied by the footings. The ground actually under the footings should have the same pressure per square foot throughout the building; but as the construction of the building causes certain parts to be more heavily loaded than others, for example under partitions and chimneys, the footings must be designed to give equal unit pressure on the soil, no matter what load they may receive from the structure itself. To accomplish this, various forms of footings are in use.

1518. **Square or circular footings** carry concentrated single loads, such as an interior column. The circular shape (Fig. 1518B) is theoretically most economical, but the square shape (Fig. 1518A) is more common on account of greater ease in arranging grillage beams.

Elongated rectangular footings are used when a square footing would overlap a property line, or for some similar reason. The same total area is used as for a square footing, with a strengthening in the longer dimension.

Offset footings where the wall itself must abut on the property line, have the additional complication of eccentric loading, where the center

of pressure is not coincident with the center of gravity, and consequently the condition of uniform unit pressure over the foundation bed cannot obtain. These footings are more liable to failure than the ordinary kind.

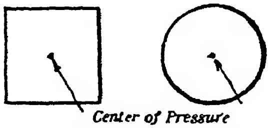


Fig. 1518 A Fig. 1518 B

1519. **Footings for walls** extend uniformly the same distance beyond the wall on both sides for the whole length of the wall, except as noted in preceding paragraph. Where concentrated loads occur along the center line of the wall, the footing is spread out on both sides so that the unit pressure on the foundation will be uniform. If a concentrated load is not on the center line of the wall, the enlarged footing should extend equally on each side of the vertical plane through the center of gravity of the combined load of wall and burden.

1520. **Special footings** for difficult foundation beds may take the form of reinforced concrete slabs over all or part of the area under the building, the load on the piers, etc., being distributed by means of inverted arches

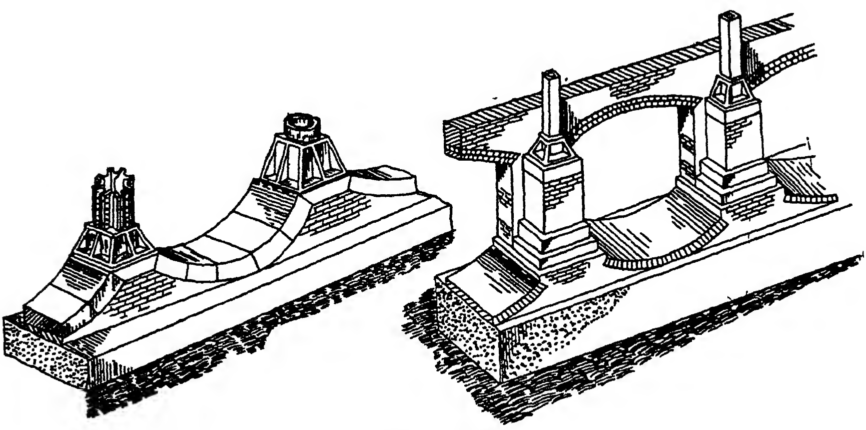


Fig. 1520 A

(Fig. 1520A). Foundations which move under loads may be enclosed by sheet piles. In soft soil, round piles may be driven to support the footings, the tops of the piles being imbedded in the concrete of the footings.

1521. **Problem.**—A brick wall with a footing 4′ wide, has an added concentrated load at its center line, the load requiring a footing area of 36 sq. ft. to support it. Find the dimension of the footing of the wall at the point where the concentrated load is placed. (Fig. 1521A).

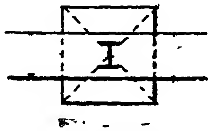


Fig. 1521 A

Solution.—Let D be the side of the square footing which will support both the wall and the concentrated load.

Then,

$$D^2 = 36 + 4D$$
$$D^2 - 4D = 36$$
$$D - 2 = \sqrt{36 + 4} = \sqrt{40}$$
$$D = 2 + 6.3 = 8.3'$$

Hence 8.3′ is the dimension of the side of the square footing at the point where the concentrated load is placed on the wall.

1522. **Problem.**—A floor of 15′ span is to be loaded with canned goods in cases. The floor complete weighs 26 pounds per square foor of floor space. The walls which support the floor are calculated to sustain for this floor a load of 1680 pounds per linear foot of wall, including the weight of the floor. Case goods average 58 pounds per cubic foot How high may the cases be piled?

Solution.—Each linear foot of floor can support

$15 \times 58 \times h$ pounds of canned goods.

Hence

$$\frac{15 \times 58 \times h}{2} + \frac{15 \times 26}{2} = 1680$$
$$h = 3.4'.$$

FRAMES OF BUILDINGS.

1523. The framework of a building is the means whereby the dead loads, live loads, and wind loads are transmitted to the footings. Wooden and steel frames consist of numbers of pillars or columns on which are hung the sides, partitions, and floors. Masonry walls, on the contrary, support the loads directly along their complete length, being specially strengthened where heavy loads are received.

Masonry Walls.

1524. **Walls.**—The least thickness of a brick wall is 8″, which is the thickness of the walls of a narrow and low dwelling. The usual rule for higher walls is to make the first 25′, from the roof downward, 12″ thick in dwellings and 16″ thick in other buildings; the next 25′ is made 4″ thicker, and this rate of increase is continued to the ground-line. From

the ground-line to the footing, the increase in thickness is 4″ for every 10′. The change from one thickness to another is usually made at the floors.

The building laws of Boston require the walls of an eight-story building to be 28″ in the first story, 24″ in the second, 20″ in the third to the seventh story, 16″ in the eighth.

1525. A long wall is weaker than a short one, and must be made correspondingly thicker, say 4″ for every 25′ over 100′. Walls with many openings must be made thicker. Likewise, walls of warehouses and stores must be thicker than those of dwellings and offices. Stone walls are generally 4″ thicker than brick walls.

1526. **Brickwork.**—The strength and durability of any piece of brickwork will depend upon the quality of the bricks, the quality of the mortar, the way in which the bricks are laid and bonded, and upon whether or not the bricks are laid wet or dry.

The strength and stability of a wall, arch, or pier also depend upon its dimensions and the load it supports.

The kinds and quality of mortars used for laying brickwork are described in Chapter VIII. The majority of modern brick buildings in this country are built with Portland cement mortar. For brickwork below ground, cement mortar is absolutely necessary.

1527. Common brick should be laid in a bed of mortar at least $\frac{3}{16}''$ thick, and every joint and space in the wall not occupied by other materials should be filled with mortar. The usual way of specifying the thickness of a joint is by the height of eight courses of brick measured in the wall. This height should not exceed by more than 2″ the height of eight courses of the same brick laid dry.

Pressed brick are quite true and smooth, and can be laid with a $\frac{1}{8}''$ joint, and it is often so specified. A $\frac{3}{16}''$ joint is probably better, however, as it permits filling the joint better.

1528. The best method of building a brick wall is to first lay the inner and outer courses. In each of these courses, the mortar is spread with a trowel along the edges of the last course of brick to form a bed for the brick to be laid. The brick to be laid are then pressed into place with a sliding motion, which forces the mortar to completely fill the joint.

The inner and outer courses of brick having been thus laid, the space between the courses is filled with a thick bed of soft mortar and other brick of this course pressed into this mortar with a downward diagonal motion, so as to press the mortar up into the joints.

1529. Mortar, unless very wet, will not adhere to a dry porous brick, because the brick robs the mortar of its moisture, thus preventing its

proper setting. On this account brick should never be laid dry, except in freezing weather; and in hot dry weather it is impossible to get the brick too wet. When using very porous brick the wetting of the brick is more important in obtaining a strong wall than is any other condition.

Brickwork should never be laid when the temperature is below 32°, and if it is below 40° and likely to fall below 32° at night, salt should be mixed with the mortar, the brick heated before laying, and the top of the wall covered with boards and straw at night.

1530. Figs. 1530A, B, C, D show many details of construction of a brick building. Brick walls should be tied to every floor at least once in every 6 linear feet, either by iron anchors built solidly into the wall and spiked to the floor joist, or by means of a box anchor or joist hanger.

The forms of iron anchors most commonly used for this purpose are those shown in Fig. 1530E. If the wall is a side or rear wall, where the appearance is not of much consequence, it is better to have the anchor pass through the wall, with a plate on the outside, as such an anchor gets a much better hold on the wall than is possible when it is built into the middle of the wall. All of these anchors should be spiked to the side of the joist or girder, near the bottom, as shown in Fig. 1530F.

1531. **Walls to be Carried Up Evenly.**—The walls of a building should be built up evenly, no part being built up more than 3′ above the rest, except where there is an opening. Building one part of a wall ahead of the rest produces unequal settlement and, the joints in the higher part settling before the rest is added to it, the work laid last is apt to settle away from the other and weaken the wall, besides marring its appearance. Whenever it is necessary to carry one part of a wall higher than the rest, the end of the high part should be stepped back, and not run up vertically.

The location of all door and window openings in brick walls should be carefully considered, not only with regard to convenience, but also as to their effect in weakening the wall. The total width of the openings in any bearing wall should not much exceed one-fourth of the length of the wall, and as far as possible the openings in the different stories should be over each other. No openings should be placed under a pier or similar loaded portion. If windows must be used in these positions, steel beams should be used for lintels over the windows, as a stone lintel or brick arch would be quite apt to crack from the combined effects of the load and the settlement of the joints in the masonry on either side of the window.

1532. **Concrete walls** follow the same general rules as brick walls as to thickness. For windows, it is usually best to reinforce the concrete rather than to use arches, though the latter are often used for architectural effect. Floor stringers, etc., are set into the concrete walls in the same

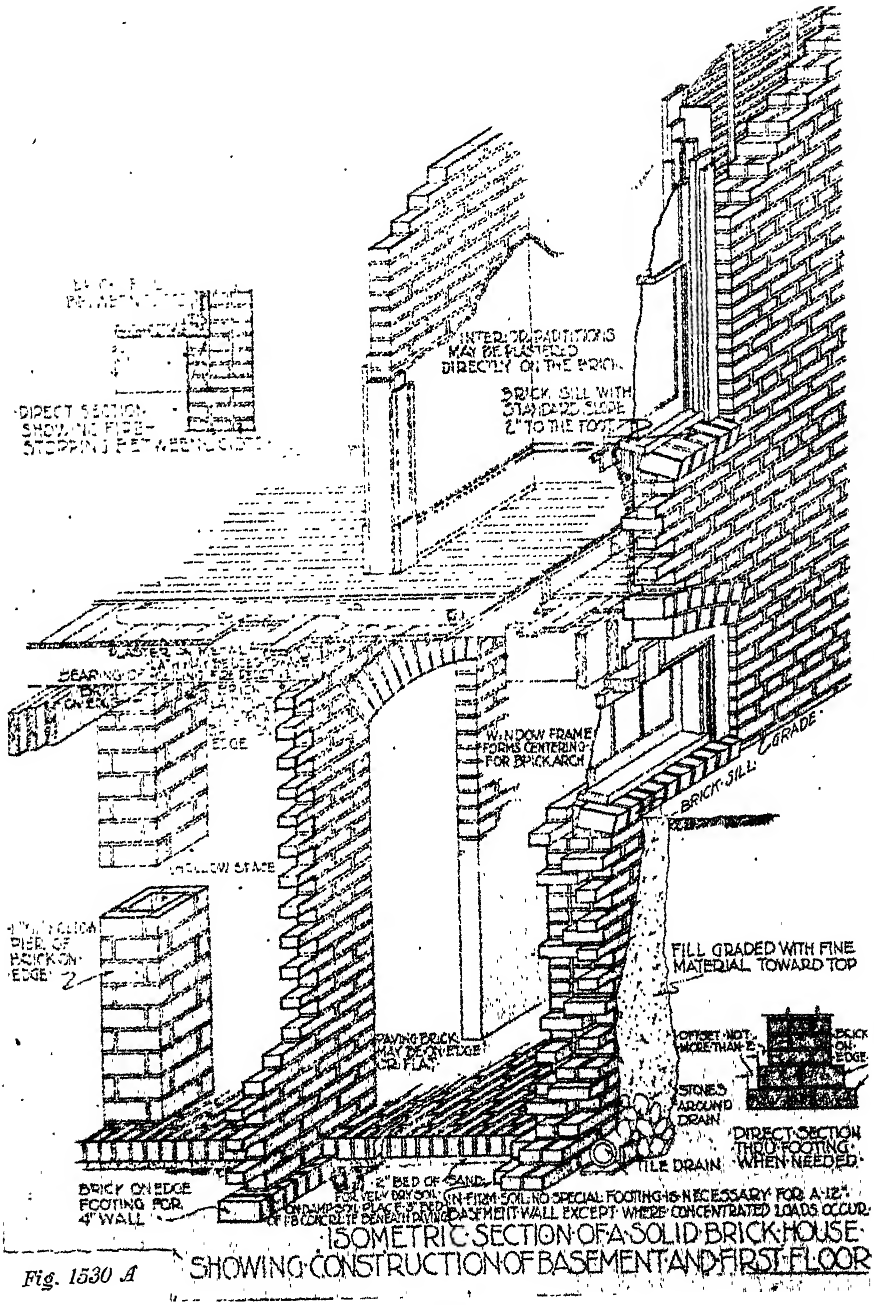

INTERIOR PARTITIONS MAY BE PLASTERED DIRECTLY ON THE BRICK
BRICK SILL WITH STANDARD SLOPE 2" TO THE FOOT
DIRECT SECTION SHOWING FIRE-STOPPING BETWEEN JOISTS
WINDOW FRAME FORMS CENTERING FOR BRICK ARCH
BRICK SILL
GRADE
HOLLOW SPACE
EDGE
FILL GRADED WITH FINE MATERIAL TOWARD TOP
OFFSET NOT MORE THAN 2"
BRICK ON EDGE
STONES AROUND DRAIN
DIRECT SECTION THRU FOOTING WHEN NEEDED
TILE DRAIN
PAVING BRICK MAY BE ON EDGE OR FLAT
2" BED OF SAND
BRICK ON EDGE FOOTING FOR 4" WALL
IN FIRM SOIL NO SPECIAL FOOTING IS NECESSARY FOR A 12" BASEMENT WALL EXCEPT WHERE CONCENTRATED LOADS OCCUR
ISOMETRIC SECTION OF A SOLID BRICK HOUSE SHOWING CONSTRUCTION OF BASEMENT AND FIRST FLOOR

Fig. 1530 A

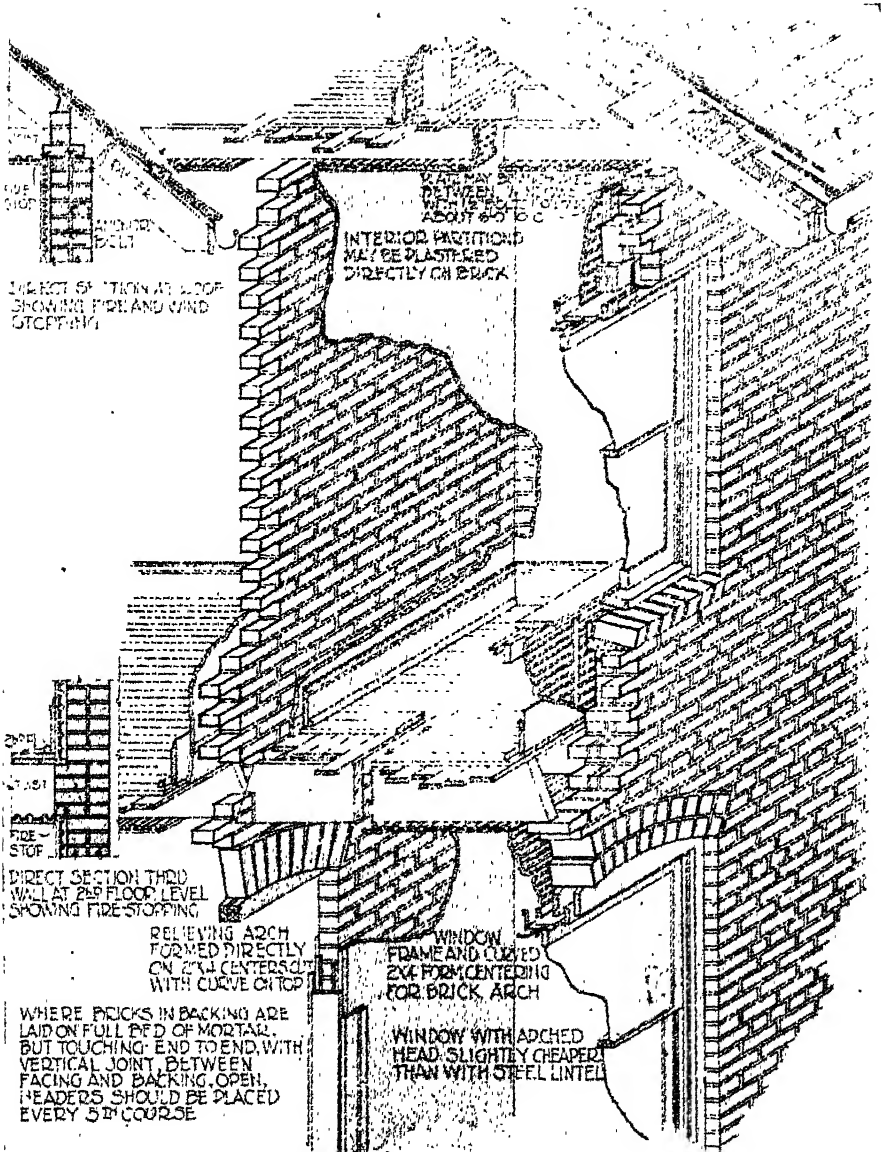


Fig. 1530 B ISOMETRIC·SECTION·SHOWING·CONSTRUCTION·OF·A·SOLID·BRICK·HOUSE·UPPER·PART·OF·FIRST·FLOOR·SECOND·FLOOR·AND·ROOF·

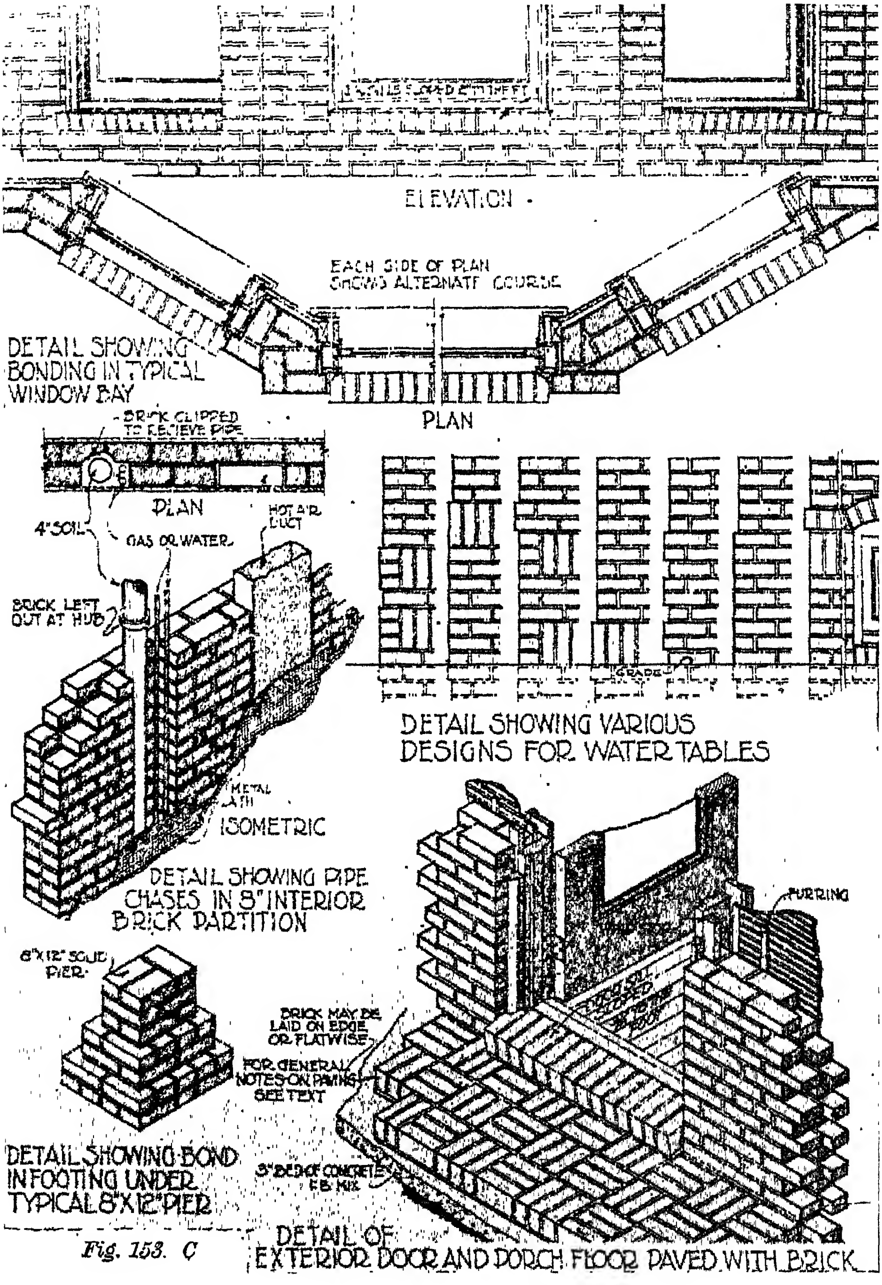

ELEVATION
EACH SIDE OF PLAN SHOWS ALTERNATE COURSE
DETAIL SHOWING BONDING IN TYPICAL WINDOW BAY
PLAN
BRICK CLIPPED TO RECIEVE PIPE
PLAN
4" SOIL
GAS OR WATER
HOT AIR DUCT
BRICK LEFT OUT AT HUB
METAL LATH
ISOMETRIC
DETAIL SHOWING PIPE CHASES IN 8" INTERIOR BRICK PARTITION
DETAIL SHOWING VARIOUS DESIGNS FOR WATER TABLES
GRADE
FURRING
8"X12" SOLID PIER
BRICK MAY BE LAID ON EDGE OR FLATWISE
FOR GENERAL NOTES ON PAVING SEE TEXT
DETAIL SHOWING BOND IN FOOTING UNDER TYPICAL 8"X12" PIER
DETAIL OF EXTERIOR DOOR AND PORCH FLOOR PAVED WITH BRICK

Fig. 153. C

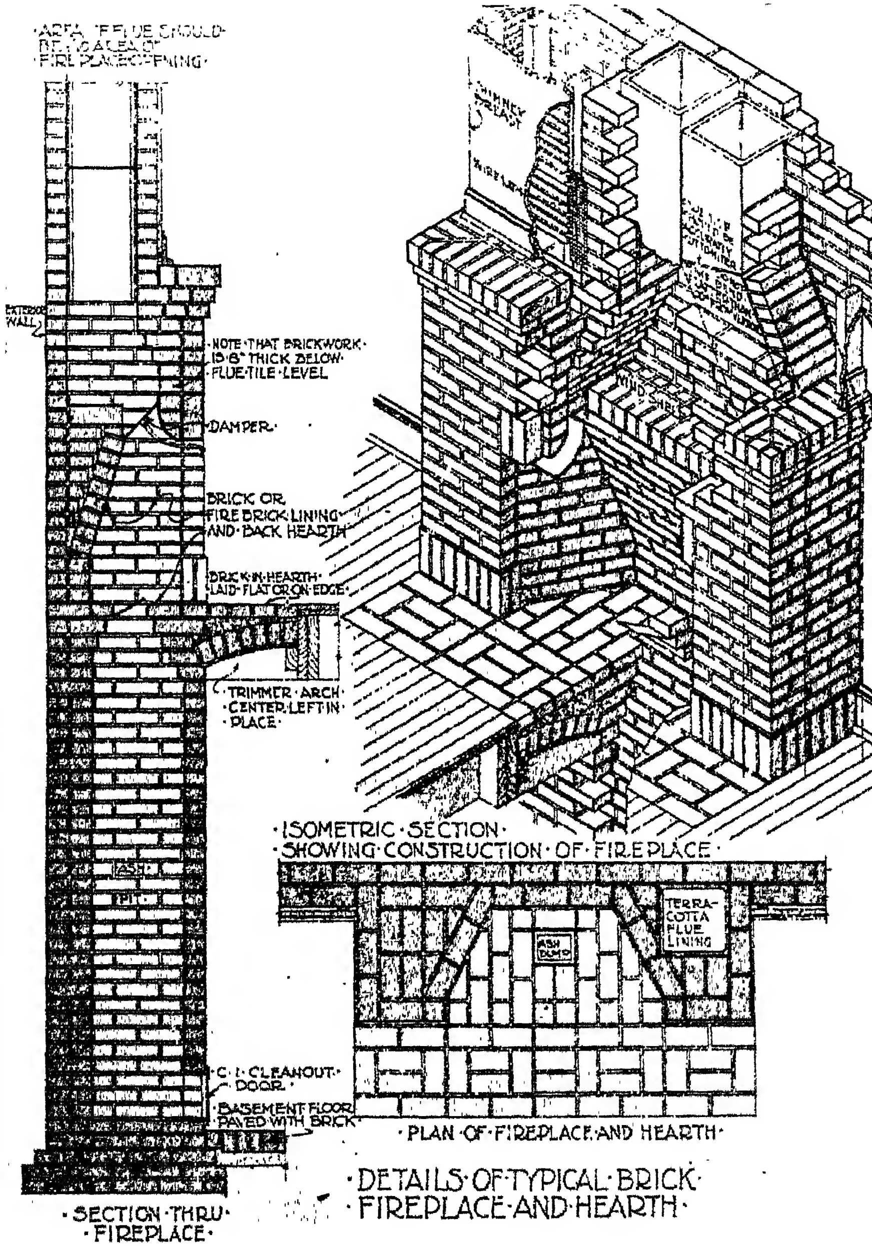

EXTERIOR WALL
NOTE THAT BRICKWORK IS 8" THICK BELOW FLUE TILE LEVEL
DAMPER
BRICK OR FIRE BRICK LINING AND BACK HEARTH
BRICK IN HEARTH LAID FLAT OR ON EDGE
TRIMMER ARCH CENTER LEFT IN PLACE
ASH PIT
C. I. CLEANOUT DOOR
BASEMENT FLOOR PAVED WITH BRICK
SECTION THRU FIREPLACE
WIND SHELF
ISOMETRIC SECTION SHOWING CONSTRUCTION OF FIREPLACE
TERRA-COTTA FLUE LINING
ASH DUMP
PLAN OF FIREPLACE AND HEARTH
DETAILS OF TYPICAL BRICK FIREPLACE AND HEARTH

Fig. 1530 D

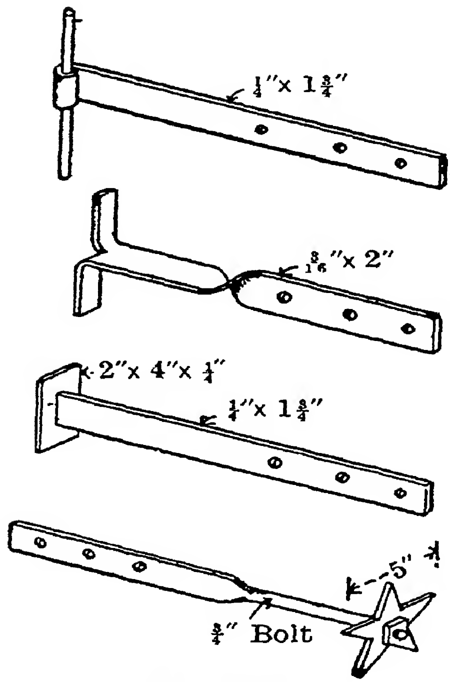


Fig. 1530 E

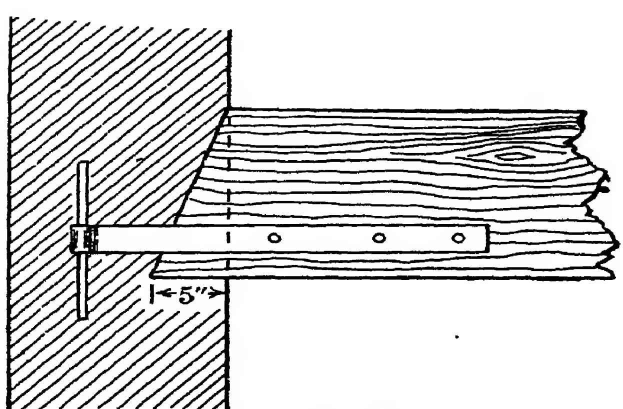


Fig. 1530 F

way that they are set into brick walls. Concrete work does not require skilled labor as stone and brick masonry do, but skilled supervision is necessary to insure correct construction of forms, proper placing to secure bond and finish, and prevention of freezing in cold weather.

1533. **Stone walls** also follow the same general rules as brick walls. Stone is more expensive to obtain, and, on account of variations in size, more expensive to place than brick, consequently brick is generally preferred except for architectural effect.

Wooden Frames.

1534. In framing the walls of wooden buildings, two distinct methods may be followed; these are called braced framing, and balloon framing.

The braced or full frame, the only kind in vogue before about 1850, consists of heavy timbers, mortised and pinned together, and braced

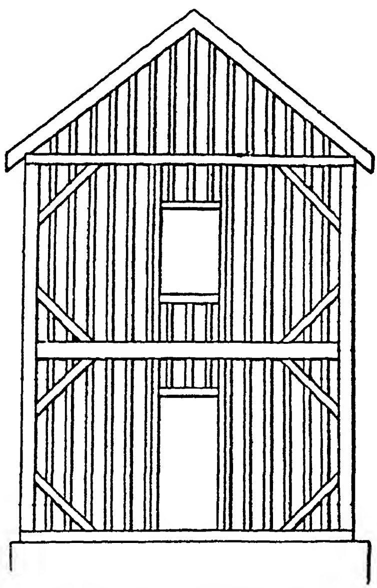


Fig. 1535 A Braced Frame

at every junction of wall and floor by mortised and pinned diagonal braces. The posts, sills, girts, and braces form a substantial, slow-burning, heavy framework, and the studding is in each case but one floor high, thus allowing vermin stops to be placed at each floor level (see Fig. 1535A).

The balloon frame, the common type for the ordinary American frame building, is composed of much lighter pieces than the braced frame, is

nailed in most places where the other is mortised, does not have heavy girts at each floor as part of the framing, but allows the studding to run the entire height of the building without break. The second floor joists rest on a board set into the studding (girt strip, ribbon, or ledger board) instead of on a girt (Fig. 1535B).

1535. The construction of a small two-story balloon frame house is described below, and shown in Figs. 1535B to D.

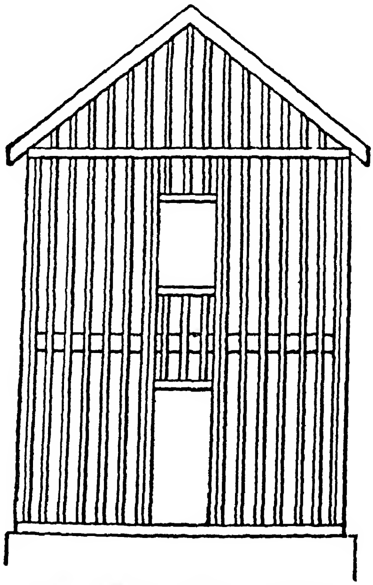


Fig. 1535 B **Balloon Frame.**

The **sill** is the first part of a frame to be set in place. It rests directly on the underpinning and extends all the way around the building; it is usually 6″ × 8″. It is placed so as to allow 1″ on the outside face for the water table. A bed of mortar in which the sill is to rest should be laid on top of the underpinning, and the under side of the sill should be painted with linseed oil to prevent it from absorbing moisture from the masonry. In many cases long bolts are set at intervals of 8′ or 10′ in the masonry extending up through the sill, to fasten the sill, and consequently the whole frame, securely to the underpinning.

The **joists** of the first floor are supported by the sills, and may be fastened to them in different ways: the best method is the patent joist hanger shown in Fig. 1535E where S is the sill, J the joist, H the hanger. As this is too expensive a method for ordinary work, the sill is usually mortised to receive a tenon cut in the joist (Fig. 1535F). These mor-

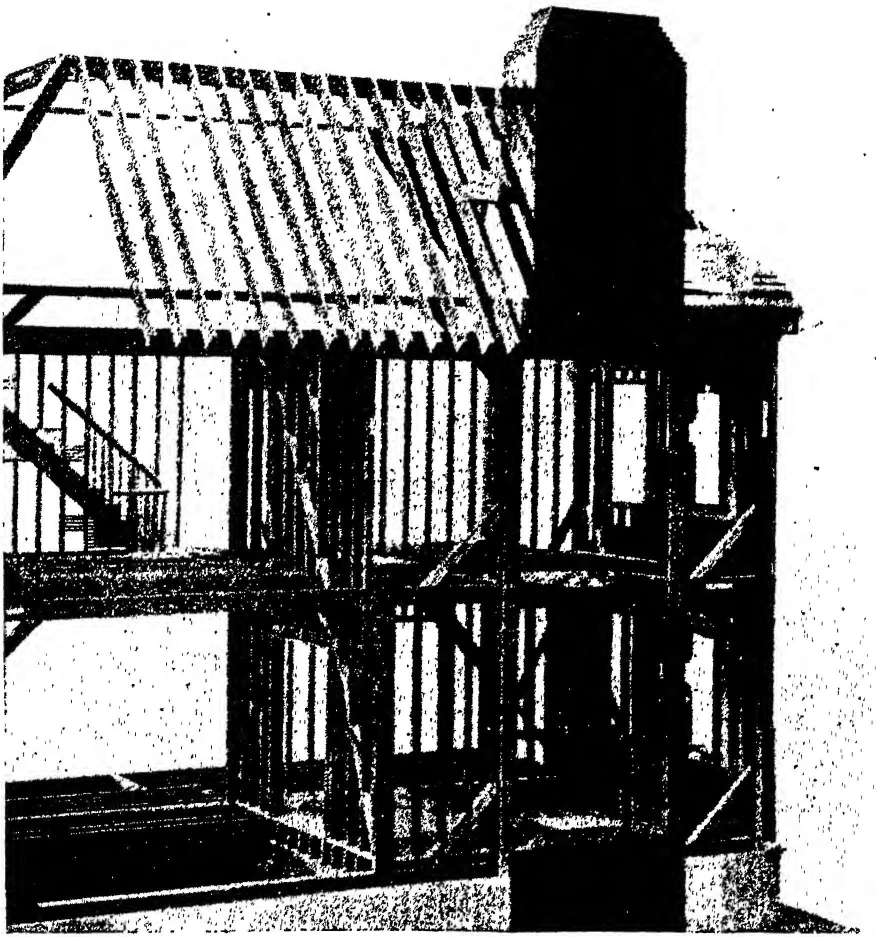


Fig. 1535 C Braced Frame

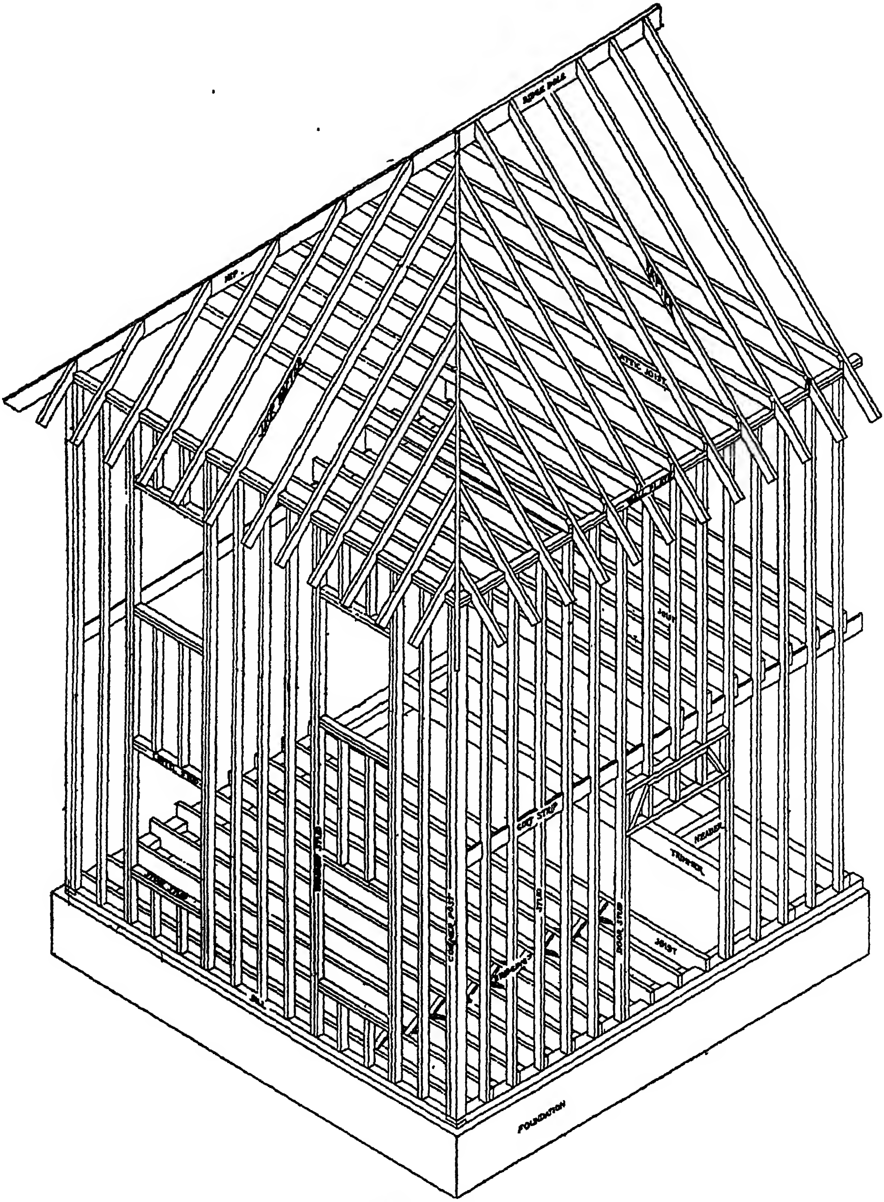


Fig. 1535 D Balloon Frame

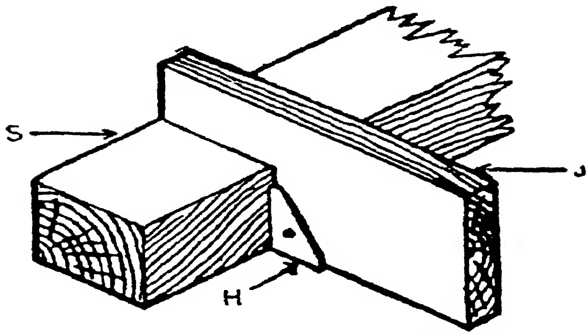


Fig. 1535 E Patent Hanger

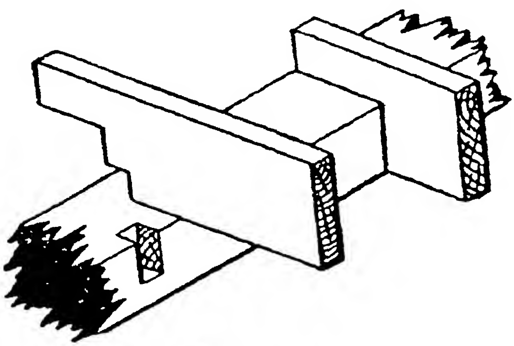


Fig. 1535 F Mortise in Sill.

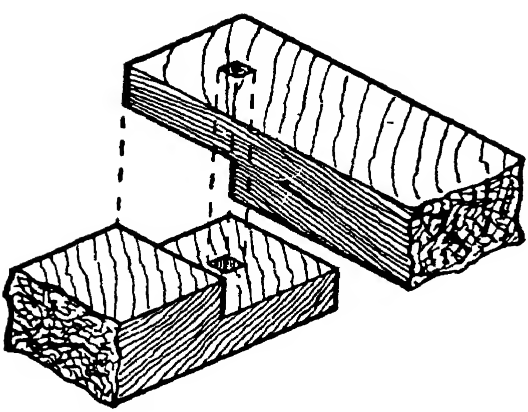


Fig. 1535 G Joint of Sills at Corner.

tises are usually 4″ deep, and cut 2″ into the width of the sill. Joists are usually 2″ × 10″ or 2″ × 12″. Sills are usually halved and pieced together at the corners (Fig. 1535G) but sometimes they are fastened together by means of a mortise in one sill and a tenon in the other.

1536. **Corner posts** must be long enough to reach from the sill to the top plate. The posts are braced (Fig. 1536A); the braces are often nailed in place, but it is much better to cut mortises in the posts for them.

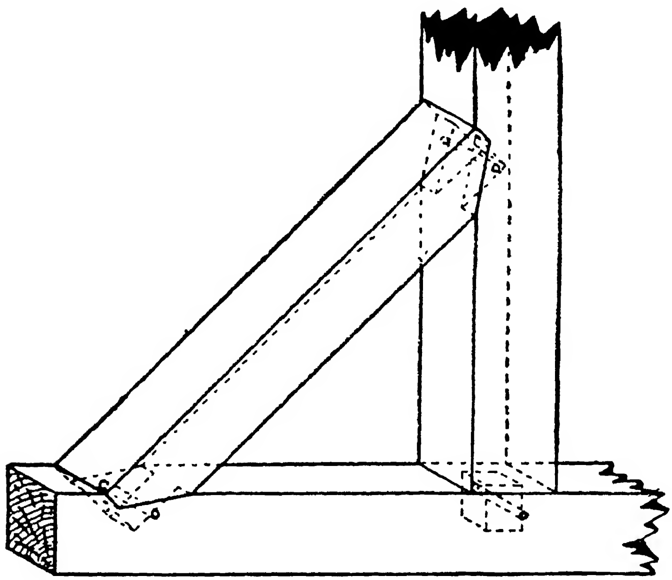


Fig. 1536 A Brace for Post and Sill

The plate is usually fastened to the posts by means of spikes only, but it may be mortised to receive a tenon cut in the top of the post. Corner posts are usually 4″ × 6″; braces 2″ × 4″.

1537. The **girts** support the floor and are made of the same width as the corner posts, and 8″ deep, being flush with the face of the post both outside and inside; 6″ timber may sometimes be used, but the girts are usually 4″ × 8″. A tenon at each end fits into a mortise cut in the post, and the whole is secured by a pin ($\frac{7}{8}$″). It is evident that if the girts in two adjacent walls are framed into the corner posts at the same level, the tenons on the two girts which run parallel with the floor joists will have to be raised above the girts on which these joists rest. The floor joists are carried by the lower (dropped) girts, and the raised girts are so placed that they are just flush on top with the joists to which they

are parallel. Heavy girts are used only in the braced frame. In the balloon frame, ledger boards are used instead; these boards are 2″ thick and 6″ deep, and are placed in notches cut into the studding instead of being framed into them as in the braced frame. The floor joists which the ledger boards carry are notched over them and spiked to them and to the studding. The disadvantage of the ledger board is that it is not strong enough as a tie between the corner posts; consequently a wall with ledger boards is not as stiff as one with girts.

1538. The **plate** at the top of the wall is usually 4″ × 6″ laid flat. The plate serves to tie the studding together at the top and also to furnish a support for the lower ends of the rafters. (See Fig. 1538A.)

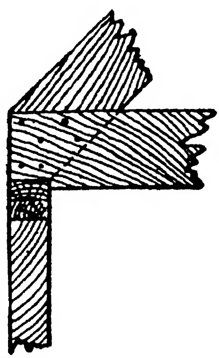


Fig. 1538 A

It thus connects the wall and the roof, just as the sill and the girts connect the floors and the walls. Sometimes the plate also supports the attic floor joists, and acts in this case like a girt, but this arrangement is unusual, as the attic floor joists are usually supported on a ledger board. The plate is spiked to the corner posts and to the top of the studding; but at the corner, the two adjacent plates are connected by a framed joint usually halved together in the same way as the sill. No cutting is done on the plate except at the corners, and the rafters and the attic floor joists are cut to fit it. **Braces** serve to stiffen the wall, to keep the corners square and true, and to prevent the frame from becoming distorted by lateral forces, such as wind. In a full-braced frame, a brace is placed wherever a sill, girt, or plate makes an angle with a corner post. Braces make angles of 45° with the post, and frame into the corner post at heights from one-third to one-half the height of the story (Fig. 1536A). The braces are made the same width as the posts and girts so as to be flush with these pieces both outside and inside. In a balloon frame, there are no permanent braces, but light strips are nailed across the corners

during construction to keep the frame in place. As soon as the outside boarding is in place these are removed; though sometimes light braces are used as permanent parts of even a balloon frame, being notched into them and spiked.

1539. When the sill, posts, girts, plates, and braces are in place, the only step that remains to complete the rough framing of the wall is the filling in of this framework with studding. The studding is of two kinds, the heavy pieces which form the frames for the door and window openings, and the stops for the partitions; and the lighter pieces which are merely filling-in studs. The doors and windows are usually made from 4″ × 4″ pieces (see Fig. 1539A), one on each side of the opening. If

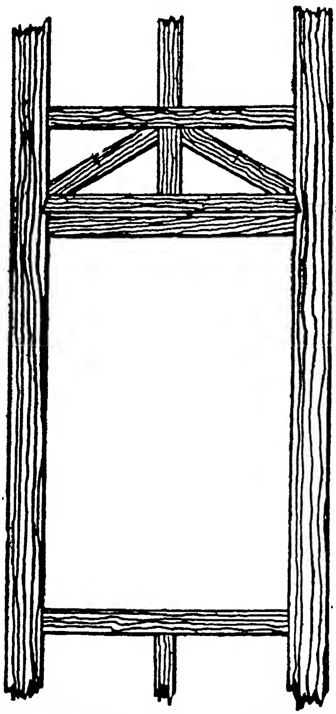


Fig. 1539 A Truss over Window

necessary, a small truss is made as shown to receive any weight which comes from studding directly above the opening and to carry it to either side of the opening whence it is carried down to the sill. In the balloon frame, the door and window side studs are always made of two 2″ × 4″ pieces placed close together, and connected as shown in Fig. 1539B. If there is an opening in the wall of the second story, and no corresponding

opening in the wall of the first story, the door and window studding must be carried double down to the sill, thus wasting material. In designing balloon frames, therefore, it is desirable to have the openings come in

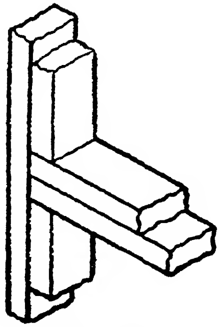


Fig. 1539 B

corresponding positions in all stories. When a partition meets an outside wall, it is necessary to provide a nailing surface for the partition. This is done as shown in Figs. 1539C, D, E. The studs shown in the figures are 4″ × 8″, two 4″ × 4″, and two 2″ × 4″.

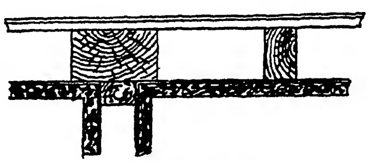


Fig. 1539 C

The "filling-in" studs are used merely to fill up the frame of the other heavier pieces, and afford a nailing surface for the boarding on the outside and the lathing on the inside. The filling-in studs are usually

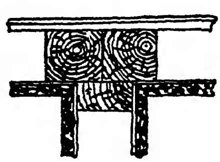


Fig. 1539 D

placed 16″ apart, measured from center to center; never more, sometimes less. The studs are the full width of the wall, usually 4″, and are usually 2″ thick, 2″ × 4″ being the ordinary dimensions for "filling-in" studding. In the braced frame there is necessarily much cutting of the

"filling-in" studding; but in the balloon frame the "filling-in" studding extends clear up from the sill to the plate, and no cutting is necessary. The studding used in partitions is usually 2″ × 4″. The partition walls are usually made 4″ wide, the same as the outer walls, except in the case of the so-called "furring" partitions, which are partitions built around

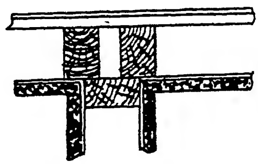


Fig. 1539 E.

chimney breasts to conceal the brickwork and furnish a surface for plastering. As it is usually specified that no woodwork shall come within one inch of the chimney, the studding is placed flat so as to provide a 1″ space between the brickwork and the furring wall. If the outside walls are of brick or stone, wooden "furring" strips are usually placed just

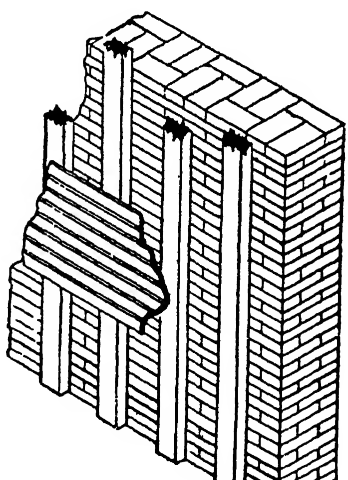


Fig. 1539 F
Furring, Outside Wall

inside of the outer walls to furnish a surface for laths and plastering for the inside finish. The studding for these walls is 2″ × 4″, set close up against the masonry walls and preferably spiked to them (see Fig. 1539F). All the partition walls are finished at the top and bottom by horizontal pieces, called respectively the **cap** and the **sole.** The sole rests directly on the flooring whenever there is no partition under the

one being built; but if there is a partition in the story below, the sole of the upper partition is used as the cap for the one below. The sole is usually made about 2″ thick and $5\frac{1}{2}$″ wide, which leaves a nailing surface of $\frac{3}{4}$″ on each side. The cap is usually made of the same width as the studding, and 2″ thick, so that a 2″ × 4″ piece may be used in most cases. In order to stiffen the partitions, short pieces of studding are placed between the regular pieces on each side of it and make the columns shorter. Thus, if one piece is for any reason excessively loaded, it will

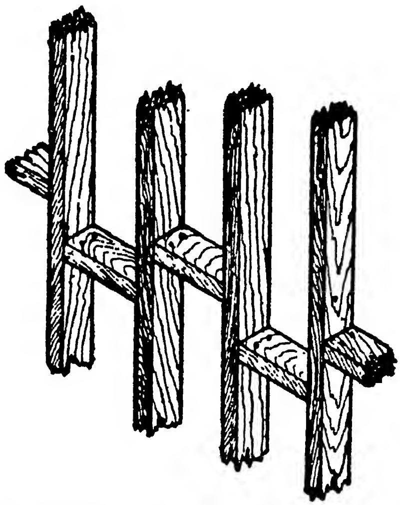


Fig 1539 G Horizontal Bridging

not have to carry the whole load alone but will be assisted by other pieces. This operation is called " bridging," and is shown in Fig. 1539G. The pieces are 2″ thick and the full width of the studding; and in addition to strengthening the wall, they prevent fire and vermin from passing through and are also utilized as a nailing surface for an inside finish such as wainscoting or chair rails. All partitions should be bridged at least once in the height of each story, and the bridging pieces should be securely nailed to the vertical studding at both ends.

Steel Frames.

1540. In large office buildings, modern construction tends to the use of steel frames. Of these there are three types based on the method of sustaining the loads, viz.:

a. The exterior walls carry their own weight and some of the adjacent floor load, the remainder being borne by the interior columns of steel or cast iron.

b. The beams and columns at the edge of the wall sustain the entire weight of the floor and roof, allowing the wall to support itself.

c. The well known " cage " construction in which all loads, including the weight of the wall, are carried at each floor level by the steel frame alone.

The first type is the common type up to five stories. Above this limit, the necessary wall thickness becomes so great that a large portion of the available space is absorbed. The windows of high buildings must be narrow in order to conserve the strength of the masonry, and, with the resulting thick walls, the light is poorly distributed. Likewise, if the unit bearing capacity of the ground under the footing below is small, a large amount of masonry footing will be necessary to properly distribute the pressure over the soil; this is expensive and takes much valuable space, even though the latter disadvantage may be overcome by using the grillage foundation.

The second type is often used for buildings up to seven stories in height. The difficulties of construction are not great; but the second type has been generally discarded in favor of the " cage " type because of the disadvantages of designing and constructing a steel frame separate from the walls of the building.

The third type, the " cage " construction steel frame, which practically eliminates weight-carrying walls from consideration in high buildings, has three important advantages:

(1) Windows may be used for almost the entire exterior area without lessening the stability.

(2) The structure, if properly designed, is proof against hurricanes and earthquakes.

(3) As it is necessary to fireproof these buildings, they are thus made safer against a conflagration than ordinary structures.

1541. **Description of Steel Cage Construction.**—Exterior and interior footings are designed for the building, and from these footings the columns are carried vertically to the top of the building. Connecting with these columns at each floor are two sets of beams, at right angles to each other, the floor stringers and the floor girders which support the floor stringers. At the top, trusses or a similar set of beams sustain the roof of the structure. The exterior wall rests upon the outside beams and stringers, and supports itself for the height from one beam to the one above it. Between the columns and in the same vertical plane is placed the wind bracing, which stiffens the structure and prevents any distortion by the wind. Inside, the building is divided by partitions, into rooms of suitable size.

1542. Steel Cage Columns.—The columns are usually fabricated in two-story lengths, sometimes, however, in three-story lengths. Splices are placed $1\frac{1}{2}'$ to $2'$ above the floor and they are staggered, so that only half the splices occur together at each alternate floor. There are two ways of making this splice:

a. Cap Plate.
b. Continuous.

In the cap plate splice each piece is manufactured at proper length, angles are set at the ends flush on two or four sides and a cap plate is riveted thereto (Fig. 1542A) and later bolted to a similar cap plate of

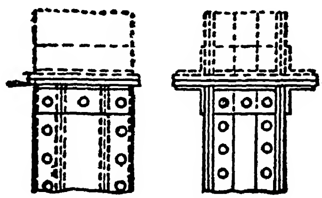
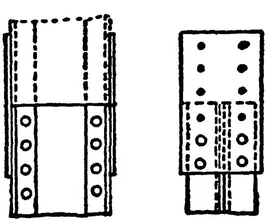


Fig. 1542 A
Cap Plate Splice

the continuing piece. However, this type is unsatisfactory because of its lack of stiffness. In the continuous splice, plates are shop riveted to the lower piece and field riveted to the upper (Fig. 1542B). Size of



1542 B
Continuous Splice

plates and number of rivets are determined by making the bending strength of the splice equal to that of the full column.

It is seen that the loads on the columns increase from the top to the bottom floors. Therefore, the strength of the columns must increase similarly. This necessary increase of strength is obtained by adding plates to the back of the channels of the columns, usually retaining the channel of the same size from top to bottom and adding plates one at a

time as necessary from top story to basement. Fig. 1542C shows cross-sections of a column. A list or schedule is made, showing for each floor the size and numbers of each of the pieces composing the column for that floor.

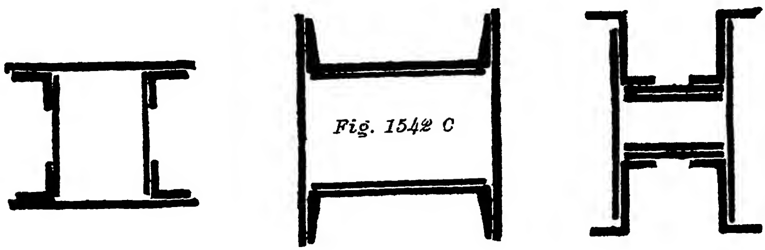


Fig. 1542 C

The column is the most important element of the building. It supports the entire construction work above. Thus a defect in a single column may be the cause of disaster. The usual form is that shown in Fig. 1542C, consisting of channels, reinforcing plates, and lattice bars or box plates. The Bethlehem H-columns are being used a great deal. They are similar to I-beams except that the flange is made considerably wider in proportion to the web than is the case in the I-beam. The purpose of this is to allow substantial thickness to which to bolt or rivet I-beam girders. H-columns are erected one on the top of another and connected together by splice plates. They are heavy, and their thick web is not economical, but their advantages in practical construction work are considered sufficient to offset these disadvantages.

These columns must of course be protected from fire. The best way to accomplish this is to surround them by hollow tile blocks. These are laid with " broken " joints and are bonded at intervals with bent wires, metal clips, etc., placed in the mortar at the joints. The tile should always extend from fireproof floor to fireproof ceiling with no unprotected spaces between them.

In place of hollow tile, gypsum is sometimes used to protect the steel. Brick would admirably serve this purpose except for its weight, due to which it is practically never used.

Another method of protecting steel columns is by plaster or metal lath. In this case wire laths should be separated from the steel. Plaster itself is not to be depended upon alone for fireproofing. Therefore, such columns are constructed with two layers of plaster and an air space between. A better method is to put up the metal lath away from the column, pour a cinder concrete fill between the lath and the column, and then plaster the metal lath.

A cause of failure of the protective covering of columns has often been

the inclosing of water pipes, conduits, etc., within the covering. These pipes expand more quickly than the steel column, with the result that they buckle, thus breaking the column covering and leaving the column exposed.

1543. **Exterior Walls.**—The purpose of these walls is to exclude rain and weather, to retain heat, and to protect against fire. At the same time, windows must be provided for the admission of light and air, and doors for entrance. The walls are built of terra cotta, brick, or stone. The first two are best; stone is too expensive and lacks fire-resisting qualities. The outside columns are commonly set in the wall a little outside of the center. The protection for the outermost steel of the columns should be not less than 4″, and more is desirable. An ideal theoretical position is one in which the center of gravity of the columns coincides with the resultant of the applied loads. The beams which carry the wall should not only be amply strong, but also stiff enough to prevent cracking of the masonry. The deflection of the beam should not exceed $\frac{1}{360}$ of the span. When the wall beams are parallel to the stringers, they are so placed that they support the portion of the flooring adjacent to the walls. When parallel to the girders, their position is determined by the necessity of supporting the stringers.

1544. **Steel Shop Buildings.**—Steel framed buildings are extensively used as machine shops, garages, storehouses, and steel mills. Fig. 1544A shows a building of this type. They are usually one-story, and built entirely of metal. The roof is usually made of corrugated iron, the sides of corrugated iron or steel plate, windows and doors are often simply openings in the walls, and the floors are of concrete or brick or earth. All the fastenings are of metal.

For each roof truss, there is a corresponding column. Horizontal beams connect adjacent trusses and columns and stiffen them. If a large overhead crane is to form part of the shop equipment, the horizontal beam running from column to column for this purpose is actually a large plate girder specially designed to support the overhead crane.

FLOORS.

Loads.

1545. **Dead Loads.**—The weight of a wooden floor including its joists is about 10 pounds per square foot; with the attached ceiling 20 pounds per square foot. The weight of a brick floor arch in pounds per square foot is about four times the depth of the arch in inches, and may vary from 25 to 45 pounds per square foot. The weight of tile block varies from about 25 to 75 pounds per square foot, depending upon the thick-

ness of the block. With reinforced concrete construction, including the wooden flooring, the total weight is between 40 and 70 pounds per square foot. The weight of floors supported by I-beams is between 120 and 300 pounds per square foot. All of the above weights are for a floor system built in accordance with usual requirements. For specially large spans, requiring specially strong construction, the weights will be much greater than those given above.

Fig 1544 A Steel Shed

1546. Live Load.—The Building Code of the National Board of Fire Underwriters prescribes that:

"Each floor of every building shall be of sufficient strength in all its parts to bear safely the weight to be imposed thereon, in addition to the weight of the floor itself. It shall safely support a minimum live load per square foot of area as specified in the following table."

	Ground and Lower floors	Upper floors
Foundries, Light and Power Plants, Printing and Lithographing Houses, Railway Freight Depots	250	250
Car barns, Garages	150	120
Warehouses	200	200
Fire Houses	150	60
Armories, Ball Rooms, Dance Halls, Exhibition Buildings, Factories, Gymnasiums, Work Shops, Lofts, Markets, Stables	120	120
Stores, Public Halls, Restaurants	120	120

Railway Passenger Stations	120	90
Office Buildings	120	75
Court Houses	100	100
Churches, Libraries, Museums, Theaters	90	90
Schools and Colleges	90	75
Asylums, Bath Houses, Club Houses, Detention Buildings, Dormitories, Hospitals, Hotels, Lodge Rooms, Lodging-Houses, Studios	90	60
Tenement Houses and Dwellings	60	40

"Every floor beam in factories, lofts, office buildings, printing houses, restaurants, stables, stores, warehouses and workshops shall be capable of standing a live load at its center of at least 4000 pounds."

In estimating the live load transmitted from the floors to the columns of high buildings, a reduction of 5 per cent of the maximum total live load is made for each floor below the top one until this reduction reaches 50 per cent, as stated in paragraph 1513.

Wooden Floors.

1547. Wooden floors consist of **flooring, joists** (stringers), and **girders.** (Fig. 1547A.) The essential qualities are strength and stiffness. The flooring is made of tongued and grooved boards supported by joists: the joists are supported by beams called girders, which are in turn supported by the walls of the building or by columns.

Problem.—Design the floor of a warehouse to carry 400 pounds per square foot, assuming that the joists are 2′ apart and that they are supported every 12′. Flooring and joists are of longleaf yellow pine.

Solution.—The flooring has a span of 2′, and it is seen from the tables of the engineering handbook that a 2″ thickness of flooring is much more than is needed.

The joists have a 12′ span, and carry in addition to their own weight (which is included in the figures in the tables), the weight of the floor and the live load. The floor is $\frac{1}{6}$′ thick, and each joist carries a section 2′ wide and 12′ long, total, 4 cubic feet per joist. At 44 pounds per cubic foot, this weighs 176 pounds. The live load weighs 400 pounds per square foot of floor, and each joist carries 2′ × 12′ of the floor, or 9600 pounds of the live load. Adding, the total load on the joist is 9776 pounds. Entering the table under 12′ span, we find that a beam 12″ deep and 1″ wide will support 1733 pounds. Hence a beam 12″ deep and 6″ wide will support 10,398 pounds, which is ample to carry the load.

Therefore 2″ flooring and 6″ × 12″ joists will be satisfactory.

To secure stiffness as well as strength, the depth of the joists and girders is made large as compared with their width. The width of joists varies from 2″ to 3″, and their depth from 6″ to 14″. They are spaced from 12″ to 16″ center to center. To hold the joists in position, they are **bridged** at the middle of short spans and at intervals of 8′ in long ones. The bridging consists of cross-bracing composed of 1″ by 3″ pieces, inserted between consecutive joists. If the upper surfaces of the joists and girders are to lie in the same horizontal plane, the joists are notched into the girders, or supported by iron stirrups fastened to the girders.

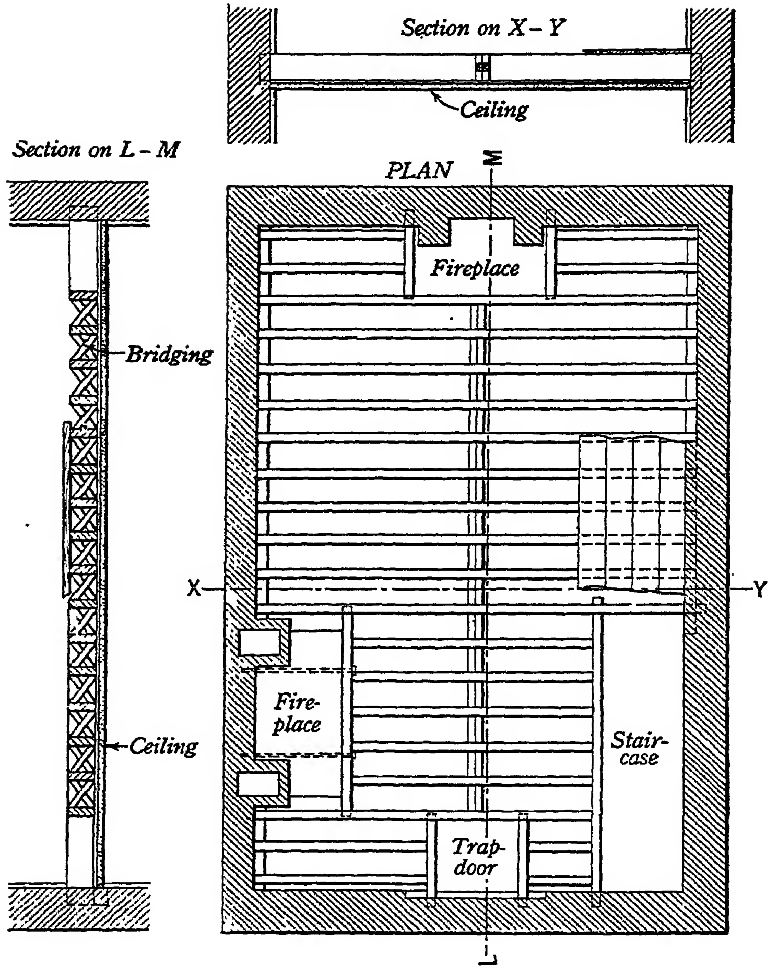


Fig. 1547 A Wooden Floor.

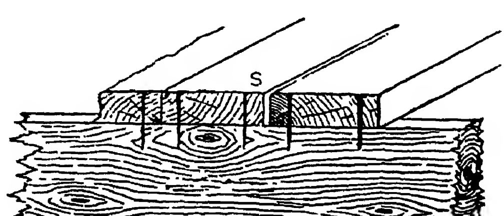


Fig. 1547 B Plain Jointed Floor

The latter method is the better, as the total cross-sections of both joists and girder are thereby preserved. Floor boards are laid in several different ways, of which the more usual are:

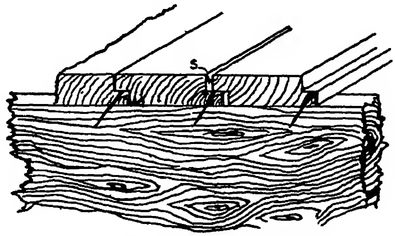
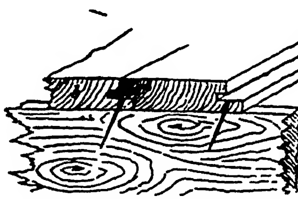


Fig. 1547 C Rebated Floor

a. Plain Jointed.—The boards are simply laid side by side, as close as possible (see Fig. 1547B), a nail or generally two being driven in through the boards into each joist. The inevitable shrinkage of the boards, as at *S*, will cause openings in this type of floor.



Rebated Tongued and Grooved Floor
Fig. 1547 D

b. Tongued and Grooved.—One board can first be nailed as shown in Fig. 1547D and the other board, upon being slipped into it, will be kept down by the form of the joint. Thus the nails are prevented from appearing on the surface of the floor.

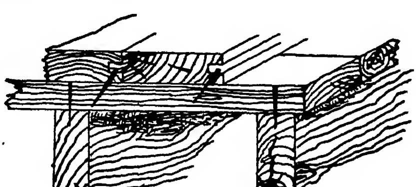


Fig. 1547 E Tongued and Grooved Floor

Fireproof Floors.

1548. Wooden floors are not at all fire-resisting. The need of floors with great fire-resisting qualities has led to the use of tile and concrete in floors. Steel I-beams are somewhat fire-resisting; and since they

must be used for long spans, they are covered with some fire-resisting material.

I-beam Floor.—In this floor the joists are steel I-beams; the girders are steel I-beams, plate girders or box girders (Fig. 1548A). The joists

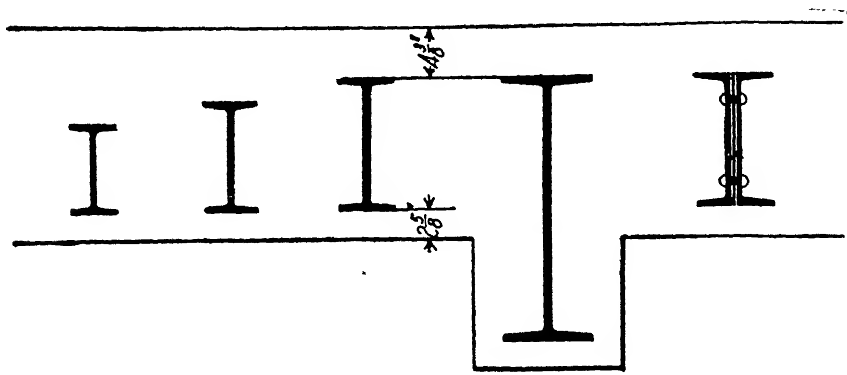


Fig. 1548 A Various forms of Steel Floor Beams

and girders are fastened together by angles riveted to the webs of the girders. The joists are connected by arches or slabs of incombustible material such as brick, tile blocks, or concrete. Above the arches or slabs the floor is leveled with cinder concrete and in this concrete bed are placed wooden strips to which the flooring is nailed.

The first effective fire-resisting floor was constructed of heavy brick arches between wrought-iron beams (Fig. 1548B). This method was

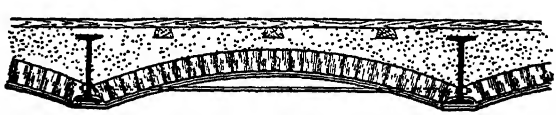


Fig. 1548 B Steel and Brick Floor

abandoned, however, because of its great weight and the consequent necessity for heavy columns and heavy foundations. It was replaced by the use of hollow tile, constructed in segmental arches between steel I-beams. These arches were constructed in spans of from 5′ to 12′ and above, the rise or " crown " of the arch being usually $1\frac{1}{2}''$ to every foot of span, with tie-rods connecting the beams in order to take the lateral thrust of the arches. They were found to have many disadvantages. The segmental arch materially increased the thickness of the floors and hence the expense. Moreover, in the longer spans the tie-rods were necessarily exposed because of the high crown of the arch. They were

also difficult to construct and the flanges of the beams were left unprotected. The objections were overcome by the skew types as shown in Figs. 1548C and 1548D. The last type has now completely replaced

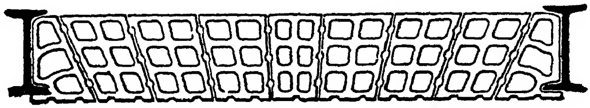
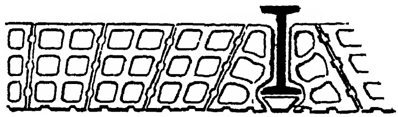
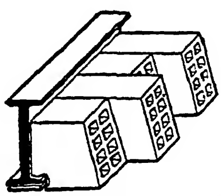


Fig. 1548 C Steel and Hollow Tile Floor



Steel and Hollow Tile Floor
side construction
Fig. 1548 D

the segmental arches in most modern construction. Its spans vary from 4′ to 6′.



Steel and Hollow Tile Floor
end construction
Fig. 1548 E

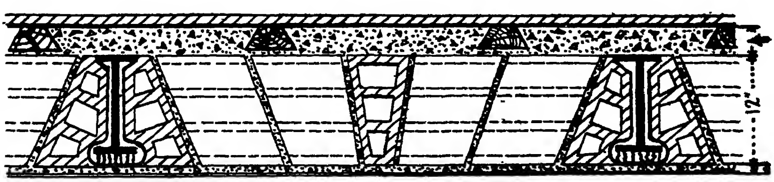


Fig. 1548 F Steel and Hollow Tile Floor

Hollow Tile Flat Arch.—The hollow tile flat arch is constructed in three principal ways; side construction (Fig. 1548D), end construction (Fig. 1548E), and combination construction (Fig. 1548F).

In the side construction the tiles can be more easily laid; they can be laid with broken joints; the skew backs have better bearing on the beams than in the end construction; and good protection is obtained for the bottom of the beam.

End construction is somewhat stronger than side construction. Ties are run through the hollows of the tiles. In this method, however, the joints are not broken; the binding is not good; the bearing of the skew against the beam is poor, and it is almost impossible to protect the bottom of the beam properly.

Combination construction consists of a side construction skew and key, and end construction lengtheners. It is the type of flat arch most commonly used at the present time.

The typical method of laying hollow tile floors is shown in Fig. 1548F. The forms are suspended from the I-beams. The tiles are then put in place and slushed with Portland cement mortar. Cinder concrete is poured in to the level of the top of the beam; nailing strips are set in place and the concrete fill is continued to the top of the nailing strips.

The forms should remain in place for 48 hours. They may then be removed and used over again. It is very important that tile work should be properly laid. Such work should therefore be entrusted only to experienced men.

Advantages of hollow tile over concrete floor:

Hollow tile floors are lighter in weight.

Forms are less expensive and can be used over again.

There is no drip through the forms as in concrete slab floors; construction can therefore go on rapidly underneath.

Woodwork of floors can be commenced very soon after pouring the cinder concrete.

In two days' time the tiles are practically dry.

Building goes up very rapidly.

Disadvantages of hollow tile:

Thicker floors than concrete are required.

They are more seriously injured by fire.

They are not readily repaired when destroyed by fire.

They cannot be made to fit around irregular spaces.

Both concrete and tiles are not sufficiently sound proof.

Concrete Floors.—Fig. 1548G shows the cross-section of a type of concrete floor often used. It is designed as a non-continuous beam. The floor shown in Fig. 1548H is designed as a continuous beam. This floor is reinforced and is much stronger than the floor of Fig. 1548G. It

will be noticed that the reinforcement is always placed on the tension side of the concrete.

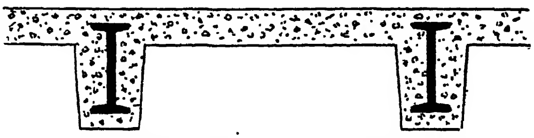


Fig. 1548 G Steel and Concrete Floor

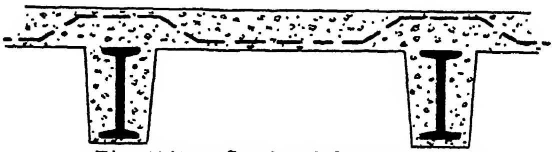


Fig. 1548 H Steel and Concrete Floor

Forms for this type of floor may be constructed as shown in Fig. 1548 I. This form is erected with very few nails and so constructed that it can be pulled down all at once. Soft soap or oil is used on the forms

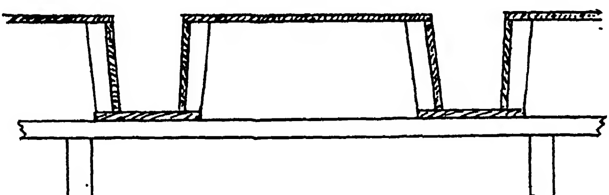


Fig. 1548 I Forms for Concrete Floor

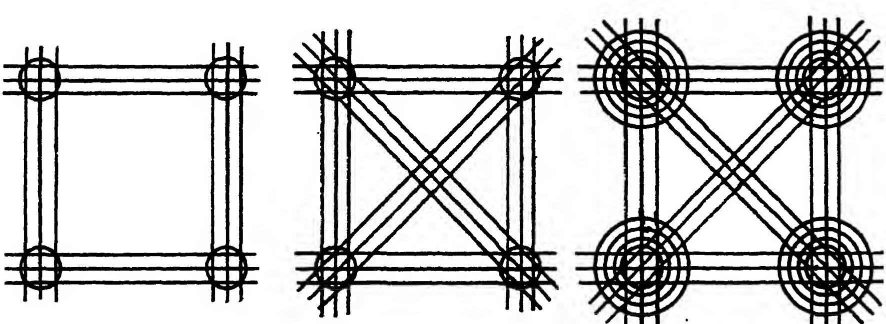


Fig. 1548 J Reinforcement for Concrete Floor Slabs

before pouring the concrete, in order to prevent adherence of the concrete to the forms.

Reinforced concrete slab floors generally have neither girders nor beams, but consist of concrete so reinforced as to consolidate the entire floor and columns into a monolithic mass. Fig. 1548 J shows several standard methods of reinforcing floor slabs.

Advantages of reinforced concrete:

It gives the strongest floor possible.
It allows the shallowest floor.
It fits around irregular spaces.
It can be plastered over, if injured by fire.

Disadvantages:

It is heavier than hollow tile.
The forms used to hold the concrete are more expensive than for plain concrete.
The forms must be kept up at least two weeks.

Composite Flooring.—There are on the market several compositions used for flooring. Among these are *granolithic,* a floor finish composed of cement, sand, and granite chips in a 1 : 1 : 1 ratio, applied like concrete and polished with pumice stone; *terrazo,* composed of cement, sand, and marble chips in a 1 : 1 : 1 ratio; and *asbestolith,* composed of cement, sand, and cork and asbestos in a 1 : 1 : 1 ratio, put down hot and smoothed with a trowel.

THE STAIRWAY.

1549. In frame buildings, a means of egress — sometimes the sole means of egress — from the upper floors in case of fire is the stairway. The importance of the stairway cannot therefore be over-emphasized. It must be of ample strength and proportions to safely withstand all loading conditions which it may be required to meet in any eventuality. These proportions, in their variations to meet the requirements of different classes of buildings, may be found in Building Codes.

There are three members in a stairway: the **stringers,** the **risers,** and the **treads.** These are shown in Fig. 1549A. The important thing to be ascertained by the stair builder is the space which the stairs are to occupy (run and rise). The "run" is the horizontal distance — that is, the length of floor — covered, while the "rise" is the vertical distance from the top of the lower floor to the top of the upper floor. (Fig. 1549B.)

There is always one less tread than riser in a stairway, as the top of the last riser is on a level with the upper floor. The sum of the width of tread and height of riser should be approximately 16″. Thus with a 6″

riser, the tread is usually made 10″. This relation between the tread and riser should be adhered to as closely as possible. The riser should not be made less than 6″.

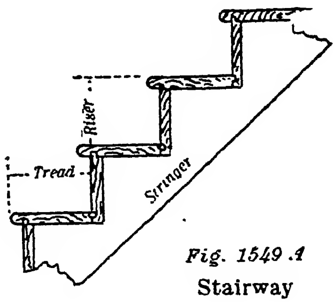


Fig. 1549 A
Stairway

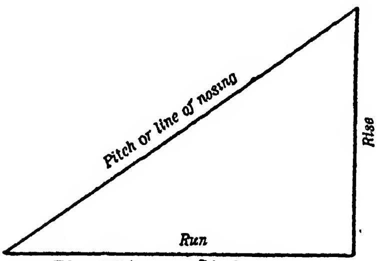


Fig. 1549 B Stairway

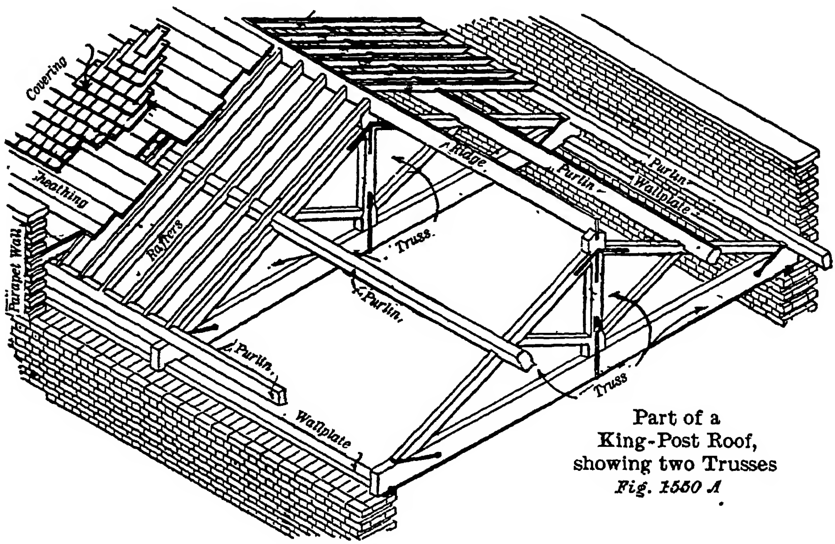
ROOFS.

1550. A complete roof consists of covering, sheathing, rafters, purlins, and roof truss (Fig. 1550A). The **covering** is the outer or weather-resisting coating of the roof. The materials employed for this coating are shingles, slate, tiles, asphalt, tin, copper, lead, and corrugated iron. The **sheathing** is the layer of boards or other material to which the covering is attached. The **rafters** are the inclined beams which support the sheathing; they correspond to the joists in floors. The **purlins** are the horizontal beams which support the rafters and correspond to the floor-beams of bridge floors. The **roof-trusses** are the frames which support the roof and transmit its weight to the walls or columns of a building.

The **wall plates** are beams which are laid on top of the wall to distribute the weight transmitted by the trusses.

The **pitch** of a roof is the ratio of the rise to the span.

The **ridge** is the highest horizontal line of the roof. The **eaves** are the lowest horizontal lines of the roof.



Part of a King-Post Roof, showing two Trusses
Fig. 1550 A

1551. According to their surfaces, roofs are classified as:

a. **Shed** roof or **lean-to,** which has a single plane surface (Fig. 1551A).

b. **Gable** roof, which has two plane surfaces intersecting in the ridge (Fig. 1551B).

Fig. 1551

Shed Roof Gable Roof Gambrel Roof Hipped Roof

c. **Curb** or **gambrel** which has four plane surfaces; the two on each side of the ridge have different inclinations and intersect in a line parallel to the ridge (Fig. 1551C).

d. **Hipped** roof, which has the same inclination at the ends of the building as the sides; the inclined lines of intersection of its slopes are the **hips** (Fig. 1551D).

A **valley** is the line of intersection of roof surfaces making a reëntrant angle, as the line of intersection of two gables which are perpendicular to each other.

Loads.

1552. The **dead load** of a roof comprises the weight of the roof covering, rafters, trusses, etc., and any permanent weights which the roof may have to support. Weights for ordinary types of roofing, trusses, etc., may be found in engineering manuals. The following typical figures are quoted:

Shingles, $2\frac{1}{2}$ pounds per sq. ft.
Slate, $\frac{3}{16}''$ thick, $7\frac{1}{4}$ pounds per sq. ft.
Tin, including felt, 1 pound per sq. ft.
Tile, 11 to 14 pounds per sq. ft.
Corrugated iron, 1 to $3\frac{1}{2}$ pounds per sq. ft.
Sheathing, 3 pounds per sq. ft.
Rafters, hard pine, 2 pounds per sq. ft. for $2'' \times 4''$ rafters, $16''$ apart; 4 pounds per sq. ft. for $2'' \times 10''$ rafters $20''$ apart, etc.
Purlins, are figured the same as rafters.
Wooden trusses, 3 pounds per sq. ft. of roof for $36'$ span, $\frac{1}{2}$ pitch; 9 pounds per sq. ft. for $100'$ span, flat, etc.
Steel trusses, $5\frac{3}{4}$ pounds per sq. ft. of roof for $40'$ span, $\frac{1}{2}$ pitch; 13 pounds per sq. ft. for $100'$ span, flat, etc.

The following formulas may be used for determining the weight of roof trusses:

$$W = 0.05 + 0.075\,L \text{ for wooden trusses,}$$
$$W = 0.05\,L + \frac{12}{A} \text{ for steel trusses,}$$

in which W = weight of trusses in pounds per sq. ft. of horizontal projection of the roof supported, L = span in ft., A = distance in ft. between trusses.

1553. The **live load** of a roof comprises the weight of the snow, wind, and any temporary weights which it may have to support. No roof should be designed for a total load of less than 40 pounds per square foot except flat roofs in warm climates. **Wind loads** are usually considered for roofs with a greater pitch than 5 inches to the foot, but as snow will not remain on the side of a roof against which a strong wind is blowing, both snow and wind loads are frequently calculated together. For computing wind pressure alone, several empirical formulas are in use. Usually the component of the wind assumed normal to the roof surface is given a value of 30 pounds per square foot against a vertical surface, as

would be caused by a wind of nearly 100 miles per hour. With such a wind, the pressure on the windward side and suction on the leeward side make it necessary that the roof covering be securely fastened down, that all joints in the roof be designed to take both tension and compression, and that roof trusses be securely fastened down.

The formula of Colonel Duchemin of the French Army is considered to give the most reliable results. The formula is:

$$P_n = P\frac{2\sin\phi}{1+\sin^2\phi} \qquad (1553A)$$

where P_n = component of unit pressure normal to the roof, P = unit pressure in pounds per square foot on a surface perpendicular to the direction of the wind, and ϕ = angle which the inclined surface makes with the

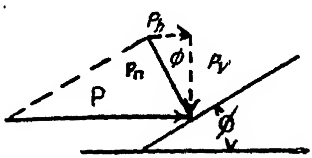


Fig. 1553 A

horizontal. The vertical and horizontal components of P_n in Fig. 1553A are given by the formulas:

$$P_h = P\frac{2\sin^2\phi}{1+\sin^2\phi} \quad \text{and} \quad P_v = P\frac{2\sin\phi\cos\phi}{1+\sin^2\phi} \qquad (1553B)$$

where P_h and P_v are respectively the horizontal and vertical components of the unit pressure in pounds per square foot.

The normal wind loads given by this formula for a 30-pound wind vary from 5.1 pounds for a 5° roof, to 24 pounds for a 30° roof, and to 30 pounds for a 90° roof.

The **snow load** to be provided for is a variable quantity, depending on the slope of the roof, the latitude, and the humidity. Dry freshly fallen snow weighs 8 pounds per cubic foot and may be 3′ deep or even more on flat roofs; packed or wet snow weighs about 12 pounds per cubic foot, but is seldom more than 18″ deep.

1554. Allowance for wind and snow loads per square foot of roof surface is given in the following table:

Location	Snow Load Alone			Wind and Snow Combined		
	Pitch of Roof			Pitch of Roof		
	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$ or less	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$ or less
Northwest States	12	30	45	30	30	45
New England	10	25	40	30	25	40
Rocky Mountains	10	25	35	30	25	35
Central States	5	20	30	30	25	30
Southern States, Pacific States	0	5	5	30	25	20

Variations in Loading for which Stresses should be Found.—To determine the maximum stresses under any possible condition of loading, stresses should be found for the following cases:

(1) Stresses due to permanent dead loads,
(2) Snow covering only one side of roof,
(3) Snow covering entire roof,
(4) Wind on side of truss nearer the expansion-end,
(5) Wind on side of truss nearer fixed end.

It is generally assumed that the maximum wind pressure and the snow-load cannot act on the same half of the truss at the same time; hence the combinations for maximum stress will be either cases 1 and 3 or cases 1, 2, and 4 or 5.

Roof Trusses.

1555. The trusses are made of (1) wood (Fig. 1550A); (2) wooden compression and steel tension members (Fig. 1555A); (3) wholly of steel (Fig. 1555B).

The upper chord is in compression and corresponds in general outline to the roof itself. The lower chord is in tension and may be a straight line or a series of straight lines. Fig. 1555C shows that the compression members may be vertical and the tension members inclined, the compression members may be inclined and the tension members vertical, or the upper chord may be trussed. The panel points of the upper chord are usually at the purlins.

In wooden trusses the members are all of the same thickness and differ only in depth; they are united by mortise and tenon or notched joints, strengthened by straps or bolts. Steel trusses may have pin-connected joints along the lower chord or they may be riveted throughout; the latter is the more common construction. In the riveted truss all members are usually angles in pairs riveted to connecting plates and fastened at intervals to each other.

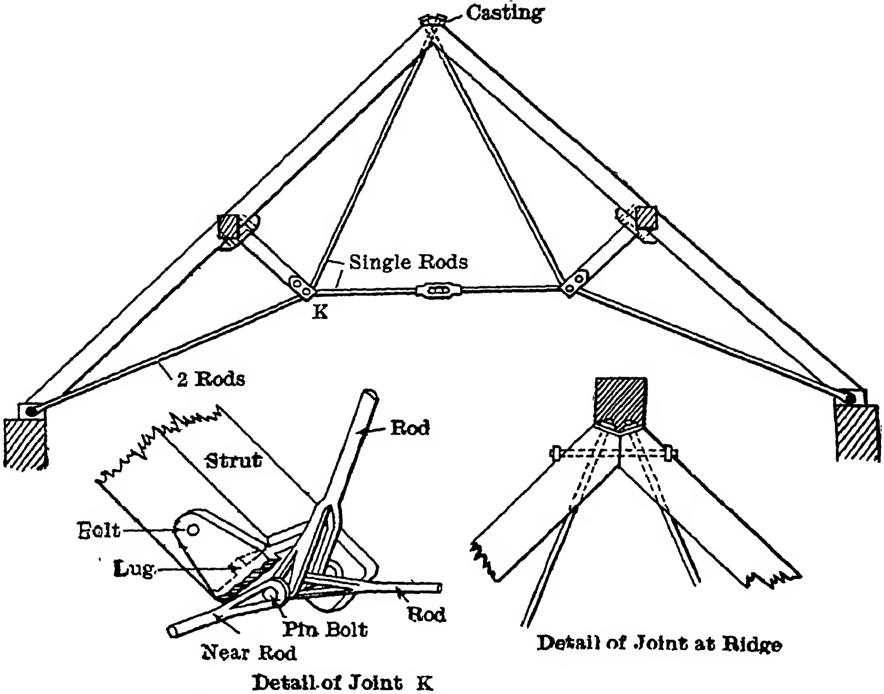


Fig. 1555 A Wooden Truss with Iron Ties. Spans up to Thirty-six Feet

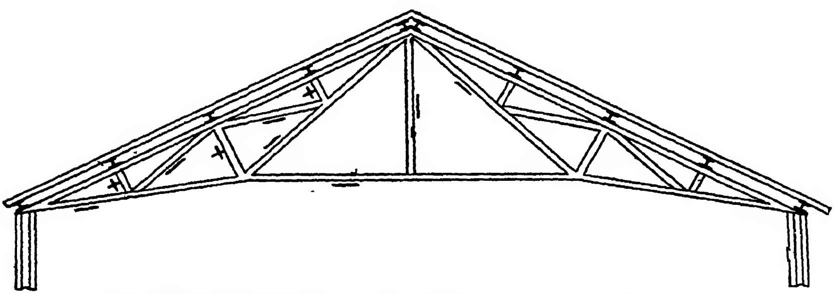


Fig. 1555 B Steel Truss. Spans from Forty to Eighty Feet

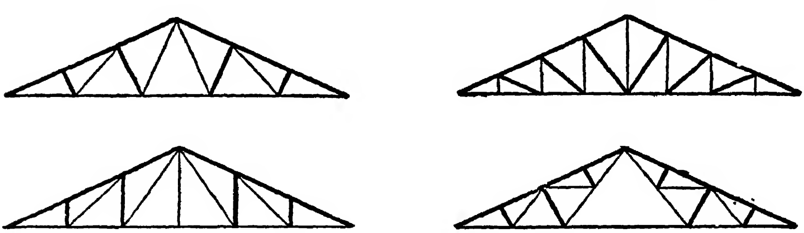


Fig. 1555 C Roof Trusses

Construction.

1556. The kind of roof covering varies with the character of the building, the architectural effect desired, and the pitch of the roof. If the pitch is small, there is greater tendency toward leakage, and consequently, the covering should be a continuous sheet, as asphalt, tin, etc.; if the pitch is large, the covering may be composed of overlapping plates, as slates, tiles, etc. The sheathing is usually inch boards, covered by layers of tarred paper or similar materials; when a fireproof construction is desired, the sheathing consists of slabs of terra cotta or other fireproof material supported by a metal framework.

Wooden rafters are beams 2″ to 3″ thick and 4″ to 10″ deep. They are spaced from 16″ to 24″ center to center, for ordinary loads. If the wooden rafters rest directly upon the wall-plates and not on purlins, the opposite rafters are connected about midway between the wall-plates and ridge by a horizontal brace called a **collar-beam.** In metal roofs the rafters are I-beams or other structural steel forms. The I-beams are spaced at wider intervals than wooden rafters, and in fireproof construction are connected by inverted tees which support the fireproof sheathing. The purlins are usually spaced about 8′ to 10′, as wider spacing requires the rafters and purlins to be inconveniently large. They are either square wooden beams or one of the forms of structural steel beams.

The economical spacing of trusses depends to a great extent upon the kind of roofing that is used, and also upon the span. As a general rule, however, the most economical spacing is about as follows:

For wooden trusses under 80′ span, from 12′ to 16′ on centers.
For wooden trusses over 80′ span, from 16′ to 24′ on centers.
For steel trusses under 80′ span, 16′ to 20′ on centers.
For steel trusses over 80′ span, from 20′ to 40′ on centers.

When the distance between trusses exceeds 16′ for wooden roofs or 20′ for steel roofs, it is generally necessary to use trussed purlins.

Having decided upon the kind of truss to be used, the spacing of the trusses and the roof-construction, a section-drawing of the roof should be made, showing the elevation of the truss, the points at which the purlins are to be supported, the manner of supporting the ceiling, and any other loads that are to be supported by the trusses. The section and truss-drawing, with the tables of the weights of roofing-materials, will furnish the necessary data for computing the loads at each joint. Until the stresses have been determined, the sizes of the members computed, and the joints detailed, an exact drawing of the truss cannot be made; but in order to compute the loads and stresses, it is necessary to know the positions of the joints, and these can be indicated with sufficient accuracy before the exact sizes of the members are obtained.

In fireproof construction, roof materials may be the same as those used for floors. Gypsum blocks are used to a considerable extent because of their light weight. Where concrete roofs are constructed, as in many of the flat roofs of business buildings, the principles are exactly the same as those for floor construction. The concrete or gypsum is not left exposed. It is covered with metal, tar and gravel, or some other standard roof covering.

STRESSES IN ROOF TRUSSES.

1557. It is assumed that the loads or forces transmitted by a truss are coplanar forces which act in the plane containing the axes of the members; and the loads are so placed that the forces act only at the panel points.

In all trusses resting on end supports, the reactions at the supports are taken as acting along lines parallel to that of the resultant of the other external forces, unless the action lines of the reactions are fixed by special conditions. If the truss rests on a roller at either end, to allow for the expansion and contraction of its members due to changes in temperature, the reaction at that end is assumed to be normal to the surface on which the roller rests.

The two reactions and all the other external forces will therefore always form a system either of parallel or concurrent forces in equilibrium. The determination of the stresses in a truss therefore consists: (1) in finding all the external forces acting on a truss, this usually being a question of finding reactions; (2) in finding the resultant stress in each member of the truss as caused by these forces and the stresses in other members. After obtaining the reactions, there are two general methods of determining the stresses in the members of a truss, the **analytic** and the **graphic.** The analytic may be sub-divided into the analytic method of concurrent forces and the analytic method by sections.

Analytic Method of Concurrent Forces.

1558. Considering each panel point as a "free body," we know that it must be in a state of rest. Therefore, all the forces and stresses acting at a panel point must be in equilibrium.

The system of forces at each panel point being a system of concurrent forces, the only requisites for equilibrium are that the algebraic sums of the horizontal and vertical components shall each be equal to zero.

In any system of concurrent stresses in a truss, the stresses acting towards the common point of application are compressive, and those acting away from the point are tensile; and, conversely, a compressive stress in a piece must be assumed as acting towards the common point of application, and a tensile stress as acting away from it.

1559. Let it be required to determine the stresses in the roof truss shown in Fig. 1559A.

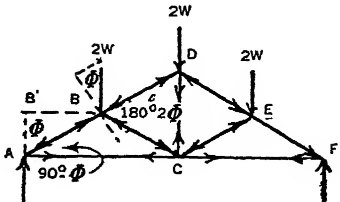


Fig. 1559 A

Panel Point *A*.—Considering the panel point as a "free body," we see that the vertical component of AB must be compressive and equal to 3 W in order to produce equilibrium of vertical forces.

$$\frac{AB'}{AB} = \cos\phi$$

$$AB = \frac{AB'}{\cos\phi} = \frac{3\,W}{\cos\phi}$$

The stress on AC must be equal and opposite to the horizontal component of AB. This horizontal component of AB is obtained from the equation

$$\frac{BB'}{AB} = \sin\phi$$

$$BB' = AB\sin\phi = \frac{3\,W\sin\phi}{\cos\phi} = 3\,W\tan\phi.$$

Thence the stress on AC is tensile and equals 3 $W \tan\phi$. In tabulated form, these stresses are shown below, the signs indicating direction of action of the component of the force in accordance with the system stated in paragraph 109:

Stress	Vertical Component	Horizontal Component	Total Stress	Character
3 W	$-3\,W$	0	3 W	Reaction
AB	$+3\,W$	$-3\,W\tan\phi$	$\frac{3\,W}{\cos\phi}$	Compressive
AC	0	$+3\,W\tan\phi$	$3\,W\tan\phi$	Tensile

Panel Point *B*.—Considering the panel point as a "free body," we see that both BD and BC are unknown and each has a vertical component; hence it is not possible to determine the two vertical components. However, if we consider only the components which are either normal or

parallel to line ABD, we find the equations of equilibrium will be sufficient, as we have two equations and two unknown quantities.

Considering components normal to line ABD, we have

Normal component of $2W = 2W \sin \phi$

Hence, " " " $BC = 2W \sin \phi$ and is compressive.

Knowing that the angle DBC equals $180° - 2\phi$, the actual stress in BC is obtained from the equation

$$\frac{2W \sin \phi}{BC} = \sin(180° - 2\phi) = \sin 2\phi$$

$$BC = \frac{2W \sin \phi}{\sin 2\phi} = \frac{2W \sin \phi}{2 \sin \phi \cos \phi} = \frac{W}{\cos \phi}$$

The parallel component of BC is obtained from the equation

$$\frac{\text{Parallel Comp.}}{BC} = \cos(180° - 2\phi) = -\cos 2\phi$$

$$\text{Parallel Comp.} = (BC)(-\cos 2\phi) = -\frac{W \cos 2\phi}{\cos \phi} = \frac{W(\sin^2 \phi - \cos^2 \phi)}{\cos \phi}$$

The stress in BD is equal and opposite to the resultant of all other parallel components. It is obtained from the equation

$$BD = \frac{3W}{\cos \phi} - 2W \cos \phi - \frac{W(\sin^2 \phi - \cos^2 \phi)}{\cos \phi}$$

$$= \frac{3W - 2W \cos^2 \phi - W \sin^2 \phi + W \cos^2 \phi}{\cos \phi}$$

$$= \frac{3W - W \sin^2 \phi - W \cos^2 \phi}{\cos \phi} = \frac{2W}{\cos \phi}$$

In tabulated form these stresses are shown below:

Stress	Normal Component	Parallel Component	Total Stress	Character
$2W$	$-2W \sin \phi$	$+2W \cos \phi$	$2W$	Applied Load
AB	0	$-\frac{3W}{\cos \phi}$	$\frac{3W}{\cos \phi}$	Compressive
BC	$+2W \sin \phi$	$+\frac{W \sin^2 \phi - W \cos^2 \phi}{\cos \phi}$	$\frac{W}{\cos \phi}$	Compressive
BD	0	$+\frac{2W}{\cos \phi}$	$\frac{2W}{\cos \phi}$	Compressive

Panel Point *D*.—Here the two unknown stresses are those in *DE* and *DC*. As *DE* makes with the horizontal an angle equal to that of *BD* with the horizontal, it must be equal to *BD*, because their horizontal components must be equal.

The vertical component of *BD* is obtained from the equation

$$\frac{\text{Vert. Comp. of } BD}{BD} = \cos\phi$$

$$\text{Vert. Comp. of } BD = BD\cos\phi = \frac{2\,W}{\cos\phi}\cos\phi = 2\,W$$

The stress in *DC* must be equal and opposite to the resultants of the other vertical components acting at point *D*. Hence

$$DC = -(-2\,W + 2\,W + 2\,W) = -2\,W$$

In tabulated form, the stresses are shown below.

Stress	Vertical Component	Horizontal Component	Total Stress	Character
$2\,W$	$-2\,W$	0	$2\,W$	Applied Load
CD	$-2\,W$	0	$2\,W$	Tensile
BD	$+2\,W$	$+2\,W\tan\phi$	$\frac{2\,W}{\cos\phi}$	Compressive
DE	$+2\,W$	$-2\,W\tan\phi$	$\frac{2\,W}{\cos\phi}$	Compressive

Panel Points *C*, *E*, and *F*.—The truss is symmetrical and symmetrically loaded and the stresses in one-half of it have been found. The stresses in the members of the remainder will equal those of the corresponding members in the half already considered.

Analytic Method by Sections.

1560. The solution by sections depends upon the principle that since every member or combination of members is in equilibrium, a portion remaining after another portion has been cut off by a plane will be in equilibrium if equivalent forces are substituted to take the places of the members cut off. The portion remaining is considered as a "free body," and forces, known or unknown as may be necessary, represent the stresses in the members severed by the section. This method is often called the "method of moments" because the stresses in the unknown members are found by a series of equations of moments. The center of moments is preferably taken at the junction of certain unknown members, so that their lever arms will be zero and the equation can be more quickly solved.

1561. Represent by Fig. 1561A the same roof truss with same loads as in paragraph 1559, and let it be required to determine the stress in the member AC by the method by sections.

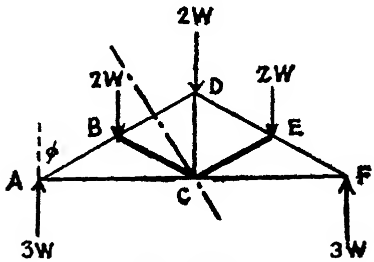


Fig. 1561 A

Pass a plane as shown, through the members BD, BC, and AC. Take the portion to the left as a "free body": it will be acted upon by the forces as shown in Fig. 1561B, of which the natures and intensities are not yet determined.

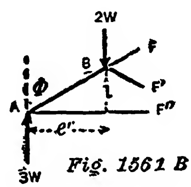


Fig. 1561 B

Taking moments around B, selected as center of moments because F and F' will have lever arms of zero, we obtain the equation of moments:

$$3W \times l' - 2W \times 0 + F \times 0 + F' \times 0 + F'' \times l = 0$$

$$F'' = \frac{-3W \times l'}{l} = \frac{-3W \times AB \sin \phi}{AB \cos \phi} = -3W \tan \phi$$

This value of F'' is the same as determined as the stress in AC by the method of concurrent forces. As it causes a negative moment, it produces tension at panel point A, which is also as determined for AC by the method of concurrent forces.

Method by Graphic Statics.

1562. **The Graphic Method** is the simplest and in most cases the quickest method of determining the stresses in a roof-truss; and it has, besides, the additional advantage of being applicable to any true truss form or any arrangement of loads. There is also less chance of making a mistake in the graphic method than in the method of numerical computations, as an error in the graphical analysis becomes evident because the lines do not meet, the polygon does not close. When the principles are understood, **stress diagrams** can be very quickly drawn without the aid of books or tables.

1563. The graphic method is founded upon the composition and resolution of forces as learned in mechanics, from which is derived the basic principle of graphic statics, namely:

If any number of forces acting at a point can be represented in magnitude and direction by the sides of a closed polygon taken in order, they are in equilibrium.

Applying this principle to the determination of stresses in roof-trusses, we find by graphics a number of closed polygons, which represent the equilibrium of forces at a panel point or on a portion of the truss, or on all of the truss.

1564. In the detailed work of drawing the graphical representations of the forces, it is not always necessary to draw every force; as a matter of convenience, parts of one polygon of forces join to the parts of one or more other polygons of forces, so that the principle of equilibrium of the polygon of forces is often not considered. But if the polygons were each completed, each would represent forces in equilibrium.

1565. **Representation of Forces.**—In the graphic method the intensity of force in terms of the unit of force is expressed by the length of a right line drawn to a scale whose unit of length represents a unit of force. The direction of a force is indicated by an arrow head written on its action line; if the force acts towards a material point, the arrow head is directed towards the point. If the force acts away from the point, the arrow head is directed away from the point. The complete figure of all the forces is called the **force polygon.**

1566. **Graphic Nomenclature.**—Before the stress diagram for a truss can be drawn, it is necessary to make a skeleton drawing of the truss, representing the central or median lines of the members. This diagram, called the truss diagram, should be drawn on the same sheet of paper as the stress diagram, for the convenience of drawing the latter. The truss diagrams should also have all of the loads which come on the truss indicated by arrows and figures.

After the truss diagram is drawn, it is convenient to letter it according to the method known as Bow's Notation, which allows a ready comparison of the truss diagram and the stress diagram, and also makes it easier to draw the stress diagram and immediately to determine the character as well as the magnitude of the stresses. The essential principle of this method is the lettering of each space on each side of every external force and of every member of the truss, so that on the truss diagram a truss member or external force is denoted by the letters on each side of it. In the stress diagram the same letters come at the ends of the lines representing the external forces and the stresses in the truss members. Capi-

tals are commonly used for the truss diagrams, and small letters for the stress diagrams.

1567. As in the analytic method, the determination of stresses consists in finding: (1) the external forces acting on the truss, the reactions being usually the forces which must thus be determined; (2) the stresses in each member.

Ordinarily, the force polygon of the external forces is first drawn and the reactions are determined. This force polygon is used as the framework for the stress diagrams of the different members of the truss. As there must be equilibrium at each panel point, the force polygon must close; and the direction of each stress can be determined by going around the perimeter of the polygon in proper order.

1568. **Problem.**—Find the intensity and action line of the resultant of the concurrent system of coplanar forces in Fig. 1568A.

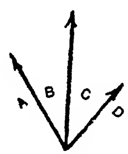


Fig. 1568 A

$AB = 10 \qquad BC = 13 \qquad CD = 8$

Solution.—From any point, as a (Fig. 1568B), draw a line ab parallel to AB and make it equal to ten linear units; from b, draw a line bc parallel to BC and make it equal to thirteen linear units; from c draw the line cd parallel to CD and make it equal to eight linear units; complete the polygon by drawing the line ad. If through the

Fig. 1568 B

common vertex of AB, BC, and CD, we draw a line parallel to ad, it will be the action line of the resultant of AB, BC, and CD, and its intensity will be the number of linear units in the length ad. This results from the principle of the parallelogram of forces,

since by construction ac must be equal and parallel to the resultant of AB and BC, and ad must be equal and parallel to the resultant of ac and CD. Following the perimeter around in proper order (ab, bc, cd, da), we see that the force with direction and intensity of da will establish equilibrium at the point.

1569. **Problem.**—Determine the resultant of the system of non-concurrent coplanar forces in Fig. 1569A, when the action lines of forces intersect within the limits of the drawing.

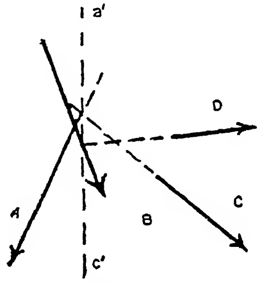


Fig. 1569 A

Solution.—Draw the force polygon (Fig. 1569B) in which ab, bc, and cd are parallel and equal to the forces AB, BC, and CD. Then ad gives the intensity and direction of the resultant.

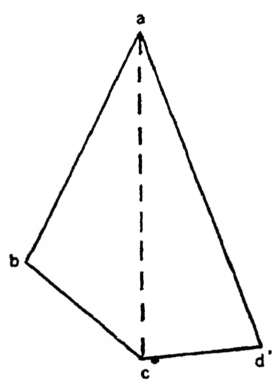


Fig. 1569 B

Produce the action lines of AB and BC until they intersect. Through the point of intersection draw their resultant, which will be parallel to ac. Produce this resultant until it intersects the action line of CD; their point of intersection will be one point on the action line of the resultant, and the resultant itself will be AD, drawn through this point parallel to ad.

1570. **Problem.**—Determine the closing force of the system of non-concurrent coplanar forces in Fig. 1570A, when the action lines of the forces do not intersect within the limits of the drawing.

Solution.—Draw the force polygon (Fig. 1570B) in which ab, bc, and cd are parallel and equal to the forces AB, BC, and CD. Then da gives the intensity and direction

of the closing force. Assume a point O, called the *pole* of the force polygon, at any convenient point in the construction plane, and draw the lines ao, bo, co, and do. These lines may be taken as the intensities and directions of a new system of forces in which ao and ob are together equal to and may be substituted for AB, bo and oc are together equal to and may be substituted for BC, etc. If from any point on the action line of

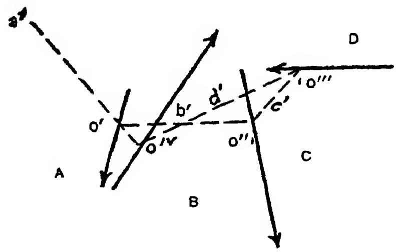


Fig. 1570 A

the force AB, as o', we draw the lines $o'a'$ and $o'b'$ equal and parallel to ao and ob, the original force AB may be replaced by its components $o'a'$ and $o'b'$. Finding the point o'' where the line $o'b'$ intersects the action line of the force BC, we may replace the force BC by its components $o''b'$ and $o''c'$, equal and parallel to bo and oc. Finding the

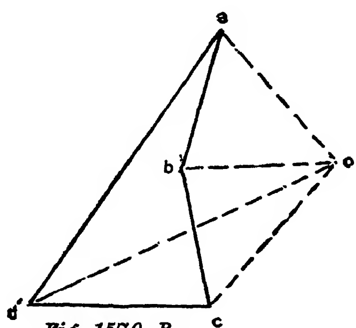


Fig. 1570 B

point o''' where the line $o''c'$ intersects the action line of the force CD, we may replace the force CD by its components $o'''c'$ and $o'''d'$, equal and parallel to co and od.

The resultant of these six components, $o'a'$, $o'b'$, $o''b'$, $o''c'$, $o'''c'$, and $o'''d'$, is the resultant of the original forces AB, BC, and CD.

The components $o'b'$ and $o''b'$ are equal and directly opposed; hence their resultant is zero. For the same reason, the resultant of $o''c'$ and $o'''c'$ is zero; hence the resultant of $o'a'$ and $o'''d$ must be resultant of the system. The resultant of the system, AD, and the closing force, DA, therefore act through the point o^{IV}, the point of intersection of $o'a'$ and $o'''d$, and is parallel to ad. This is evident from the force polygon, since ao and od are the components of ad in that polygon. The lines $o'a'$ and $o'''d'$ are in this case called the closing lines of the polygon $o'o''o'''o^{IV}$.

The polygon $o'o''o'''o^{IV}$ is called the **equilibrium polygon.**

1571. **Problem.**—Determine the reactions at the two points of support of the roof truss in Fig. 1571A, with live and dead loads as shown.

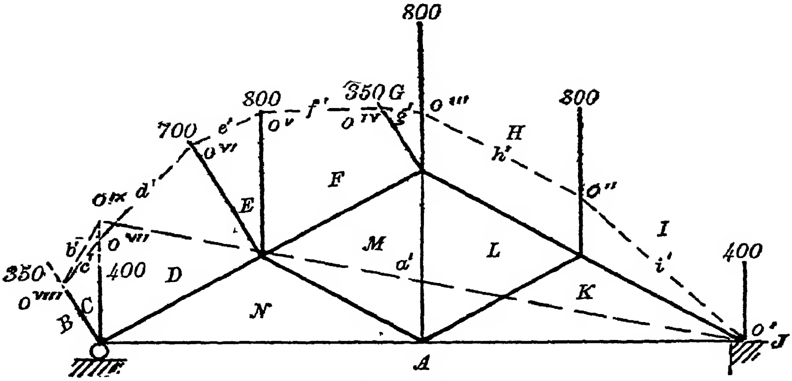


Fig. 1571 A

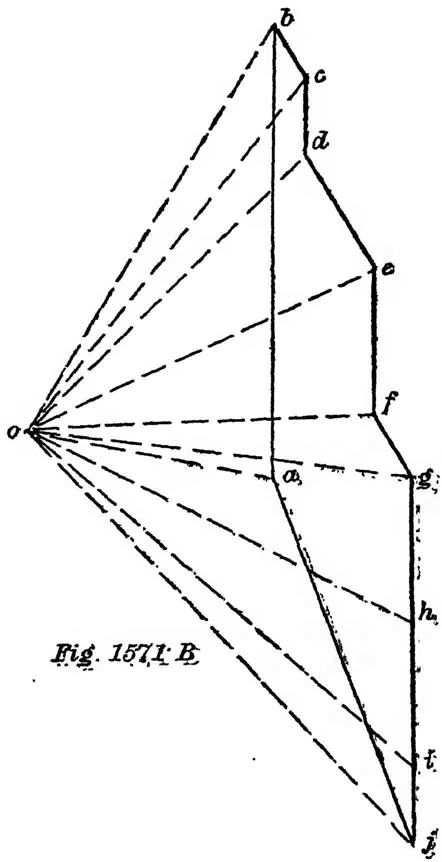


Fig. 1571 B

Solution.—As there is a roller under the left support, we know that the reaction at that support is vertical. The problem therefore consists in finding the intensity of the left reaction, and the intensity and direction of the right reaction.

Draw the lines *bc, cd, de,* etc., of the force polygon (Fig. 1571B), parallel to the external forces *BC, CD, DE,* etc. Assume the pole at the point *o,* and draw the component forces *ob, oc, od,* etc.

To construct the equilibrium polygon, we start at the right support because this is the only point known of the reaction *AJ,* and the equilibrium polygon must therefore pass through this point. At once it is evident that the component force, $o'j'$, is equal in length to zero in this particular equilibrium polygon because *IJ* and *JA* meet at the point where the line $o'j'$ is to be drawn from one to the other.

Draw the equilibrium polygon lines $o'i'$, $o''h'$, $o'''g'$, etc., parallel to the component force lines *oi, oh, og,* etc. The line $o^{\text{VIII}}b'$ intersects the force *AB* produced at o^{IX}.

Connect o^{IX} and o' by the equilibrium polygon line $o^{\text{IX}}a'o'$, and draw the component force line *oa* parallel to it in the stress diagram. This line *oa* meets the force line *ab,* drawn parallel to the force *AB,* at the point *a*: *ab* is therefore the intensity of the reaction *AB* at the left support.

Connect *a* and *j* in the force polygon: *ja* gives then the direction and intensity of the reaction at the right support.

1572. **Problem.**—Determine the stress in each member of the roof-truss in Fig. 1572A (same as 1571A), with forces and reactions as shown.

Solution.—Draw the force polygon *ab, bc, cd, de, ef, fg, gh, hi, ij* as shown in Fig. 1572B.

Panel Point *ABCDN*.—Starting at the left support, we have the forces *ab, bc,* and *cd* known. From *d,* draw *dn* parallel to *DN* of the truss diagram. Similarly, from *a* draw *an* parallel to *AN* of the truss diagram. Their point of intersection *n* gives the lines *dn* and *na* which represent the directions and intensities of the stresses in the members *DN* and *NA*. Following the perimeter of the polygon in the direction of the forces and stresses *AB, BC, CD, DN, NA,* we find that *DN* acts towards the panel point and its stress is therefore compressive, *NA* acts away from the panel point and its stress is therefore tensile.*

Panel Point *DEFMN*.—Here the lines *fm* and *mn* are drawn parallel to *FM* and *MN,* thus completing the stress diagram *defmn.* Following the perimeter of the polygon around in proper order, we find both of the members *FM* and *MN* to be in compression.

In the same manner, the stress diagram is drawn for each panel point. The stress is the length of the line in the stress diagram and the character of the stress is obtained by following the perimeter of the polygon in order as indicated by the known forces or known stresses.

UTILITIES.

Plumbing.

1573. The plumbing of a building consists of two complete systems:

a. The water-supply system.
b. The house-drainage system.

Although entirely distinct, to save space and to render all pipes more

* With colored chalk, compression is shown in red, tension in blue, forces in yellow.

easily accessible, the pipes for the two systems are generally placed close together in the building.

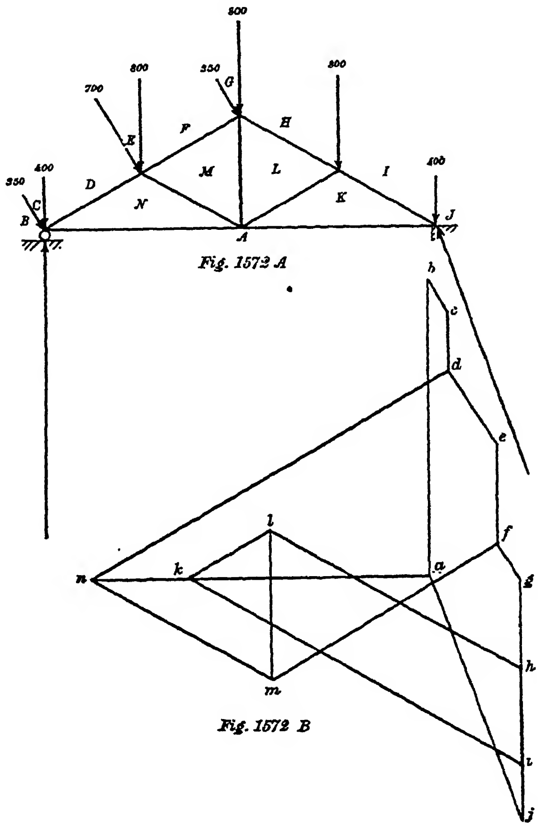


Fig. 1572 A

Fig. 1572 B

1574. **Water-supply System.**—The amount of water needed and consequently the size of the supply pipe is based upon the probable number of fixtures from which water will simultaneously be drawn. It is usually taken at about 40 gallons daily per person, but waste may triple the amount.

In residences and other buildings with comparatively few fixtures, the supply pipes should be proportioned to supply all the fixtures simultaneously. In hotels, apartment houses and like buildings, however, such ample provision is deemed unnecessary; and if provision be made to supply at the same time one fixture in each group within such a building, the pipes will be of sufficient capacity to meet all requirements. The largest pipe used to supply any fixture is $\frac{3}{4}''$ in diameter and the usual size $\frac{1}{2}''$ in diameter, though often $\frac{3}{4}''$ pipes are used throughout. Faucets

and cocks for $\frac{3}{4}''$ pipes seldom have an unobstructed waterway larger than $\frac{1}{2}''$ in diameter, while the waterway of $\frac{1}{2}''$ faucets and cocks seldom exceeds $\frac{3}{8}''$ in diameter; hence, if in all water pipes an allowance is made of the capacity of a $\frac{1}{2}''$ pipe for one fixture in each group, the system will be so proportioned that an adequate supply of water at low velocity will be had at all fixtures.

If the water is obtained by connection with the street main, the matter is further complicated by the fact that the largest tap permitted by most water companies is $\frac{3}{4}''$ in diameter. Hence, when larger sizes of service pipes are required, connections for them must be made either by inserting a special fitting or by means of a multiple service connection as shown in Fig. 1574A. With a moderate sized street main, connections

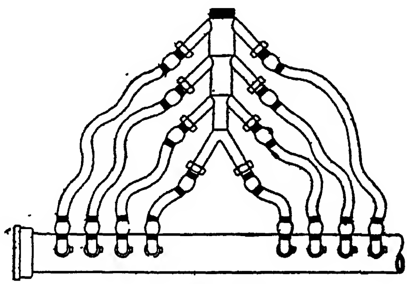


Fig 1574 A

for supply pipes over $2\frac{1}{2}''$ in diameter should be made by means of a special fitting; for all smaller sizes of service pipes when the water main is extremely large, connections may be made by means of a multiple connection. Galvanized iron is probably more extensively used than any other material for water supply pipes in buildings. Brass piping is preferable to galvanized iron or lead for conveying hot water, and is largely used in the better class of buildings.

1575. Whether water is obtained from a nearby tank or from the street main, the distribution should start from some central location. With such a system, all supply lines can be controlled by valves located at one end of the building; provision can then be made to drain the pipes when the water is shut off, a better distribution of water will be effected, and systematizing work will so simplify the construction that it can readily be understood and more cheaply installed. In dwelling houses, branches to the several fixtures or groups of fixtures may be taken from the hot and cold water supply mains at some convenient point in the kitchen. A suitable place for the grouping of valves, branches, and drains is over the kitchen sink. Another convenient place is back of

the kitchen range boiler as shown in Fig. 1575A. This system makes it possible to cut off quickly the supply of water from any branch that is out of order. In the installation of water supply systems in large buildings, the distributing manifold is usually located in the basement or

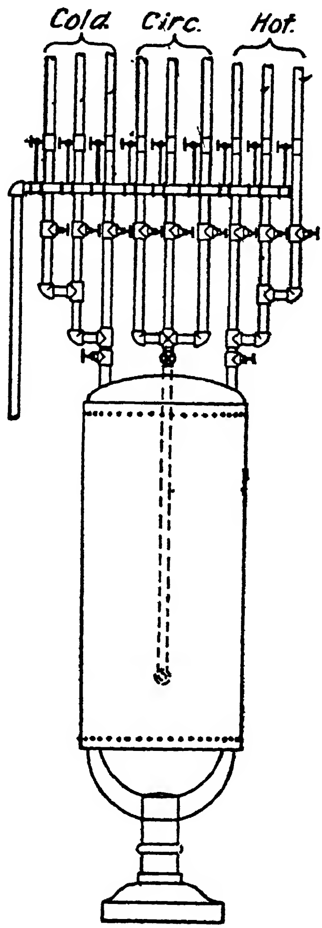


Fig. 1575 A
Water Distribution from
Range Boiler

cellar, and a separate line of supply pipes taken from the manifold to supply each rising line and separate group of fixtures in the building.

After leaving the distributing manifold, the hot and cold water pipes are usually together in the system.

Economy in the design of a building requires that the pipe outlets be located reasonably near each other. In dwellings the outlets are preferably over each other and over the kitchen outlets. Outlets in adjacent rooms are placed on opposite sides of the same partition. If more than one set of supply pipes is needed, the second set also has its outlets over each other or on opposite sides of a partition. Proper design of the water supply system of a building will make it unnecessary later to correct this error by running exposed pipes inside a room.

1576. **House-drainage System.**—The general principles governing house drainage are to insure the necessary slopes to pipes, to make the system of piping simple, short, and readily accessible, and to provide for their ventilation and for the removal of obstructions.

The supply of water must be sufficient in volume and pressure to flush all the fixtures at the same time. The fixtures must be of porcelain enamel or some other clean non-absorbent material. The pipes must be such as will neither corrode easily nor be affected by sudden changes of temperature.

1577. The general system of house drainage (Fig. 1577A) consists of one or more vertical soil and waste pipes which receive the wastes

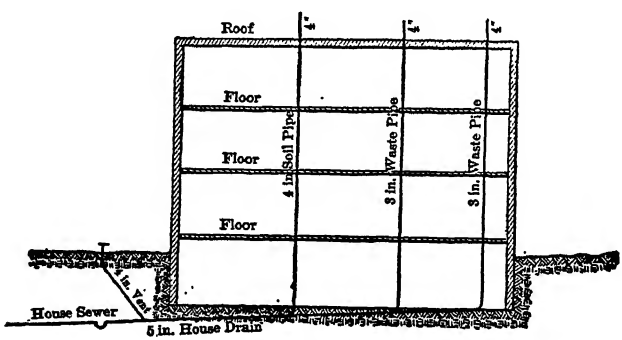


Fig. 1577 A

from the fixtures and which empty into the house-drain. The **house drain** usually is a heavy cast-iron pipe with leaded joints. For ordinary houses, it is 5″ in diameter and has a fall of at least 1″ in 4′. The house drain is provided with cleaning holes at intervals, and a short distance beyond the exterior walls of a house empties into a stoneware pipe called the **house sewer,** which is drained by the street sewer.

A **soil pipe** is a vertical pipe about 4″ in diameter, which receives the discharge of a water-closet. It is a heavy metal pipe, usually cast-iron, which extends from a house drain to a point about 2′ above the roof.

At its base it is supported on a masonry pier; its upper end either is left open or is closed merely by a wire cage to prevent the ingress of materials which might obstruct it.

A **waste pipe** is a vertical pipe which receives waste from other sources than water-closets. It is supported below and terminated above like the soil pipe. The diameter of a waste pipe is usually 3″, but the length above the roof is enlarged to 4″ in diameter to prevent its being closed by frost. The pipes which carry the waste from the fixtures to the soil and waste pipes are called **branches.**

1578. **Traps.**—A trap is any contrivance which, inserted in a pipe, will automatically prevent a passage of air or gas. The ordinary form of trap is a U-shaped pipe filled with liquid. (Fig. 1578A.) The depth

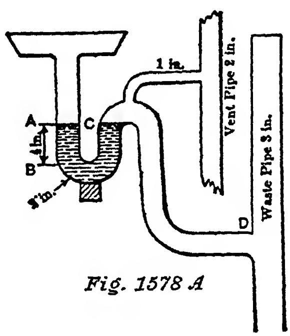


Fig. 1578 A

of the seal is the difference of level of A and B, which does not ordinarily exceed 4″. It is evident that no air can pass the trap so long as the U-shaped pipe is filled above the level of B. The water seal may be removed in one of the following ways:

a. By evaporation. This occurs in houses which are unoccupied for some time. To prevent this, the water may be replaced by some oily liquid like glycerine, which is not easily evaporated, or the inlet may be plugged if the fixture is not too frequently flushed.

b. By capillary attraction. If a piece of waste or cloth is caught in the water of the trap and the other end extends into the branch CD, the water will be slowly drawn into the branch CD by the capillary attraction.

c. By difference of pressure. If the pressure due to air or gas is enough greater in CD than at A, the water will be forced out of the tube AB and the air or gas will escape.

d. By siphonage. If the pipe from C to D runs full, a siphon will be formed which will drain the trap. It can be prevented by making an air inlet above C as shown in the figure. The vent pipe to which the

air inlet is connected is carried to a point above the roof, or is connected to a soil or waste pipe above the highest branch.

Ventilation.

1579. In a properly designed heating system, provision must be made for raising to room temperature the cold fresh air required for ventilation. A study of heating must therefore include a study of ventilation, and, in practice, it will be found that the apparatus used for ventilation is always a part of the heating system.

1580. **Amount of Ventilation Required.**—Authorities differ on the amount of air required for ventilation. However, in practice, the minimum quantity of fresh air per occupant that should be furnished for various types of buildings, under average conditions, may be taken as follows:

Schools and churches................	1800 cu. ft. per hour
Theaters and auditoriums............	2000 cu. ft. per hour
Hospitals..........................	4000 cu. ft. per hour

When objectionable gases or contagious diseases have to be considered, the values given above must be increased considerably.

1581. **Ventilating Methods.**—**Natural ventilation** includes all ventilation produced without positive means for admission and escape of air. In buildings of ordinary construction, any room will receive some ventilation whenever its temperature is above or below that of the surrounding air, by the leakage around the doors, windows, etc. In a room heated by an ordinary stove, it is found that the air will be changed from one to three times an hour, even when no air is purposely admitted for ventilation. For private residences, especially if the principal rooms are provided with fireplaces, heating by hot air or indirect radiation will generally afford sufficient ventilation, and only in exceptional cases will it be necessary to resort to mechanical devices for the admission and escape of air.

Systematic ventilation is necessary in buildings in which people congregate, such as churches and theaters; and provision must be made for the admission and escape of air through flues or definite openings and for power for moving the air. (See Fig. 1581A.) The air may be moved by expansion due to heat or by fans. The latter method is the one most used since it affords ventilation in summer as well as winter. Ventilation by fans may be accomplished by forcing the air into a room or exhausting air from it. The exhaust system is not recommended. The quantity of air passing through a flue or opening is measured by multi-

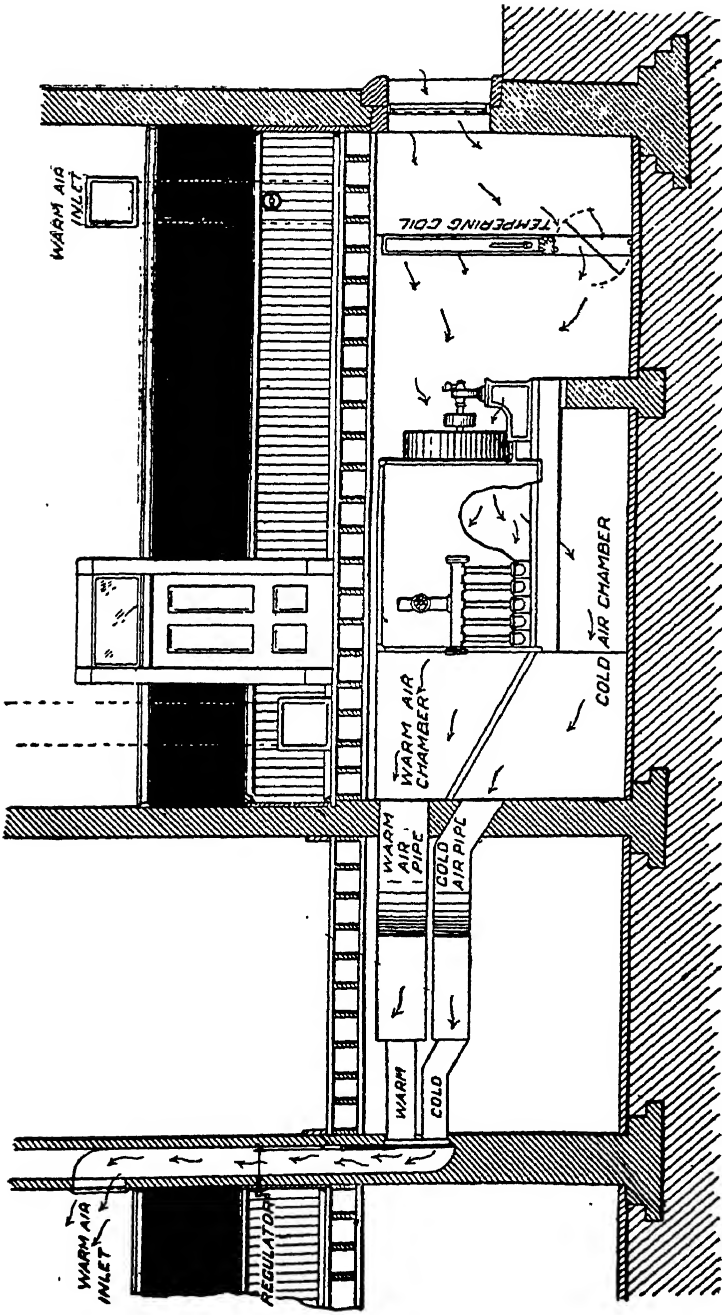


Fig. 1581 A Warming System Combining Tempering Coil, Disc Fan, Second Heater, and Double Ducts

(American Technical Society)

plying the net area of the opening by the velocity of the air. In order to prevent objectionable drafts, the velocity of the air through an inlet register for a room should not exceed five feet per second.

Heating.

1582. **Amount of Heat Required.**—In order to maintain standard room temperature, the heating apparatus must supply heat to replace that lost through the walls, floors, and ceilings, and, in addition, the heat necessary to warm the cold fresh air used for ventilation. Heat is lost by conduction through walls, floors, and ceilings, and by diffusion through cracks around doors, windows, etc.

The **heat lost by conduction** can not be determined exactly. It is a function of the difference in temperature between the inside and outside air and depends upon the thickness and character of the walls, their surface areas, and the presence or absence of air spaces. Carpenter's formula, used extensively and successfully for estimating the loss of heat by conduction from a building is as follows:

$$H = \left(A_o + \frac{A_w}{4}\right)(T_r - T_o) \qquad (1582A)$$

In which:

H = Amount of heat lost in B.T.U. per hour.
A_o = Area of windows and outside doors in square feet.
A_w = Area of exposed wall surface in square feet.
T_r = Room temperature in degrees F.
T_o = Outside temperature in degrees F.

In computing exposed wall surface (A_w in the formula), ceilings and partitions adjacent to unheated rooms should be taken from 30 per cent to 50 per cent exposed.

The **heat lost by diffusion,** by leakage through cracks around doors, windows, etc., is even more indeterminate than that lost by conduction, since it depends upon the care with which the building was constructed and upon the exposure. Expressed in B.T.U., the heat lost in escaping air is equal to the weight of the air lost times its specific heat times the difference in temperature between the room and the outside. It takes $\frac{1}{55}$ of a B.T.U. to raise the temperature of one cubic foot of air, at ordinary pressure, one degree F. Using 0.02 for this quantity, Carpenter gives the following formula for the heat lost by leakage, or diffusion:

$$H = 0.02\, nV\ (T_r - T_o) \qquad (1582B)$$

in which

H = Amount of heat lost in B.T.U. per hour.
n = Number of times air in room is changed per hour.

V = Volume of room in cubic feet.
T_r = Room temperature in degrees F.
T_o = Outside temperature in degrees F.

For the direct heating of residences, n should be taken as 3 for halls, 2 for first floor rooms, and 1 for rooms on the upper floors.

1583. **Heating Fresh Air Used for Ventilation.**—Where the heating apparatus furnishes fresh air for ventilation, the amount of this air can be computed. Its temperature on entering the room must be high enough so that on cooling to the desired room temperature it will give up sufficient heat to supply that lost by conduction through the walls.

The following table, giving the properties of dry air, will be useful in making calculations:

PROPERTIES OF DRY AIR

Temp. degrees F.	Cubic feet per pound	B.T.U. per pound above 0° F.
0	11.58	0.0
70	13.35	16.90
80	13.60	19.32
90	13.86	21.74
100	14.11	24.16
110	14.36	26.58
120	14.62	29.00
130	14.88	31.42
140	15.13	33.85

1584. **Contract Requirements.**—Most contracts for heating plants contain specifications which state that the apparatus shall be capable of heating the house (or rooms) to a stated temperature when the external temperature is · · · ° F. The external temperature specified is the lowest that is likely to prevail for several consecutive days in the locality in question. The following desirable room temperatures are quoted from Greene's Elements of Heating and Ventilation:

Bathrooms	72° F.
Hospitals	72° F.
Shops for light work	64° F.
Entrance corridors	60° F.
Gymnasiums and workshops	55° F.
Houses, Offices, and Schools	70° F.
Lecture halls	66° F.

1585. **Systems of Heating.**—Leaving stoves and fireplaces out of consideration, the systems ordinarily employed for heating may be classified as follows:

a. Hot air.
b. Steam.
c. Hot water.

1586. **Hot Air Systems.**—In a hot air system, heated air from the furnace is introduced through leaders, stacks, and registers into the room. This air is at a higher temperature than the room, and, in flowing across the ceilings and down by the walls, heat is abstracted until it is eventually cooled to the desired room temperature. Fresh warm air from the furnace then forces the air that has been cooled to room temperature out of the room through cracks, fireplaces, etc. A heat balance may therefore be written as follows: the heat given up by the entering air equals the heat lost by conduction.

If we assume that the volume of the entering air is equal to that lost by diffusion, and solve the heat balance equation for the temperature of the entering air, it is usually found that this temperature is too high for comfort. Consequently in practice a reasonable value for this temperature is assumed and the heat balance equation then solved for the volume of the entering air. Carpenter recommends 120° F., and Greene 130° F. as the proper values for the temperature of the entering air.

A hot air **furnace** (Fig. 1586A) may be described as a stove surrounded by an air chamber. The air to be heated enters at the base of the air chamber, passes over the hot surfaces of the stove, and is then conducted by hot air pipes to the various rooms. The cold air supply for the furnace comes from the outside, through a suitable screen or filter, into a flue connected with the air chamber of the furnace at its base. This flue is called the " cold air box," and most authorities recommend that its area of cross-section be made equal to the sum of the areas of cross-section of all the leaders from the furnace. In addition to this supply of air from the outside, it is customary, in cold climates, to install an air duct leading from the hall down to the base of the furnace air chamber. In very cold weather the supply of cold air from the outside is partially, or entirely, shut off from the furnace and warm air from the house fed back into it by means of the air duct from the hall. This procedure, however, is apt to result in poor ventilation unless care is exercised in regulating the proportions of fresh and stale air admitted to the furnace. The capacity of a furnace must be sufficient to furnish the B.T.U. necessary to raise the total volume of air required for heating the house from outside temperature to the temperature selected for it

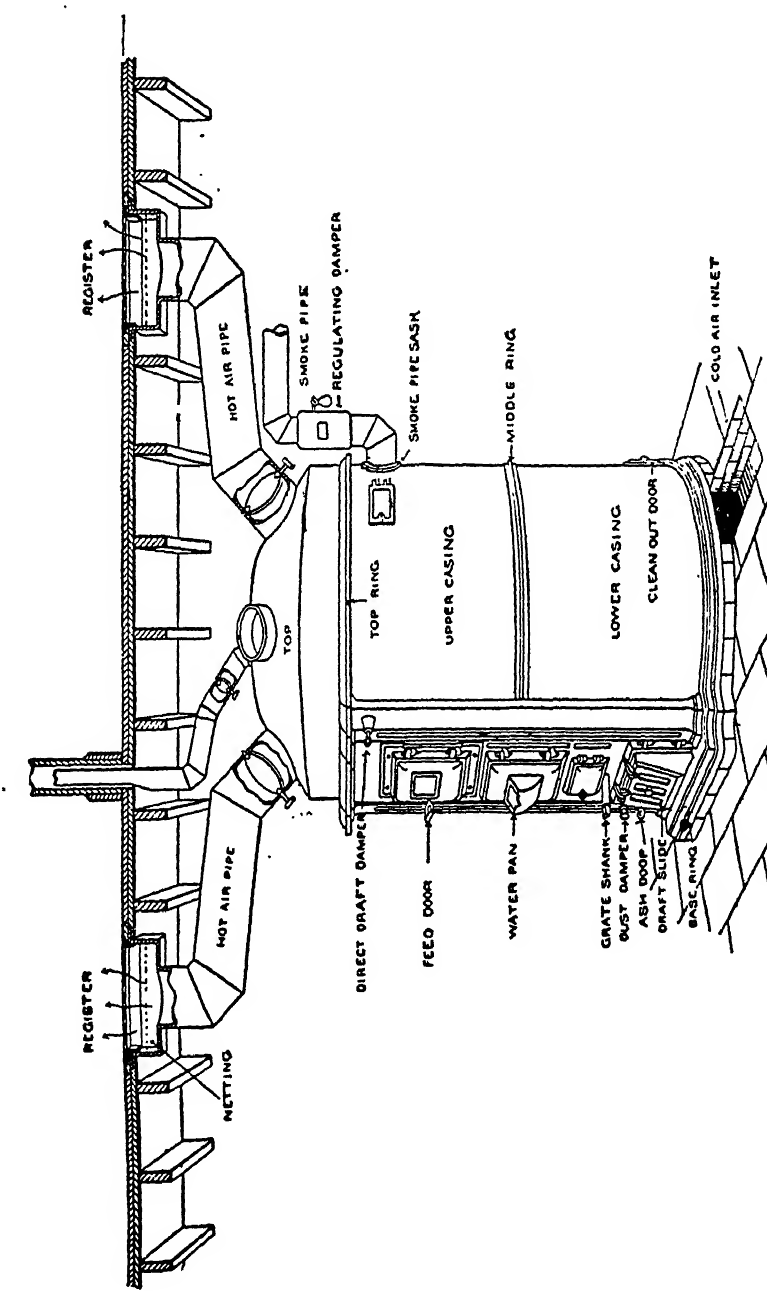


Fig. 1586 A General Arrangement of Details of a Hot-Air Furnace, with Connecting Pipes

(American Technical Society)

when it enters the rooms. The ordinary hot air furnace will burn about 3 pounds of coal per square foot of grate area per hour, and has an efficiency of about 65 per cent. The heating value of coal ordinarily used may be taken at 13,000 B.T.U. per pound.

1587. **Hot Air Pipes and Registers.**—The force which causes hot air to flow from furnace to room results from the difference in densities of the cold air outside and the warm air inside the furnace and pipes. An equation can be written for this flow, but in practice it is customary to assume velocities of flow in hot air pipes which have been shown by experience to give satisfactory results. For residence heating, Carpenter gives the following values for these velocities:

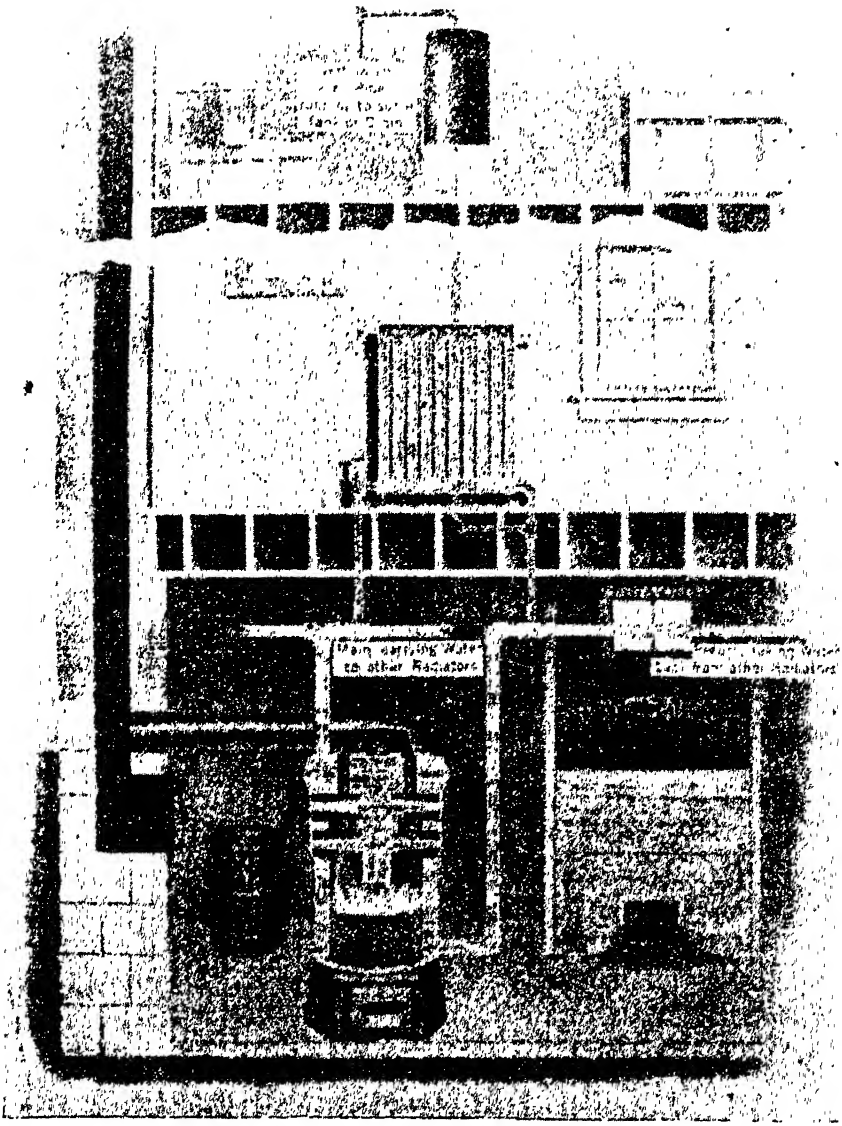
Pipes leading to 1st story rooms	3 feet per second
" " " 2nd " "	5 " " "
" " " 3rd " "	6 " " "

Using the above values, and knowing the volume of air to be supplied each room, the computation of the proper size for air pipes is a simple matter. The net area of registers is determined by the velocity with which heated air can be introduced into the rooms without causing excessive drafts. Carpenter recommends a velocity, at the registers, of about 3 feet per second.

1588. **Vent Ducts.**—Generally, in a hot air system of heating, a greater volume of air is introduced into the room than is lost through the cracks. Unless an exit is provided for this excess of air a slight pressure will build up in the room, the velocity of flow to the room will be decreased, and insufficient heat will be supplied to maintain the temperature at the desired point. For this reason, it is good practice to provide vent ducts connecting each room, by registers near the floor, with either the attic or the outside. Open fireplaces, although very inefficient as a means of heating, are valuable adjuncts to any system of ventilation, since large quantities of air are drawn through them and discharged up the chimney. Consequently in rooms having fireplaces there is no need for vent ducts.

Advantages and Disadvantages.—A hot air heating system is cheap to install, has a low cost of maintenance, and is not hard to manage. Its operating cost is little, if any, greater than that of hot water or steam system of equal capacity. Unless the installation is first class in every respect and the fire is handled carefully, overheating of the air, mixture of gases of combustion with the air, and poor distribution of heat are likely to take place.

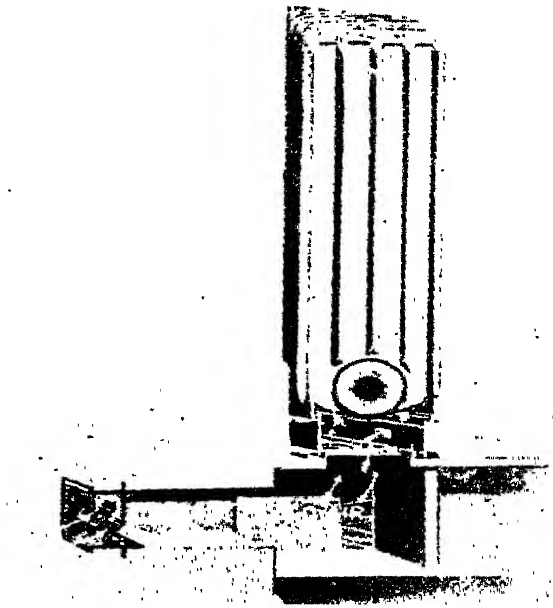
1589. **Steam and Hot Water Systems.**—A steam, or a hot water, heating plant consists essentially of the radiators, the boiler, and the



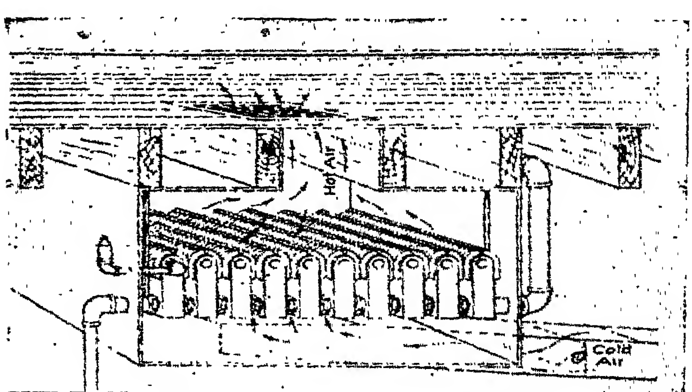
Principle of Hot Water Heating Illustrated by Transverse Sectional View
Showing Boiler, Radiator and Expansion Tank

American Radiator Company

Fig. 1589 A



Direct-Indirect Method of Warming taking a Fresh Air Supply from Outside and Passing it Upward
American Radiator Company
Fig. 1589 B



Steam Heater Placed under Floor Register — Indirect System
Fig. 1589 C

system of piping connecting the former with the latter. Steam, or hot water, from the boiler is circulated through the piping and radiators; in these the steam condenses giving up its latent heat and the water gives up some of its heat, thus warming the rooms. In the usual hot water installation, the boiler, pipes, and radiators are kept full of water at all times, an expansion tank being provided to compensate for the increase in volume of the water when heated and to prevent explosions in case of generation of too much steam (Fig. 1589A).

Radiators are generally made of iron in the form of pipes or castings, the assembly being such as to permit a free circulation of air around the outside, and of steam or hot water inside. The thickness of metal in radiators should be the minimum necessary for strength. Radiators may be divided into three classes:

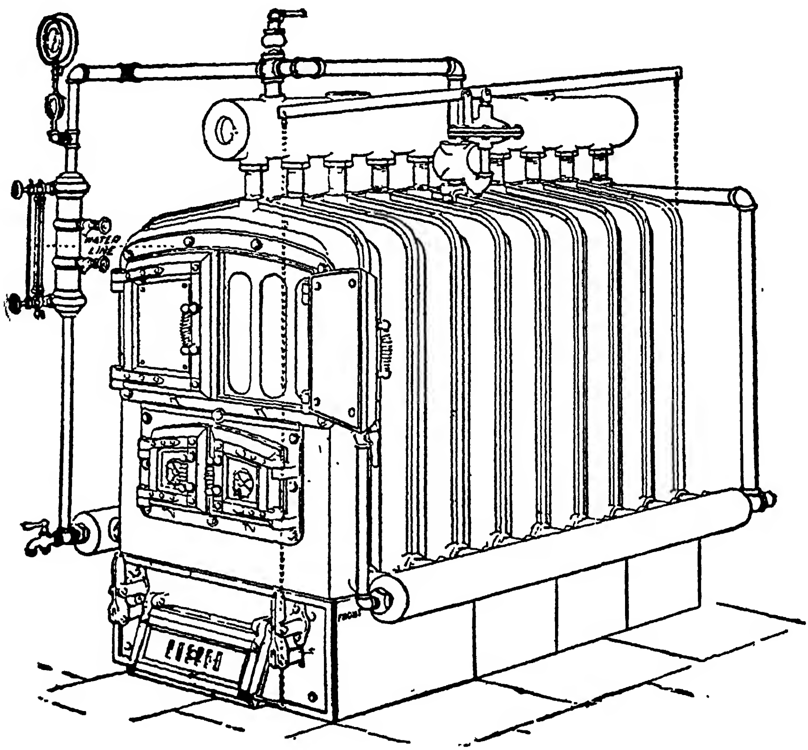
a. Direct.
b. Indirect.
c. Direct-indirect.

Direct radiators are those placed in a room to warm the air already there. Such an installation is the cheapest, and the one usually found in residences where no special provision for ventilation is necessary. Indirect radiators are those installed in a box, or casing, outside the room to be warmed. (See Fig. 1589C.) The box, or casing, is provided with an inlet for fresh air from outside the house, and a hot air flue leading to the room. Indirect radiation is used only in connection with systematic ventilation where fans are employed to force the heated air into the rooms from which it escapes through ventilating ducts. Direct-indirect radiators (see Fig. 1589B) are placed within the room to be heated, but fresh air is admitted to the room in such a manner that it first comes in contact with the radiators, is heated, then circulates around the room, and finally passes out through cracks, fireplaces, or ventilating ducts. Direct-indirect radiation requires no power for the flow of air into the rooms, and will ordinarily provide ample ventilation for living rooms and offices.

1590. **Capacity of Radiators.**—Radiators are measured by the amount of heating surface coming in contact with the air. The heating surface, measured in square feet, for different types and sizes of radiators may be found in tables published in handbooks, or in catalogues of manufacturers. The amount of heat, or the B.T.U. given off per square foot of radiating surface per hour, varies with the difference in temperature between the steam, or hot water, inside the radiator and the air in the room, also with the class of radiator, its size, shape, and type of construction. Tables for computing this value for different types of radia-

tors under varying conditions may be found in handbooks and catalogues. For ordinary house heating, it may be assumed that a steam radiator will give off 250 B.T.U. per hour per square foot of radiating surface. The capacity of hot water radiators is about one-half that of steam radiators.

1591. **Boilers.**—Although tubular boilers are still used extensively in large central heating plants, they have, for the most part, been superseded by sectional cast-iron boilers (see Fig. 1591A) for the heating of



Common Type of Cast-Iron Sectional Boiler. Note Headers at Sides and Top Acting as Drums (American Technical Society)

Fig. 1591 A

residences and public buildings. There are a great many types and makes of sectional cast-iron boilers. Most of them are capable of being used for both steam and hot water heating. They are usually rated by the manufacturers according to the amount of direct radiating surface they will supply. In practice, dependence must be placed largely upon the manufacturer's rating, but in arriving at the required rating for a

boiler, the total actual direct radiating surface that it will have to supply should be increased about 50 per cent as a factor of safety. In making computations, one square foot of indirect radiating surface should be taken as equal to one and three-quarters square feet of direct radiating surface.

1592. **System of Piping.**—For steam heating, the systems of piping usually employed are the ordinary one-pipe system and the two-pipe system. In the former, but one connection is made to each radiator,

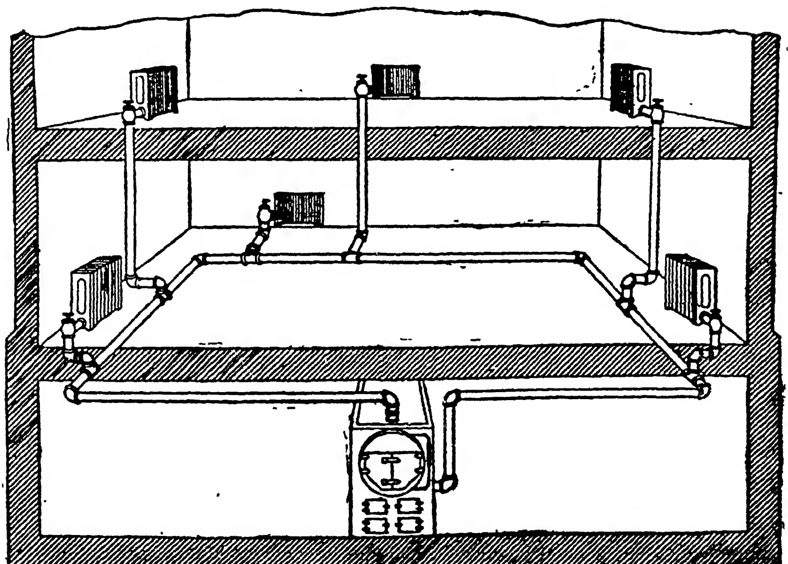


Fig. 1592 A **Arrangement of Piping and Radiators in "One-Pipe Circuit" System**
(American Technical Society)

this connection serving both as an inlet for the steam and as an outlet for the water of condensation (Fig. 1592A). In the latter, there is a supply pipe and a return pipe for each radiator (Fig. 1592B). The two-pipe system is expensive, and not used generally in steam heating except for indirect radiators which must always have two connections in order to function properly.

In hot water heating, although one-pipe systems may be used, it is considered the best practice to have a supply pipe and a return pipe for each radiator. Rules and tables for computing the size of pipe, etc., for both steam and hot water heating will be found in handbooks.

1593. **Advantages and Disadvantages.**—On account of the larger radiating surface and the more elaborate system of piping required for

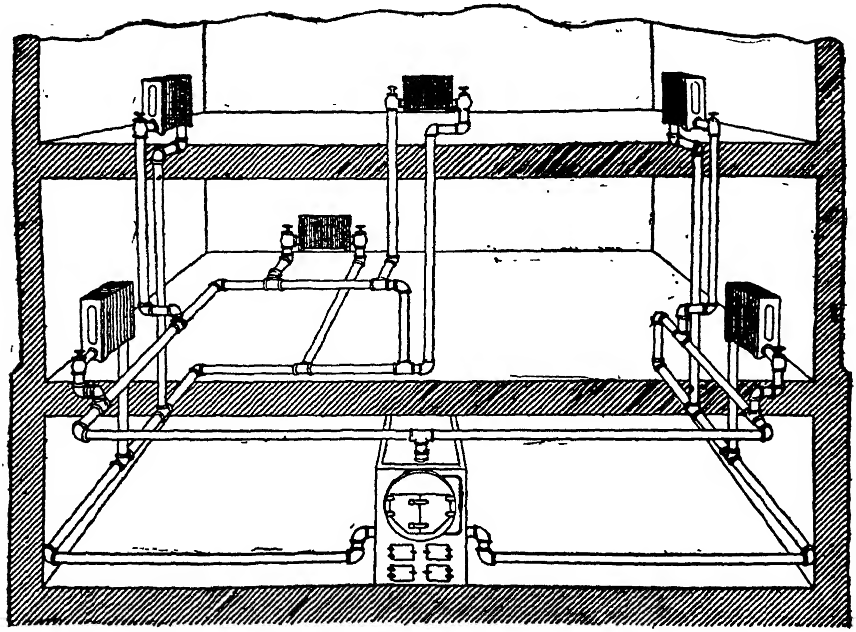


Fig. 1592 B Arrangement of Piping and Radiators in "Two-Pipe" System

(American Technical Society)

a hot water system, its first cost is usually about one-third greater than that of a steam system of the same size. In cold weather, when heating plants must be run to full capacity, steam and hot water systems have about the same operating cost, and give equally satisfactory results, but in moderate weather the hot water system consumes less fuel and affords a better regulation of heat. A building can not be heated as quickly by hot water as by steam, and it is more difficult to secure circulation in a large hot water plant than in a large steam plant. Hot water radiators require more space than steam radiators, and can not ordinarily be used for indirect radiation without danger of freezing. Steam is therefore generally employed for heating buildings provided with systematic ventilation.

In selecting a heating plant for residences there must be considered the size and type of building, the climate, the first cost, and the cost of operation. The latter, for any heating system, will depend largely on the operator. As a general rule it is believed that hot air systems are best for small dwellings, hot water (or steam) systems for large residences or apartments.

Large public buildings, hospitals, etc., as a rule, require systematic ventilation which ordinarily necessitates some form of steam heating.

If the buildings are in groups, it is the best practice to heat them from a central plant.

Lighting.

1594. Electric lighting of buildings is in general use. The use of electricity is desirable in the following cases:

Units less than 60 candle power required.
Lamps in inaccessible positions.
Lamps lighted at infrequent intervals.
Lamps placed very close to ceiling (12″ or less).
Non-rigid fixtures.

However, gas is still extensively used for lighting, especially where it is also used for other purposes in the same building. From the hygienic point of view, there is little to choose between gas and electricity for lighting. Contrary to the general opinion, experiments show that gas lighting improves the air for breathing purposes.

1595. There are three general systems of illumination, namely:

a. Direct Lighting. A system is designated as "direct" when more than one-half the light reaches the area to be illuminated by coming directly from the light-source, without being reflected from the ceiling or walls. It is the most efficient system, was the first to be used, and is still the most common. The color of the walls or ceiling has less effect in this system than in others.

b. Indirect Lighting. A system is designated as "indirect" when all the light is thrown first on the ceiling and walls, and reflected from these to the surface to be illuminated. Light finish must always be used on walls and ceiling with this system. The efficiency is usually lower than that of a direct system.

c. Semi-indirect Lighting. This system throws most of the light to the walls and ceiling, but allows a small percentage to be diffused through the reflector straight to the area to be illuminated. This system is rapidly coming into favor.

1596. The following table shows the amount of gas or of electric power required to illuminate rooms for various purposes:

Class of service	Cubic foot of gas per square foot per hour	Watts per square foot per hour
Auditorium	0.045	1.2
Factory (general illumination)	0.015	0.5
Hospital (operating room)	0.145	3.7
Hotel (restaurant)	0.065	1.6
Library (reading room)	0.055	1.4
Office (banking and accounting)	0.06	1.5
Residence (bedroom)	0.012	0.3
Residence (dining room)	0.038	0.9
Residence (living room)	0.038	0.9
School (class room or business colleges)	0.058	1.4

1597. **Gas Lighting.**—Ordinarily wrought-iron pipe, such as is used for steam or water, is suitable and proper for all kinds of gas. Galvanized malleable-iron fittings are superior to plain iron. The coating of zinc on the inside and outside effectively and permanently covers all blow-holes, makes the work solid and durable, and avoids the use of perishable cement. Before the pipe is placed in position, it should be looked through and blown through. It is not infrequently obstructed, and this precaution will save much damage and annoyance. The size of the pipe decreases (Fig. 1597A) from the main riser to the extreme outlet. Gas company rules generally require that no house riser shall be less than $\frac{3}{4}''$, no house pipe less than $\frac{3}{8}''$, and no gas range pipe less than $\frac{3}{4}''$. Flow of gas is reduced, as in the case of water, by friction, bends, and elbows in pipes. Tables are used, showing the number of inlets and distances which can be supplied with gas pipe of certain sizes. As shown in Fig. 1597A, the gas pipes may vary from about $1\frac{1}{4}''$ at the gas riser to $\frac{3}{8}''$ at the extreme outlet.

1598. **Electric Lighting.**—The electric wires should all be carried in conduits, and it is so required in fireproof buildings. The two conduits in general use are: (1) Lined mild-steel pipe; (2) flexible armored circuit. Whenever branches are taken off, junction boxes should be used.

The lamps are ordinarily placed in parallel, two-wire or three-wire systems. The voltage in common use is 110 volts at the incandescent lamp; the current is direct or alternating, the latter being used in the majority of cases in modern buildings. The National Board of Fire Underwriters issues tables showing the current-carrying capacity of wire and cables, which must be followed as a preliminary to insurance.

1599. **Problem.**—A dwelling house of wooden construction measures $30' \times 50'$ around the outside; it has 2 stories each 8′ in height; the surface of the windows and doors amounts to $\frac{1}{5}$ the total exposed surface of the building. The attic and cellar are unwarmed. If 8000 B.T.U. are utilized from each pound of coal burned in the furnace

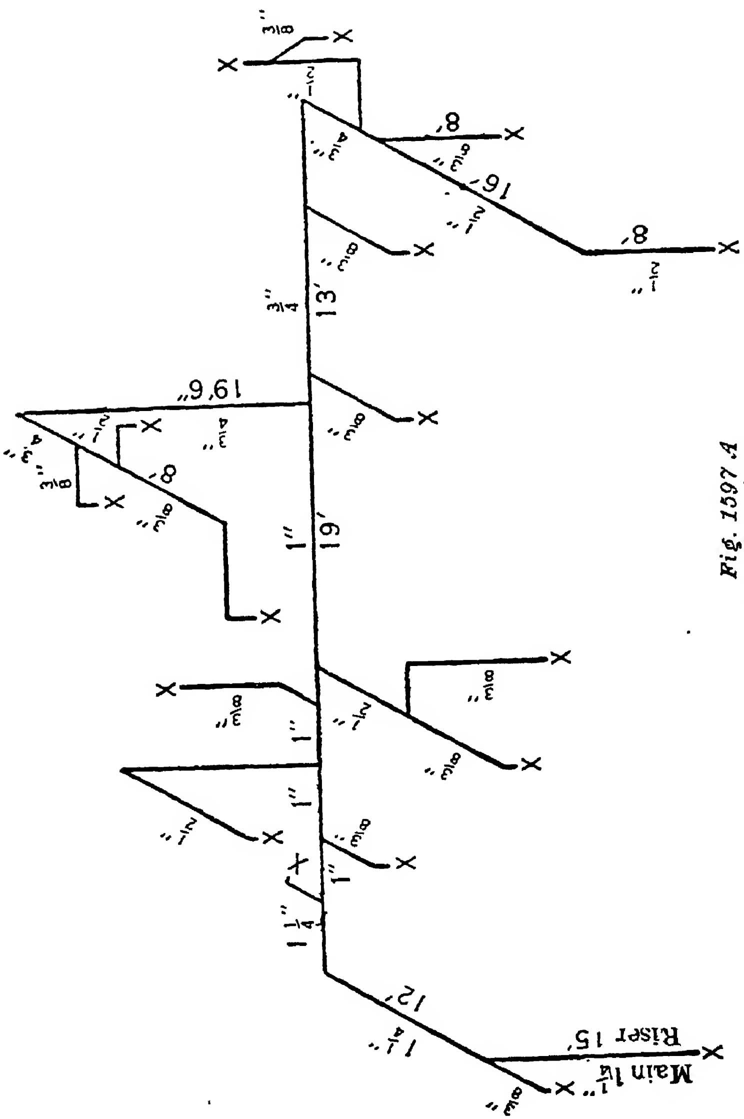


Fig. 1597 A
Diagram of Gas-piping

how many pounds will be required per hour to maintain a temperature of 70° when it is 20° above zero outside?

Solution.—Total exposure, $160 \times 16 = 2560$
Windows and doors, $\frac{1}{5} \times 2560 = 512$
Net wall exposure, $2560 - 512 = 2048$
Allowance for unheated attic, 30 per cent of surface, $= 0.30 \times 30 \times 50 = 450$
Temperature difference $= 70 - 20 = 50°$
Using conduction formula,

$$H = (A_o + \tfrac{1}{4} A_w)(T_r - T_o)$$
$$= (512 + \tfrac{1}{4} \times 2048 + \tfrac{1}{4} \times 450)\ 50 = 56{,}825 \text{ B.T.U.}$$

Using diffusion formula, $n = 2$ for 1st floor, $= 1$ for 2nd floor,

$$H = 0.02\, nV\, (T_r - T_o)$$
$$= 0.02\ (2 \times 30 \times 50 \times 8 + 1 \times 30 \times 50 \times 8) \times 50 = 36{,}000 \text{ B.T.U.}$$

Adding the two, the sum total of heat to be supplied is 92,825 B.T.U.

With coal producing 8000 B.T.U. per pound, the total number of pounds needed will be $\frac{92{,}825}{8000} = 11.6$ pounds.

For more detailed information see:

Theory of Structures, Spofford.
Foundations of Bridges and Buildings, Jacoby and Davis.
Roofs and Bridges, Merriman and Jacoby.
Tredgold's Carpentry.
Cyclopedia of Architecture, Carpentry, Building.
Concrete Engineer's Handbook, Hool and Johnson.
Treatise on Masonry Construction, Baker.
Cyclopedia of Civil Engineering.
Civil Engineering, Fiebeger.
Architect's and Builder's Pocket Book, Kidder.
International Library of Technology.

PROBLEMS.

P. 1501. A masonry wall has a footing 3′ wide. At a certain point on this wall a concentrated load, which would independently require a footing whose area is 16 square feet, is placed on the wall in such a way that its center of gravity is 8″ from the center line of the wall. Determine the dimensions of the enlarged footing necessary to support both the wall and the concentrated load at this point, and also how far the edges of the enlarged footing will be from the center line of the wall.

P. 1502. The soil under a certain wall will support 3 tons per square foot. At the end of the wall a concentrated load of 31 tons is placed on the center line of the wall; the wall has a total load of 9 tons per running foot in addition to the concentrated load. Find the dimensions of the enlarged footing at the end of the wall.

P. 1503. The footing for a wall is 8′ wide, the pressure at one edge is 20 pounds per square inch and at the other edge is 8 pounds per square inch. What is the mean pressure, and where is the center of pressure?

P. 1504. A rectangular wall of masonry is 20′ high, 40′ long, and 6′ thick; on one end of this wall is built a solid masonry tower, 6′ square and 40′ high above the top of the

wall. Weight of masonry, 150 pounds per cubic foot. Find maximum pressure per square foot on base of structure.

P. 1505. A masonry column 100′ high, of square cross-section, 10′ by 10′, is subjected to a wind pressure normal to one side of 30 pounds per square foot. Find the greatest unit pressure on the base. Masonry weighs 150 pounds per cubic foot.

P. 1506. A wrought-iron bar 18′ long and $1\frac{1}{2}''$ in diameter is heated to 400° F.; nuts on its ends are then screwed up so as to bear against the walls of a house which have fallen away from the perpendicular. Find the pull on the walls when the bar has cooled to 300° F.

P. 1507. A wooden floor beam, 20′ long, of rectangular cross-section 3″ wide, is fixed at one end and supported at the other, and bears a uniformly distributed load of 60 pounds per linear foot. What must be the depth of the beam in order that the greatest deflection shall not exceed $\frac{1}{360}$ of the span?

P. 1508. A steel I-beam, 20′ long, projecting from a wall, bears a uniformly distributed load of 200 pounds per linear foot. Select the beam that will safely bear the load.

P. 1509. Select a latticed channel bridge column 14′ long to safely carry a load of 98,000 pounds between two panel points, both ends being riveted.

P. 1510. Find the safe compression that may be safely carried by a latticed channel bridge column composed of two square ended 12″ 30 pound channels of medium steel, 28′ long.

P. 1511. Select a steel column, square at both ends and 18′ long, composed of two channels and two plates riveted to the channel flanges, which will safely bear a load of 150,000 pounds. Column of least weight required. Factor of safety equals 4.

P. 1512. Select a square ended column 18′ long, composed of plate and four angles, to safely bear a load of 150,000 pounds.

P. 1513. Design a Z-bar column of medium steel to carry 20 tons, square ended, 14′ long.

P. 1514. How much may be safely carried by a hollow circular cast-iron post 20′ long, $\frac{7}{8}''$ thick and $10\frac{1}{4}''$ interior diameter, if ends are square?

P. 1515. Design a hollow cast-iron square-ended column 22′ long, to carry 18 tons.

P. 1516. A hollow cast-iron pillar 12′ in height, with square ends, has to support a steady load of 33,000 pounds; its internal diameter is $5\frac{1}{2}''$. Find the thickness of the metal, the factor of safety being 6.

P. 1517. What is the safe uniformly distributed load on a 6″ I-beam, resting on end supports 20′ apart, if the deflection is limited to $\frac{1}{360}$ of the span? $E = 30{,}000{,}000$; $I = 24$; weight of beam equals 14.75 pounds per linear foot.

P. 1518. The joists in a warehouse floor are $8'' \times 12''$, supported every 16′, and spaced 2′ apart. The flooring is 2″ thick. Flooring and joists are of longleaf yellow pine. What uniformly distributed live load will the floor support safely?

P. 1519. A warehouse floor supports a uniformly distributed load, including its own weight, of 420 pounds per square foot. It is supported by square longleaf pine columns, 10′ long, spaced 12′ center to center in each direction. Determine proper size of columns.

P. 1520. If the floor in Problem of paragraph 1522 were reinforced by an additional support in the center of each span, how high could the canned goods be piled without exceeding the allowable load?

P. 1521. Determine the allowable floor load for a floor 3″ thick supported on joists 6″ wide, 10″ deep, spaced 2′ apart, and having a 10′ span; floor made of long leaf yellow pine.

P. 1522. Determine the size of I-beams to be used as joists for a 12′ span, in a wooden floor carrying a live load of 400 pounds per square foot. Assume spacing.

P. 1523. Make out a bill of material for a balloon frame shed (excluding the roof and floor), walls 10′ high in front, 9′ high in rear, floor dimensions 8′ deep by 12′ long, with door 6′ × 3′ in one end, and window 4′ × 3′ in the other.

P. 1524. What is the weight of a brick wall 2′ 4″ thick, 12′ long, 15′ high? How many bricks are required to build the wall?

P. 1525. (Fig. P. 1525.) By the analytical method of concurrent forces, find the stresses in the members of the truss with the loading given. Assume $AC = CD$.

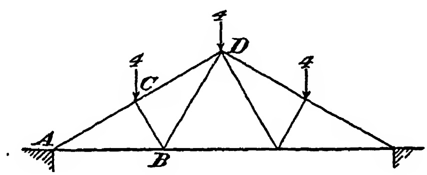


Fig. P 1525

P. 1526. (Fig. P. 1526.) By the method of sections, determine the stresses in the following members:—

(a) BD. (b) CC. (c) DC.

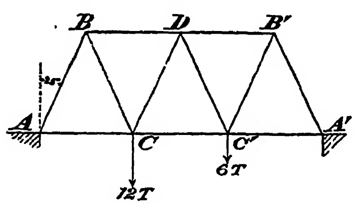


Fig. P 1526

P. 1527. (Fig. P. 1527.) By the method of sections, find the stresses in all the members with the loading given.

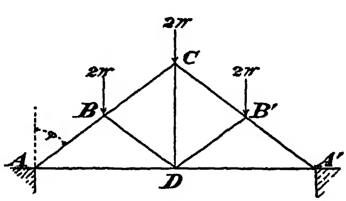


Fig. P 1527

P. 1528. (Fig. P. 1528). By the method of sections, find the stresses in the different members of the truss with the loading given.

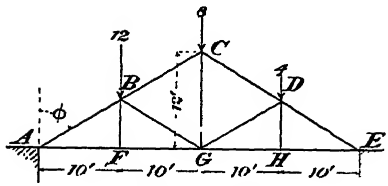


Fig. P 1528

P. 1529. (Fig. P. 1529.) The figure represents a structure attached at three points to solid rock by three hinges. Determine by graphics the reactions and the stresses in the members.

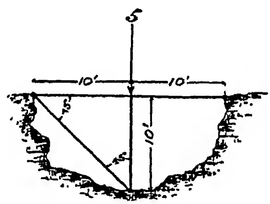


Fig. P 1529

P. 1530. (Fig. P. 1530.) AB is the reaction at one point of support of a truss, and BC and CA are the members intersecting at this point of support. Find graphically the stresses in the members BC and CA, in order that there may be equilibrium at the point.

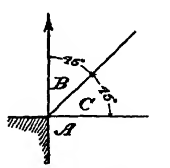


Fig. P 1530

P. 1531. (Fig. P. 1531.) The figure represents a truss and its applied load. By graphics determine the reactions and the stresses in the members.

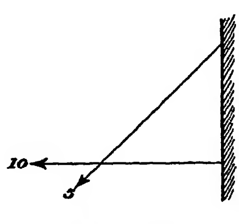


Fig. P 1531

P. 1532. (Fig. P. 1532.) With the system of concurrent forces given, find the direction and intensity of the resultant by graphical statics.

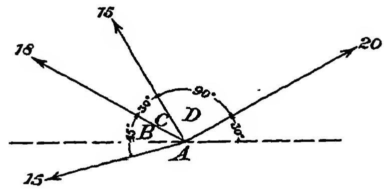


Fig. P 1532

P. 1533. (Fig. P. 1533.) The figure represents three weights suspended by a rope over two pulleys without friction. By graphics determine the position of the point S.

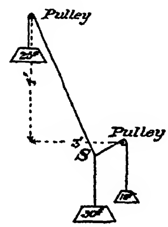


Fig. P 1533

P. 1534. (Fig. P. 1534.) This represents a beam 30′ long, resting on two supports, and acted on by three forces as shown. Find the position and the intensity of the resultant by Graphical Statics.

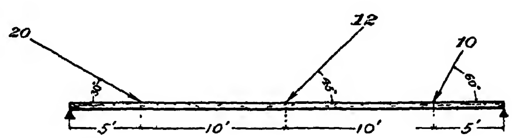


Fig. P 1534

P. 1535. (Fig. P. 1535.) (*a*) Find the resultant of the given forces by means of the equilibrium polygon.

(*b*) The reactions are parallel. Find them by means of the equilibrium polygon.

(*c*) Check by use of proportional triangle.

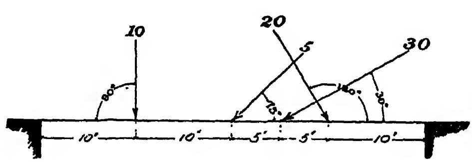


Fig. P 1535

P. 1536. (Fig. P. 1536.) (*a*) Find the intensity and direction of the resultant of the applied loads graphically.

(*b*) Find the point of application of the resultant graphically.

(*c*) Determine completely the two parallel reactions graphically.

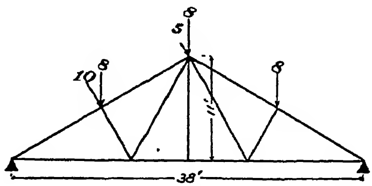


Fig. P 1536

P. 1537. Solve Problem P. 1536 analytically.

P. 1538. (Fig. P. 1538.) This represents a truss suspended at the left end by a tie and resting on a fixed support at the other end. It is subjected to the loads shown in the figure. Assuming that the resultant cannot be found without the use of the equilibrium polygon, find the reactions.

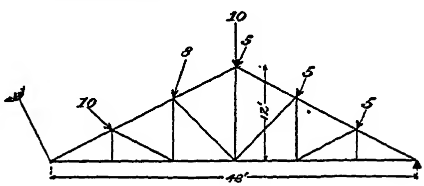


Fig. P 1538

P. 1539. (Fig. P. 1539.) This truss has a dead load of 4 tons per panel point of the lower chord and a live load of 8 tons per panel point of the lower chord. Find graphically the stress in each member when the live load covers the truss.

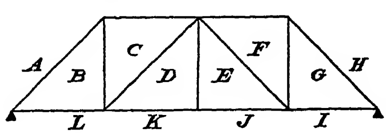


Fig. P 1539

P. 1540. (Fig. P. 1540.) The dead load per panel point of the lower chord is 9 tons and a concentrated live load of 21 tons is at the panel point *HDEFG*. Find graphically the stress in each member of the truss. All triangles of the figure are equilateral.

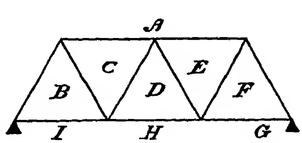


Fig. P 1540

P. 1541. (Fig. P. 1541.) Find graphically the stress in each member of the truss and the parallel reactions.

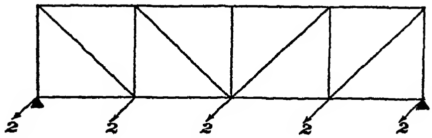


Fig. P 1541

P. 1542. (Fig. P. 1542.) Find graphically the stress in each member of the truss.

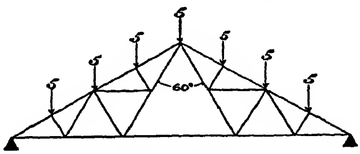


Fig. P 1542

P. 1543. (Fig. P. 1543.) By graphics find the stress in each member of the truss.

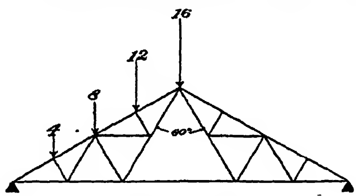


Fig. P 1543

P. 1544. (Fig. P. 1544.) By graphics determine the reactions and the stress in each member of the truss.

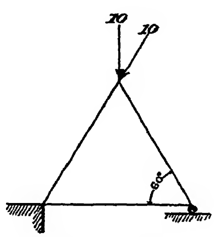


Fig. P 1544

P. 1545. (Fig. P. 1545.) Find graphically the reactions and the stress in each member of the truss.

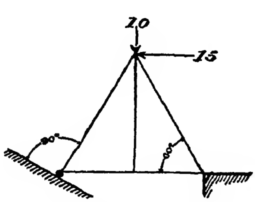


Fig. P 1545

P. 1546. (Fig. P. 1546.) By graphics determine the reactions and the stress in each member of the truss.

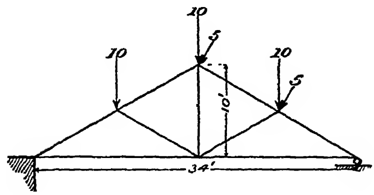


Fig. P 1546

P. 1547. (Fig. P. 1547.) By graphics determine the reactions and the stress in each member of the truss.

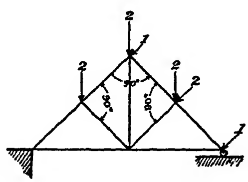


Fig. P 1547

P. 1548. (Fig. P. 1548). Find graphically the reactions and the stress in each member of the truss.

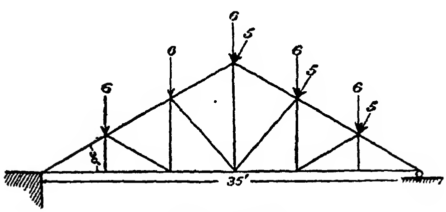


Fig. P 1548

P. 1549. (Fig. P. 1549.) Find graphically the reactions and the stress in each member of the truss.

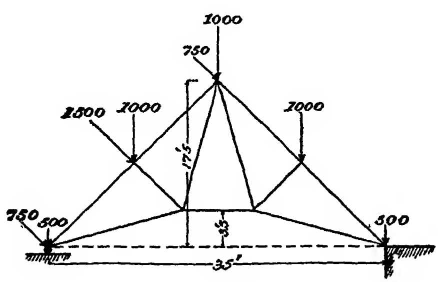


Fig. P 1549

P. 1550. (Fig. P. 1550.) The figure represents a truss and its applied load. By graphics determine the reactions and the stresses in the members.

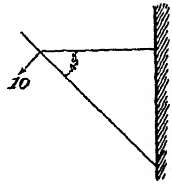


Fig. P 1550

P. 1551. (Fig. P. 1551.) Find by graphics the stresses in the members of the truss.

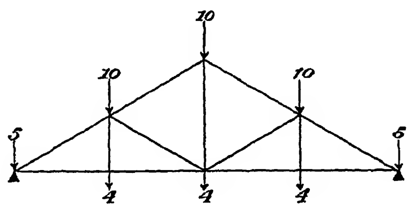


Fig. P 1551

P. 1552. (Fig. P. 1552.) Find graphically the reactions and the stress in each member of the truss.

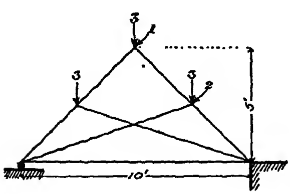


Fig. P 1552

P. 1553. (Fig. P. 1553.) Find graphically the reactions and the stress in each member of the truss.

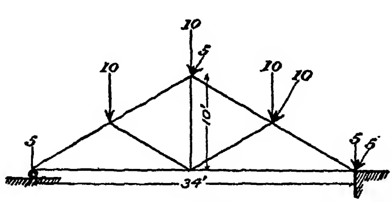


Fig. P 1553

P. 1554. (Fig. P. 1554.) Find graphically the reactions and the stress in each member of the truss.

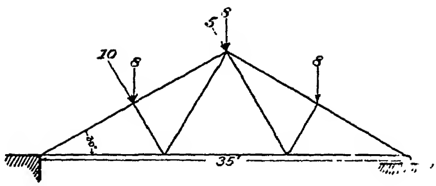


Fig. P 1554

P. 1555. (Fig. P. 1555.) Find graphically the reactions and the stress in each member of the truss.

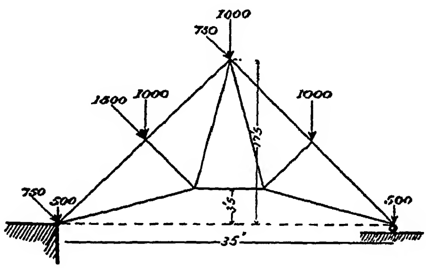


Fig. P 1555

P. 1556. (Fig. P. 1556.) Find the center of gravity of the figure by the method of graphical statics.

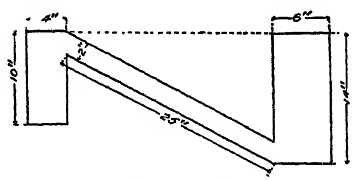


Fig. P 1556

P. 1557. (Fig. P. 1557.) Find the center of gravity of the figure by the method of graphical statics.

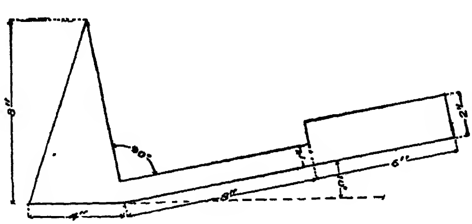


Fig. P 1557

P. 1558. (Fig. P. 1558.) Find the center of gravity of the figure by the method of graphical statics.

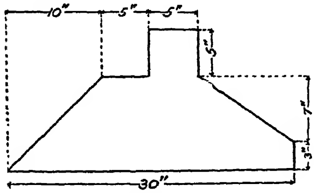


Fig. P 1558

P. 1559. (Fig. P. 1559.) The figure represents a beam of 30′ span resting on end supports and loaded as shown. Find the dangerous section and the bending moment (in foot-pounds) by the graphical method. Check the bending moment analytically.

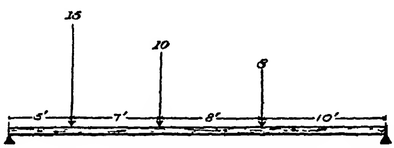


Fig. P 1559

P. 1560. (Fig. P. 1560.) (*a*) ML, LK, KN and NM are separate members attached at the point $MLKN$. Find graphically the reactions and the stress in each member of the truss.

(*b*) Omit the wind loads and assume that ML and NK form a continuous member, there being no connection between these members at the point $MLKN$. Find graphically the reactions and the stresses in the members.

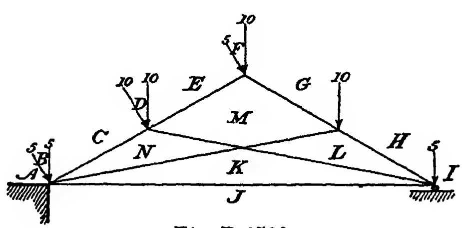


Fig. P 1560

P. 1561. In a residence heated by the hot air system, there is a second story room 14′ square with a ceiling height of 10′. The room is exposed on two sides and has four windows each 6′ by 3′. The attic over the room is unheated. Hot air is admitted to the room through a single flue and register. Determine the area of cross-section of the flue, and the net area of the register, in order that the room temperature shall be maintained at 70° F., when the outside temperature is 0° F.

P. 1562. The heating of a small dwelling by hot air requires a total of 40,000 cu. ft. of air per hour, delivered to the rooms at a temperature of 130° F. The furnace is supplied with fresh air entirely from outside. The minimum outside temperature is 0° F. Determine the grate area of the furnace.

P. 1563. The living room in a house heated by direct hot water radiation is 16′ square and has a ceiling height of 12′. It is exposed on two sides. In one of the exposed walls there is a window 6′ by 6′, in the other there is a door 3′ by 8′, and a window 4′ by 6′. What is the total square feet of radiating surface required for the room?

P. 1564. An auditorium, seating 500 persons, is 120′ long, 60′ wide, and has a ceiling height of 20′. Both of its longer sides are exposed, each containing 8 windows 4′ by 10′. It is heated and ventilated by air forced in by fans and escaping through suitable vent ducts. What should be the volume of air supplied, and what should be its temperature on entering the room when the outside temperature is 0° F.?

P. 1565. An electric heater having a resistance of 3 ohms is to supply 3000 B.T.U. per hour. What current will be required? (Assume formula, $H = .24\ I^2Rt$, H being in calories, and 1 B.T.U. = 252 calories.)

P. 1566. A room 15′ square and 10′ high has two exposed walls, one toward the north and the other toward the west. There are 4 windows, each 3′ × 6′ in size. With air changed once an hour by leakage, what will be the heat loss with an inside temperature of 70° F. when the outside temperature is 0° F.?

P. 1567. A dwelling house of average wooden construction measures 60′ × 40′ on the outside, and has 3 stories each 9′ high. Compute the heat loss when the temperature is 10° F. below zero, the inside temperature being 70° F.

P. 1568. A first-floor room has a computed loss of 27,000 B.T.U. per hour when it is 10° F. below zero. The air for warming is to enter through two pipes of equal size, at a temperature of 120° F. What will be the required diameter of the pipes?

P. 1569. The heat loss from a building is 168,000 B.T.U. per hour and the ratio of heating to grate area is such that not over 3 pounds of coal per hour can be burned per square foot of grate. What ratio of heating to grate area will be necessary, and what will be the required grate area?

P. 1570. A school has 10 classrooms, each occupied by 50 pupils. Air is to be delivered to the rooms at a temperature of 70° F. What will be the total horse power required to heat and ventilate the building at 0° F. outside temperature, if the heat loss through walls and windows is 1,500,000 B.T.U. per hour?

P. 1571. A 1½″ pipe 100′ long connected with a cistern is to discharge 35 gallons per minute. At what elevation above the end of the pipe must the surface of the water in the cistern be to produce this flow?

P. 1572. A schoolroom is to be warmed with circulation coils of 1¼″ pipe; heat loss, 30,000 B.T.U. per hour. What length of pipe will be required?

CHAPTER XVI.

BRIDGES.

1601. **Definitions.**—A **bridge** (Fig. 1601A) is a structure over a depression, usually a water-course or a valley, carrying a passageway between the two sides. A bridge over a dry depression, especially if the bridge is of considerable length, is often called a **viaduct.** If the viaduct supports an artificial channel for conveying water, it is called an **aqueduct,*** and where it crosses a stream, it is frequently called an **aqueduct-bridge.**

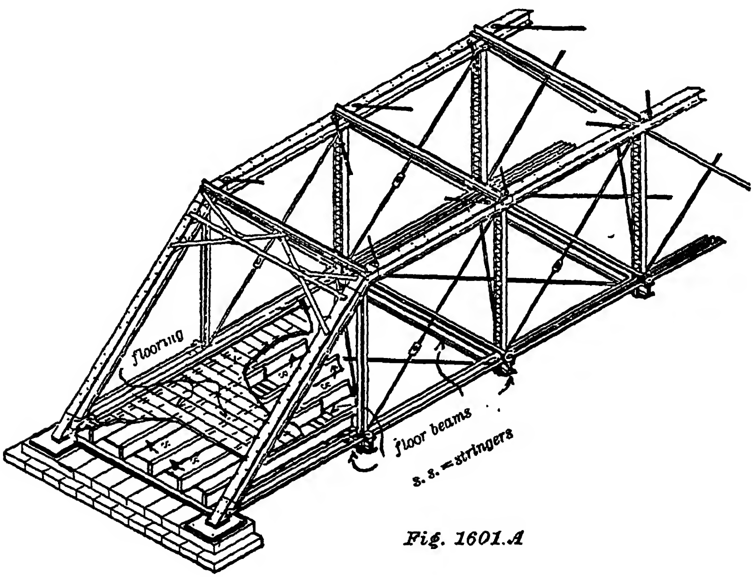


Fig. 1601.A

1602. The essential parts of a bridge are the **substructure,** by means of which the passageway is elevated, and the **superstructure.** The substructure is composed of abutments and piers: The superstructure varies with the character of the bridge, that for a truss highway bridge for example being composed of the flooring, stringers, floor beams, and trusses.

* An aqueduct need not be supported; it may be a tunnel.

An **abutment** is one of the end supports of the superstructure. It carries one end of the bridge and in addition acts as a retaining wall to the approach embankment.

A **pier** is one of the intermediate supports of the superstructure.

The **floor system** is composed of the **flooring,** over which the travel passes; the **stringers,** which support the flooring; and the **floor beams,** which support the stringers and transmit the weight of the floor to the floor carriers.

The **floor carriers** are the plate girders, trusses, arches, or cables which support the floor system and transmit its weight to the abutments and piers.

A **span** is that portion of the bridge between the centers of adjacent piers and abutments.

A **skew** bridge is one whose axis is oblique to the longer dimension of its abutments.

CLASSIFICATION OF BRIDGES.

1603. Bridges may be classified as deck, through, and pony truss. **Deck** bridges have the floor at the top of the main superstructure as in

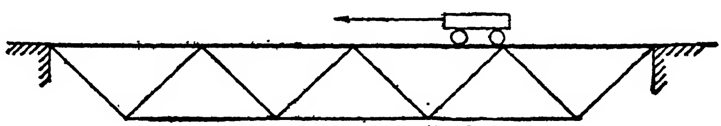


Fig. 1603 A Deck Bridge

Fig. 1603A. **Through** bridges have the greater part of the superstructure above the floor level, which gives ample room for overhead bracing to resist lateral forces (see Fig. 1603B). **Pony truss** bridges have part

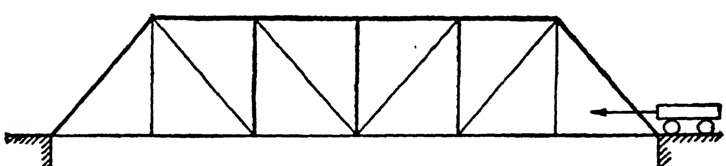


Fig. 1603 B Through Bridge

of the superstructure above the floor level but are not of sufficient height to provide overhead bracing. In this last type lateral stability is obtained by employing brackets or braces for each floor carrier.

Deck bridges are frequently used in railway construction because they are cheap, safe, and rigid. However, where a given clearance over a water course is required, they increase the cost of approaches, because the lowest element of the bridge must have the required amount of

clearance, thus making it necessary for the top of the approach to be at an increased height so as to make it the same height as the top of the truss.

BRIDGE MATERIALS.

1604. The parts of a large bridge are nearly all of fireproof material. The piers and abutments are usually of stone, concrete, steel, or combinations of these materials. The floor-carriers are generally of steel. The flooring is of wood, or of steel plates or reinforced concrete, covered with road surfacing material.

The steel used in bridges is of the regular commercial shapes as made in rolling mills. In unusual cases, special pieces are made; but the regular commercial shapes are so much cheaper, that the expense is materially decreased by the use of these shapes even if the special pieces weigh less. All steel, whether of regular size or special shapes, is given a careful inspection at the mills or on the work before being placed in the structure.

In small highway bridges, much wood is used; but steel is being used more and more in place of wood. When the expense is justified, the wood is treated with fire-resisting compounds.

LOADS ON BRIDGES.

1605. All bridges are designed to support dead and live loads. The dead load is the weight of the permanent parts of the bridge; the live load is the weight of any temporary load that may come upon the bridge.

Dead Load.

1606. The dead load on the foundation bed of a bridge is the weight of all of the permanent parts of the bridge. The dead load on any particular member is the weight of the member itself and that part of the weight of the bridge which is transmitted to it by the other members. Thus, in a highway bridge, a stringer supports not only its own weight but also the weight of an area of planking whose length is that of the stringer and whose width is the distance between centers of adjacent stringers. A floor beam supports, in addition to its own weight, the weight of the flooring and stringers in an area whose length is the distance between centers of floor beams and whose width is the width of the flooring. This area is called a floor panel.

It is impossible to determine the weight of all parts of a bridge until it is designed; yet no member can be accurately designed until its weight is known. It is therefore necessary in all cases to make use of approximate methods of solution, first assuming the weight, next designing with this assumed weight, then computing the actual weight and revising the

design. Experienced designers assume the weights so nearly correct that revision is not necessary.

A preliminary estimate of the weight of a bridge may be made by consulting charts and empirical formulas in standard hand books. The best method is to use the actual weight of a similar structure of like span which has been built. If the necessary data for this are not available, formulas may be used. Railway bridge formulas are more accurate than highway bridge formulas on account of the more uniform character of railway loads. The following formulas are typical:

For light country highway bridges, $W = 250 + 2.5\,L$ (Dufour)

For first class railway truss bridges, $W = 650 + 7\,L$ (Turneaure)

For third class railway truss bridges, $W = \frac{3}{4}\,(650 + 7\,L)$ (Turneaure)

where

W = weight of steel per linear foot of span (including both trusses, with lateral trusses, bracing, &c)

L = length of span in feet

The weight of flooring for highway bridges may be assumed as 10 pounds per square foot of floor surface. For single track railway bridges the weight of track, ties, rails, &c may be assumed as 400 pounds per linear foot of span. A complete dead load formula for a highway bridge is the following (Waddell & Hedrick):

$$W = 34 + 22b + 0.16\,bL + 0.7\,L$$

where W is the total dead load in pounds, L is the span in feet, and b is the clear width of roadway and sidewalks in feet.

Live Load.

1607. The live load on any member is that part of a moving load that may be transmitted to it. The load that passes over a bridge may be concentrated, uniformly distributed, or both. The following live loads are considered in bridge design: pedestrians, vehicles, street cars, locomotives, impact, wind, snow, traction, and centrifugal loads. In designing a member the live load is taken as placed so as to give the maximum stress in the member under consideration. The live load used in designing a bridge is the maximum that will pass over it.

1608. **Live Loads for Highway Bridges.**—The live loads for highway bridges are usually assumed to consist of a uniform live load for the trusses and a uniform live load or a concentrated live load for the floor and its supports. The purpose, situation, and circumstances of use of the bridge must be carefully studied and the bridge designed for the expected maximum loading. The customary uniform load is that of a crowd of men, which may reach a maximum of 150 pounds per square

foot. The customary concentrated load is that of a steam roller, a truck, or a crowded electric street car.

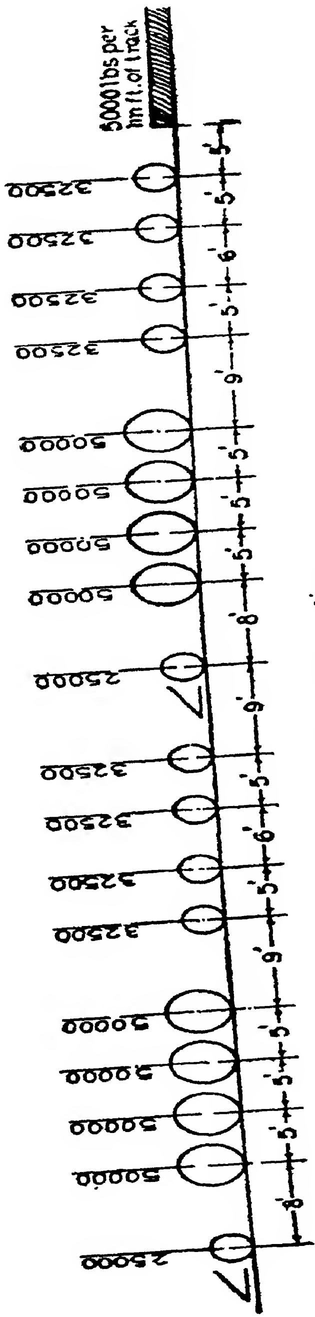
Modern highway bridges must support much heavier loads than formerly. The heaviest motor truck in common use weighs 20 tons loaded, 14 tons on the rear axle, and 6 on the front, assuming that the truck is 50 per cent overloaded, which is not uncommon. The heaviest road roller weighs about the same. Military bridges in France were designed to carry the 40 ton tank. Uniform loading is sometimes used in highway bridge specifications in place of concentrated loads, and for long spans, the uniform load is reduced because of the fact that a crowd will certainly not cover all of a long span.

1609. **Live Loads for Railway Bridges.**—The loads for any particular railway bridge are not always the same on account of variation in weights and wheel spacings of engines and cars. It is customary to design the bridge for the heaviest train in use at the time of construction, or that could reasonably be expected to be built thereafter. Most railways specify that their bridges shall be computed by using two engines and tenders followed by a train. The spacing of the wheels and the load which comes on each wheel of the engine and tender are fixed by the railway company. The train is represented by a uniform load. Formerly there was great diversity of practice among different railways in regard to the engine and train loads specified, but of late years practice has tended to standardize in accordance with Cooper's specifications for railway loadings. Cooper's Class E-50 loading is given in Fig. 1609A. The maximum live loads for railway bridges are, according to Cooper:

Engine............	9,800 pounds per linear foot
Engine and tender...	8,125 pounds per linear foot
Train..............	5,000 pounds per linear foot
Concentrated load...	120,000 pounds distributed on 4 axles, 6′ apart.

1610. **Impact.**—When a load moves over a bridge, it causes shock and vibrations, thereby increasing the stresses in the members. This shock includes not only the effect of speed, but also the effects of flat wheels, unbalanced drivers, rough track, &c. We know from paragraph 215 that the stress produced in a rod by a suddenly applied force is twice that of the same force gradually applied. An instantaneously applied force is not obtained in practice, as it takes time for a fast moving load to cross a short bridge; but although the effect is not double that of a static load, the stress is greatly increased. The American Railway Engineering Association uses the following impact formula:

$$I = S\frac{300}{L + 300} \qquad (1610A)$$



Cooper's E 50 Loading.

Fig. 1609 A Diagram of Cooper's E 50 Loading. Representing Heaviest Engines in Common Use.

(Weights indicated are for two rails. For each rail, divide above weights by two.)

in which,

I = impact to be added to the stress due to a load passing over the bridge

S = computed maximum stress due to the load that passes over the bridge

L = length in feet of the portion of the span loaded to produce the maximum stress in the member.

In the design of highway bridges, the impact on the floor may be taken as 50 per cent of the live load for a heavy truck load and 25 per cent for other live loads.

Impact is not added to stresses produced by wind, centrifugal, or traction forces.

1611. **Wind Loads.**—The pressure of the wind upon a bridge or viaduct tends to buckle the superstructure, move the structure on its supports, and overturn the piers, towers, and trestles. All these effects must be considered in bridge design.

The intensity of wind pressure on any plane surface normal to its direction is expressed approximately by the formula

$$P = 0.004\ V^2 \qquad (1611A)$$

in which, P = the pressure in pounds per square foot
V = velocity of the wind in miles per hour.

According to this formula the pressure of a wind whose velocity is 100 miles per hour is 40 pounds per square foot.

As the maximum pressure on small surfaces is much greater than on large ones, it is customary in this country to assume a pressure of 30 pounds per square foot on large surfaces, and to increase this to 45 or 50 whenever it seems desirable to increase the factor of safety, as on unloaded railway bridges.

The wind pressure is greatest when its direction is normal to the vertical plane through the axis of the bridge, as it then acts on the largest surface and produces all the effects above described. If the wind blows parallel to the axis, its effect is felt principally by the supports.

The pressure of the wind against the exposed surface of moving loads upon the bridge must also be considered. For example, in a railway bridge the most dangerous condition as regards wind is when a train of empty cars is passing over the bridge. An additional exposed surface about 14′ high (the average height of the cars) and the length of the bridge or train (if shorter than the bridge) must be added to the exposed surface of the bridge in calculation for stresses due to wind.

1612. **Snow Loads** are not ordinarily considered in the design of a bridge, as accumulated snow lessens the probable live load.

1613. **Traction Loads** on a bridge are the longitudinal forces produced by starting or stopping a truck, or electric car, or train. The stopping or braking effect is usually much greater than the starting effect, and is maximum when brakes are set and wheels sliding. Traction load is ignored in design of truss bridges, but in railway trestles it is taken as 20 per cent of the live load. In highway bridges and trestles it is so small as to be neglected.

1614. **Centrifugal Load** is the effect of centrifugal force when a railway train moves on a curved track. On a bridge the extreme effect is to make a cantilever of each pier, with the train's centrifugal force as the load at its extremity. A firm anchorage, and a gradual increase in breadth from top of bridge to bottom of foundation will be sufficient to overcome centrifugal load except in rare cases.

1615. **Problem.**—Select the steel beams for a single track electric railway bridge of 12′ span carrying a car with wheel loads (four wheels on each side) of 5000 pounds each, spaced at 5′, 15′, and 5′ apart. Assume that track and ties weigh 400 pounds per linear foot, and that each beam weighs 40 pounds per linear foot. Allow 25 per cent for impact.

Solution.—Two beams will be used; and each will carry half of the total load. On account of the relative lengths of the car and the span and the distances between wheel loads, only two wheel loads can come upon the beam at one time. Considering only one beam, we know from paragraph 411 that the maximum moment due to the live load alone will occur under one of the two wheel loads when that load is $1\frac{1}{4}'$ from the center of the beam and the other load is $3\frac{3}{4}$ on the opposite side of the center.

$\therefore$ Live load moment $= 3959 \times 4\frac{3}{4} =$ 18,805 foot pounds
Dead load moment at same point = 4132 foot pounds
Impact = 25 per cent of 18,805 = 4701 foot pounds
Total moment = 27,638 foot pounds.

To obtain the maximum shear in each beam, we know from paragraph 404 that the maximum shear for a load is at the supports when the load is at the supports; therefore, the maximum shear for two axles will be when one is at the support and the other is 5′ from the support.

$\therefore$ Live load shear = 7917 pounds
Dead load shear = 1440 pounds
Impact = 25 per cent of 7917 = 1979 pounds
Total shear = 11,336 pounds

From the table of Maximum Bending Moments in the handbook, we find that a maximum bending moment of 27,638 foot pounds requires a 9″ 30 pound I-beam. This weighs less than the assumed weight of the beam. The maximum shear of 11,336 pounds is well within the allowable limit for this I-beam.

Therefore, two 9″ 30 pound I-beams will be satisfactory.

1616. **Problem.**—Compute the total wind pressure on a plate girder deck railway bridge, due to a wind velocity of 120 miles per hour, the plate girder being 36″ deep,

and the span being 60′. Assume a train of cars 14′ high to be on the bridge, and that the track, ties, and superstructure are 1′ above the girder.

Solution.—Using formula, $P = 0.004\ V^2$, and substituting the value of V as 120, P is found to be 57.60 pounds per square foot. The total surface which the wind blows against is

$$(14 + 1 + 3) \times 60 = 1080 \text{ square feet}$$

$$1080 \times 57.60 = 62{,}208 \text{ pounds} = \text{total wind pressure on the bridge.}$$

FOUNDATIONS FOR BRIDGES.

1617. Foundations for bridges are considered in the Chapter on Foundations. The site of a bridge, unlike that of a building, may usually be selected: and in such selection, the most important consideration is the character of the foundation. Preferably, it should be of rock; and a reasonable increase in cost is justified in order to secure a rock foundation. If necessary to place on earth foundation, it may be possible to confine the earth: but, as a rule, a spread foundation is the best solution in such cases.

SUBSTRUCTURE.

1618. **Piers.**—Bridge piers should transmit the load from the superstructure to the foundation. They should interfere as little as possible with the natural flow of the water. The spacing is ordinarily a question of economy. However other considerations may govern the distance between piers; for example, the War Department requires a certain minimum spacing of piers in navigable rivers so as to cause as little danger and obstruction as possible to river traffic.

A pier with cross-section shaped like a ship interferes little with the natural flow of the water, but is more expensive than a pier made of two cylinders filled with concrete, one under each truss or girder of the bridge. This latter construction, however, obstructs the flow of the water more than the other form.

Piers should rest on a stable foundation, the base of which is well below the frost line and below any possible scouring action. When bed rock is less than 30′ below the water level, piers should usually rest on bed rock, otherwise on piles or caissons. The various methods employed in sinking bridge piers are discussed in the Chapter on Foundations.

1619. Piers are made of (1) Cribs; (2) Wooden Trestles; (3) Steel Trestles and Towers; (4) Masonry.

Crib piers are constructed of logs or beams fastened together and weighted as previously explained in the Chapter on Foundations. They are employed in the construction of hasty or unimportant bridges. If the current is strong, the cribs are so placed that they present the vertex of an angle to the current.

Wooden trestles are the simplest form of support for a viaduct. They are employed where the viaduct is on firm soil. A wooden trestle bent (Fig. 1619A) is made up of a cap sill, ground sill or mud sill, several vertical posts, (the two exterior posts may be batter posts), and two

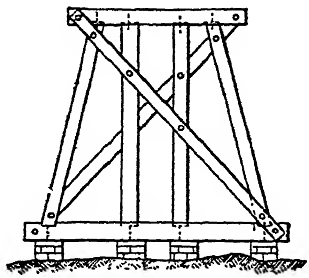


Fig. 1619 A
Trestle Bent

cross or sway braces. The sway braces may be omitted in bents less than 10′ high. In railway construction, all the members except the sway braces are 12″ × 12″ timbers: the sway braces are 3″ planks. In highway viaducts, lighter pieces are employed. The height of the simple bent is limited to 25′. If bents of a greater height are required, they are made by placing a series of bents one on top of another (Fig. 1619B). The space between consecutive horizontals is called a story

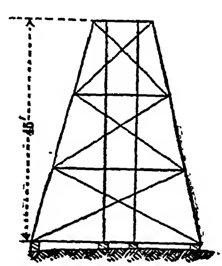


Fig. 1619 B
High Trestle Bent.

and does not ordinarily exceed 20′ in height. In high bents the posts may be continuous, or the bents may be constructed separately and superimposed. In the former type, the stories are made by bolting horizontal sills to the posts: in the second type, the superimposed sills are in absolute contact.

Simple bents are placed at intervals of 12′ to 14′; high bents, at intervals not exceeding 25′. The bents are connected horizontally by

stringers which connect adjacent caps and support the roadway. Longitudinal movement is prevented by the superstructure and by the ground. To give additional longitudinal strength, simple bents are connected by horizontal or diagonal bracing similar to the sway bracing. In high viaducts or trestles, the intermediate sills of different bents are connected by horizontal beams and the batter posts of each story by horizontal or diagonal bracing.

In a viaduct constructed on soft soil, the common trestle bents are replaced by **pile bents.** The simple pile bent consists of a cap sill supported on three or more piles. The cap is usually but a short distance above the surface: where high bents are required the pile bents are covered by one or more stories of common trestle, or are connected by sway braces.

A **steel trestle bent** consists of two posts, usually inclined, resting on masonry bases. The posts are steel columns; the cap sill is a plate girder: the ground sill is only a latticed member attached to the posts to keep them properly spaced: the sway bracing is made of steel eyebars. In low viaducts on city streets the bents consist of posts and caps only.

A **steel tower** (Fig. 1619C) is formed of four vertical or inclined posts. Each of the four faces of the tower is divided into stories and braced

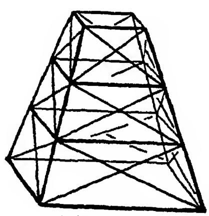


Fig 1619 C
Steel Tower

like a simple trestle bent. At intervals the diagonally opposite posts are connected by horizontal braces. High viaducts are supported by a series of steel towers. This requires less material than a series of trestle bents, as no longitudinal bracing is required between towers.

Masonry piers, or at least masonry foundations are employed in the construction of all important bridges. The general plan of the pier is rectangular, with the longer dimension parallel to the current (Fig. 1619D). The shorter faces are rounded or pointed to deflect the water and thus reduce the pressure on the pier and also the undermining action of the current. These deflectors are called **starlings.** If exposed to floating ice in a strong current, a portion of each upstream starling is inclined at an angle of 45° with the horizontal, so that the ice will be

forced up the inclined surface until it breaks of its own weight. This prevents the accumulation of ice and the consequent pressure on the pier. The inclined surface is called an ice breaker and extends from a few feet below water to a few feet above water.

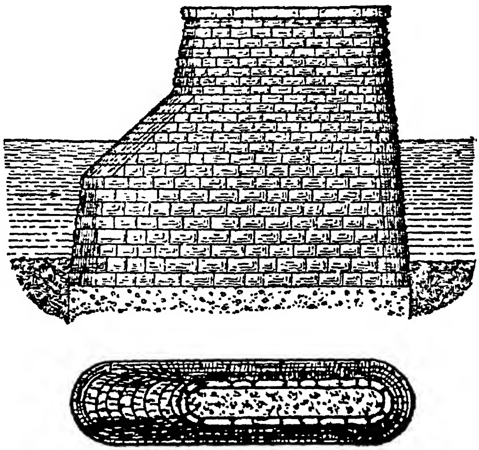


Fig. 1619 D Bridge Pier with Ice Breaker

Approaches and Abutments.

1620. The portions of a roadway at each extremity of a bridge and leading to it are termed the **approaches.** These are arranged so that the vehicles, using the bridge, may have an easy and safe access thereto. The arrangement will depend upon the locality, upon the number and direction of the avenues leading to the bridge, upon the width of these

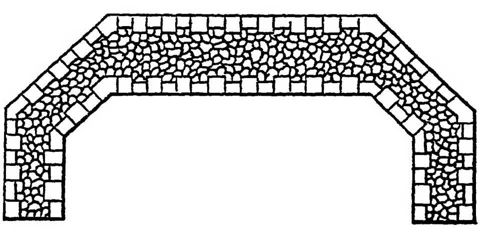


Fig. 1620 A Abutment

avenues and upon their positions, whether above or below the natural surface of the ground. When the avenue to the bridge is in the same line as its axis, and the roadway of the avenue and the bridge is of the same width, the abutment is generally made as shown in Fig. 1620A. The returns or short walls carried back parallel to the axis of the road

to flank the approach are called **wing-walls,** and are intended to sustain the embankment as well as to serve as a counterfort to the abutment.

When several avenues meet at the bridge, or it is necessary that the width of the approach shall be greater than the roadway of the bridge, the wing-walls may be given a curved shape, as shown in Fig 1620B,

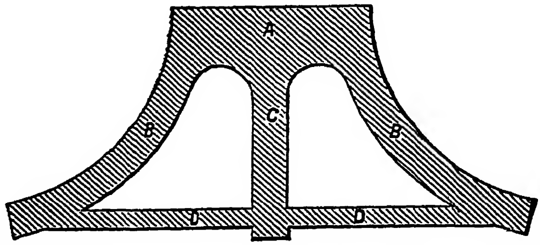


Fig. 1620 B. Horizontal section of an abutment, A; with curved wing-walls, B; B connected with a central buttress, C; by a cross tie-wall, D

in this way widening the approach. When the soil of the river banks is soft, the foundation of the wing-walls should be laid at the same depth as that of the abutment. But if the soil is firm, the wing-walls may be built in steps, thus saving considerable expense. The method of design of wing-walls is the same as for other retaining walls. It is good practice to make their length one and a half times the height of the roadway above the bed of the river, their thickness at bottom one-fourth their height, and to build them up in off-sets on the inside, reducing their thickness at the top to between 2′ and 3′.

Instead of wing-walls, a single wall in the middle is used in many cases. The plan of the abutment in such a case is that of a T. This type of abutment is often used in railway bridges. The slope of the embankment (river bank) may be the natural slope, or, if steeper, the embankment should be revetted with dry stone or sods, as shown in Fig. 1620C.

When the face of the abutment projects beyond the bank, an embankment faced with stone should connect it with points of the bank, both above and below the bridge. These are called **water-wings,** and serve to contract gradually the water-way of the stream at this point.

Where there is danger of the banks above and below the abutment being washed away or worn away by the action of the current, it is advisable to face the slope of the bank with dry stone or masonry.

Abutments are usually constructed of stone, concrete, or reinforced concrete; but for temporary or unimportant bridges they may be made of wood. A wooden abutment is usually made of squared logs placed on top of each other, and held in place by anchor logs fastened to the face logs and to transverse logs buried in the embankment. The transverse logs must be beyond the plane of the natural slope passing through

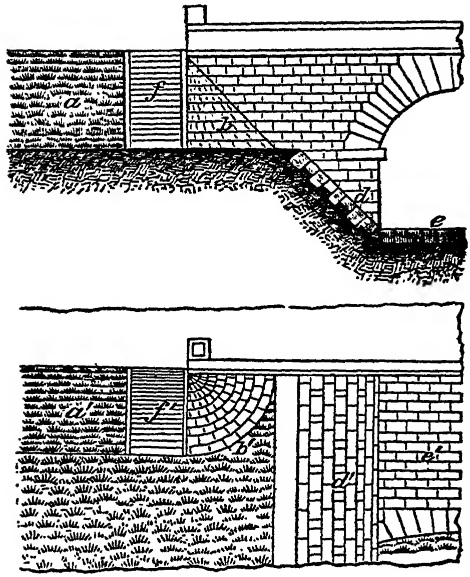


Fig. 1620 C.—Plan and elevation showing a method of arranging the embankments where there are no wing-walls.
a, *a'*, side slopes of embankment of the approach.
b, *b'*, dry stone revetment of the slope towards the water-course.
d, *d'*, dry stone facing of the slope of the bank.
e, *e'*, paving used on the bottom of stream.
f, *f'*, stairs for foot passengers.

the foot of the abutment face. If the abutment is U-shaped, the wings should be connected by anchor logs.

SUPERSTRUCTURE.

1621. **Floor System.**—In a simple highway bridge (Fig. 1621A), the flooring consists of yellow pine or white oak planks about 3″ thick which are laid transversely and securely spiked to the stringers. The opening between planks is $\frac{1}{4}''$; for convenience in making repairs, a continuous longitudinal joint is made along the center line of all long bridges having a roadway 16′ or more wide. The sidewalks are usually separated from the roadway by railings; if not thus separated, they should be elevated a few inches above the roadway. If the roadway extends to the floor-carriers, longitudinal beams at least 6″ high, called **wheel-guards**, are spiked to the floor to prevent the carriers from being injured by the hubs of the wheels. The minimum clear width of roadway for a single vehicle should be 10′ and for two vehicles should be 16′.

The stringers are of yellow pine or other wood, not less than 3″ thick and 8″ to 12″ deep; they are spaced about 2′ center to center. Stringers

of adjacent panels overlap or abut against each other on the floor-beams. If the spans are less than 25′, the stringers rest directly upon the piers and abutments and become the floor-carriers.

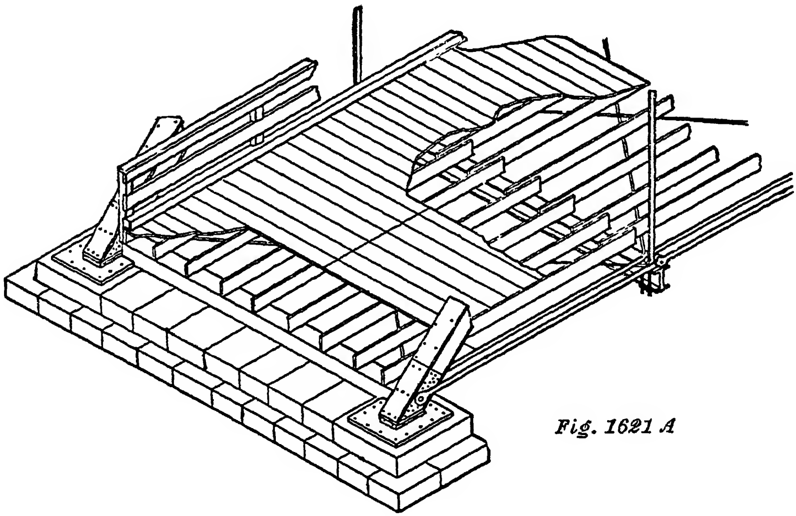


Fig. 1621 A

The **floor beams** or **cross-girders** connect the roadway carriers and support the floor; rectangular wooden beams, singly or in pairs, rolled steel I-beams, and plate girders are employed for floor beams.

When the roadway is to be paved, the pavement is supported on a **buckle-plate** floor which is riveted to steel I-beam joists. A buckle-plate is a steel plate with sides 2′ to 4′ and thickness $\frac{1}{4}''$ to $\frac{1}{2}''$, which is so bent that if placed horizontally its central point is 2″ to 3″ higher than its edges. Metal floors are also made of channel-shaped plates. Reinforced concrete is now taking the place of buckle-plates.

In **railway bridges** of less than 15′ span, the cross-ties are usually fastened to longitudinal stringers which rest on the piers and abutments; in railway bridges of longer span, these stringers rest on and are fastened to the floor beams. In railway bridges with metal floors the ties are embedded in the ballast, which is supported by the floor. For short spans, reinforced concrete floors are also used.

In order that the floor of a bridge may not become concave under its live load, the bridge is slightly arched from abutment to pier, and from pier to pier. This is called **camber**; and is theoretically so calculated that under the heaviest live load, the stringers will be horizontal.

1622. **Floor carriers.**—The different methods of supporting floors and transmitting the loads to the substructure are:

a. Beam
b. Plate girder
c. Truss
d. Arch
e. Cantilever
f. Suspension

Floor carriers of railway br dge spans under 25′ or 30′ generally consist of rolled I-beams: spans between 25′ or 30′ and 110′, of plate girders: spans between 110′ and 350′, of riveted trusses of single undivided panels: spans exceeding 350′, of pin-connected or riveted trusses with subdivided panels

In general, highway bridge spans of and below 20′ consist of rolled beams or simply wooden joists; spans from 20′ to 30′, of rolled beams; spans from 30′ to 60′, of plate girders; spans from 60′ to 100′, of plate girders or open-webbed riveted girders of single undivided panels; spans from 100′ to 300′, of riveted trusses; and spans exceeding 300′, of pin-connected or riveted trusses.

In addition to the classification in paragraph 1603, bridges may be classified according to the floor carrier, as beam bridges, truss bridges, etc.

Beam Bridge.

1623. This is the simplest form of bridge. The load is carried from flooring to stringers and then to the substructure. The stringers are the beams: and are made of wood, structural shapes, or built-up beams. The beam bridge can be used only for short spans, usually up to 20′ in railway bridges, although I-beam bridges are sometimes constructed with spans of 35′.

Plate Girder Bridge.

1624. A plate girder bridge is composed of two or more plate girders to which the floor beams are riveted. These floor beams rest either on the flanges of the plate girders, or on brackets riveted to the web. Plate girder bridges are used extensively in railway construction, where the span is short. They may be classified as (1) deck, (2) through, (3) half-through plate girders. The ties in a deck plate girder bridge lie directly on top of the upper flanges of the plate girders. The accepted limit of length of this type is about 100′, although such structures are in existence with a 30 per cent greater span.

In a half-through plate girder bridge, the floor beams are connected to the girders by means of gusset plates. Their economic length is less than that with deck plate girder due to the cost of the steel in the floor beams and stringers. In steam railway bridges, the limiting span is about 100′; in electric railway and highway bridges, the maximum span is about 75′.

The superior limit of a plate girder span is governed by shipment facilities rather than by expense. The deck plate girder bridge is cheaper than a corresponding truss bridge up to the maximum length that can be safely hauled on railways. A plate girder 130′ long would require 4 flat cars. Also, since the depth of girder is about $\frac{1}{12}$ the span, the question of clearance when passing through tunnels must be taken into consideration.

The advantages of plate girder bridges are:

a. Low cost of manufacture.
b. Low erection and maintenance costs.
c. Not as liable to injury by accident as truss bridges.
d. Rigidity.
e. Can be overstressed more than a truss bridge without danger.

Truss Bridges.

1625. A truss bridge is composed of two or more parallel trusses to which the floor beams are attached, usually at the panel points of one of the chords. Trusses are ordinarily made of wood, wood and steel, or of steel alone. Steel trusses are either riveted or pin-connected at the panel points. Although riveted trusses are less economical than pin-connected trusses, they are preferred in railway construction up to a span of 500′, due to their rigidity. In highway construction where rigidity is not so important the maximum riveted truss span is taken as about 350′. The disadvantages of long riveted trusses are the secondary stresses introduced at panel points, and the additional cost of connecting plates as compared with pins. In a 500′ span, there is 8 per cent more metal in a riveted truss than in a pin-connected truss. This per cent increases rapidly with the span.

The length of a truss is usually governed by the local conditions; it is ordinarily assumed in a bridge consisting of a number of spans that the arrangement is economical if the costs of the substructure and the superstructure are approximately the same. In medium spans, the height of the truss is about $\frac{1}{6}$ to $\frac{1}{8}$ the span; in very long trusses, this ratio is reduced.

There are many forms of trusses. The first truss was the king-post truss (Fig. 1625A) which was invented by Palladio, an Italian in 1570. Then came the queen-post truss. (Fig. 1625B.) But truss bridges were not seriously considered as the solution for long spans until the invention in 1798 of the Burr truss by Theodore Burr, an American. The Burr truss was a series of king-post trusses, with wooden tension members and counterbraces, and was later strengthened by an arch (Fig. 1625C). Howe in 1840 patented the Howe truss, which was a great improvement over the Burr: it had counterbraces; but it had metal tension members

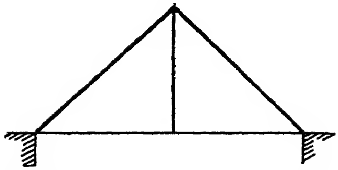


Fig. 1625 A King-Post Truss

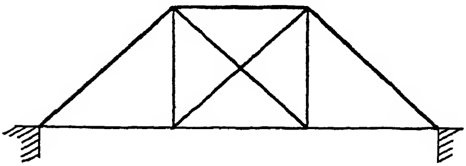


Fig. 1625 B Queen-Post Truss

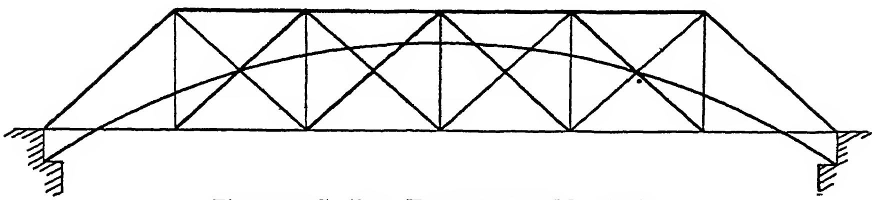


Fig. 1625 C Burr Truss Stiffened by Arch

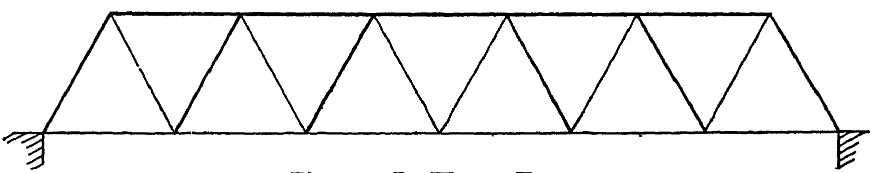


Fig. 1625 D Warren Truss

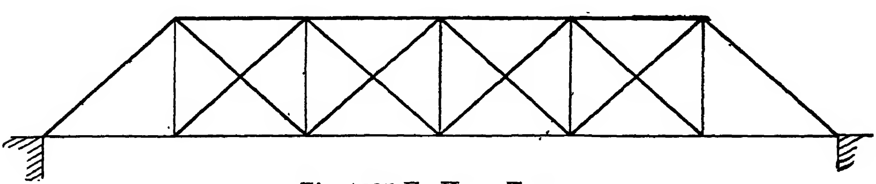


Fig. 1625 E Howe Truss

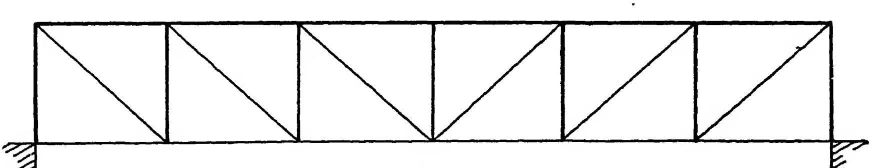


Fig. 1625 F Pratt Truss without Counterbraces

instead of wooden ones, and it omitted the arch. The types now in general use are the Warren invented in 1838 (Fig. 1625D), the Howe invented in 1840 (Fig. 1625E), the Pratt invented in 1844 (Fig. 1625F), the Bowstring invented in 1847 (Fig. 1625G), and the Baltimore invented in 1877 and generally used in the form known as the Petit truss (Fig. 1625H).

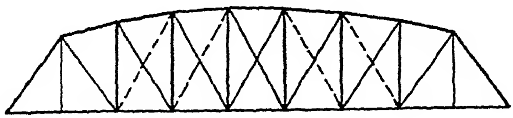


Fig. 1625 G Bowstring Truss

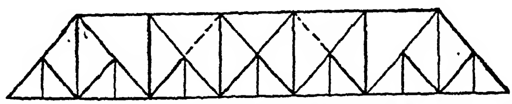


Fig. 1625 H Petit Truss with Sub-struts and Parallel Chords (Baltimore Truss).

The trusses generally used for highway bridges are as follows: for open-webbed riveted girders, the Pratt truss, or the Warren with verticals dividing the panels, the latter being employed for deck spans carrying joists resting on the top chords: for riveted spans up to about 250′, Pratt trusses with top chords either straight or inclined: for spans exceeding 250′, Petit trusses. These limiting lengths are not fixed absolutely, as the best limits will vary somewhat with the width of the bridge and the live load to be carried.

The trusses generally used in railway bridges are as follows: for deck spans having top chords supporting wooden ties, the Warren truss with verticals dividing the panels of the top chords; for other deck spans and through spans up to 300′, the Pratt truss; for spans exceeding 300′, the Petit truss. For through spans up to about 200′, parallel chords are employed; but for longer spans the top chords are generally inclined as in the Bowstring truss. These limiting lengths are not fixed absolutely, as the best limits will vary somewhat with the number of tracks and the weight of trains.

Howe Truss.—This is the type of truss usually employed when timber and simple forms of iron are to be used. It is the principal truss employed for hasty work in military operations. The chords, the main diagonals, and the counters are made of timber; the vertical braces, of iron rods. The floor beams rest upon and are supported by one of the chords. The king-post type is employed when the span does not exceed 30′; the queen-post with counter-struts when the span does not exceed 40′; and the type shown in Fig. 1625I for longer spans.

The diagonals are of uniform size, the main diagonals being in pairs, and enclosing the counters which may be single or double. Between the ends of the diagonals and the chords, blocks of hard wood or cast iron are inserted in shallow notches in the chords, as shown in the figure.

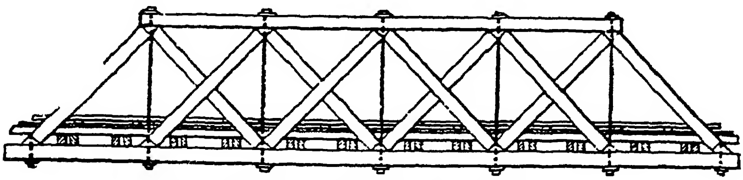


Fig. 1625 I Wooden Howe Truss

The faces of these blocks are at right angles to the axes of the diagonals. The verticals are in pairs, pass through the blocks and the chords, and are secured with nuts screwed on the ends of the rods, and kept from pressing into the timber of the chords by iron plates or washers. Sometimes a turn-buckle arrangement in the middle of the rod may be used, but this is not customary.

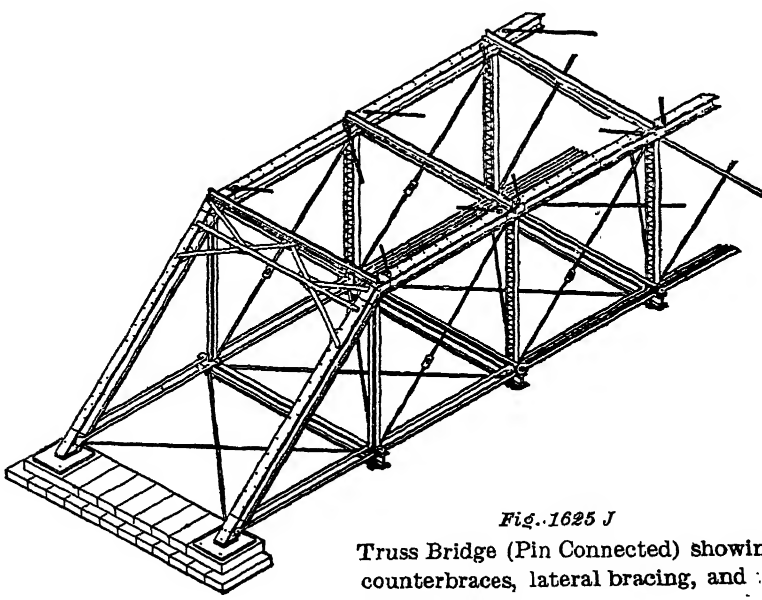


Fig. 1625 J
Truss Bridge (Pin Connected) showing counterbraces, lateral bracing, and portal bracing

Warren Truss.—This type is ordinarily used for spans from 20′ to 100′. The top chords of Warren trusses are usually made of two channels or angles, with or without cover-plate. The diagonals are made up

in the same manner. The diagonals and their connections must be designed to take both tension and compression.

In the riveted Warren truss, the general construction is the same, but the members are usually all laced or latticed pieces and are riveted together.

Pratt Truss.—This type is ordinarily used for spans from 20′ to 200′. The Pratt truss may be built with parallel chords or inclined top chords. It is simple in design and economical of metal. When spans are in excess of 175′, the top and bottom chords are seldom parallel. Choice between parallel chord and inclined chord is largely a question of expense. Since the bending moments increase from the end supports to the center of the span, increase in the amount of metal in the chord can be avoided by increasing the depth of the panels towards the center.

In a pin-connected Pratt truss of long span, the compression chord is made of two channels, either rolled or built up, connected by a top horizontal cover-plate. The inclined end posts (Fig. 1625 J) are similar in construction and are rigidly connected with the compression chord; and the vertical struts are made of two laced or latticed channels.

Bowstring Truss.—In the bowstring truss, the panel points of the upper chord lie in a curve, convex upward. If this curve is made a parabola, the dead load stress is uniform throughout the lower chord, the stress in all verticals is tensile and is equal to the panel loads at the lower ends, and the stress in all diagonals is zero. With a small live load, the stresses in the diagonals are very small, and the lower chord stresses remain almost equal. This uniformity of stress simplifies the design and construction, and is considered a great advantage of this type. It has the disadvantage that no overhead bracing is practicable near the ends of the span.

Petit (Baltimore) Truss.—The Petit truss is used for spans exceeding 250′ and the top chord is usually polygonal. It can be used for very long spans up to 1000′, although no Petit truss has yet been constructed with that span. The Petit truss is a subdivided Pratt, and has many forms. That most used is shown in Fig. 1625H.

1626. **Details of Trusses.**—A deck truss bridge consists of two or more vertical trusses which carry the floor system and vertical live loads, and of two horizontal trusses to carry the wind loads and resist swaying and vibration. The horizontal trusses of a deck bridge have much smaller web members than the two main vertical trusses; their chord members are the chord members of the main vertical trusses, and the stresses are proportionally very small as compared with the main vertical loads.

In a through truss bridge, there are the same main vertical trusses;

and there are also the same horizontal trusses for the wind loads. However, while in a deck bridge, each of the horizontal wind load trusses has support on the piers and abutments and can transmit its load directly to them, there is no support for the upper horizontal truss of a through bridge and it must transmit its wind load to the support of the lower horizontal truss. This transmission can not be made by a regular truss system because of the fact that the trains or the vehicles which use the through bridge must have a clear passage; therefore, it is possible to brace only the upper part of the space between the upper and lower trusses. Below this bracing, the effect of the wind is as though the upright columns of the main vertical trusses were cantilevers with the wind loads acting throughout. The wind loads are so small comparatively that the weight of the truss and the rigidity of its riveting is sufficient to prevent deformation of any of its parts. If a case should arise of need for such bracing, it would be placed on the outside of the main vertical trusses and would lead to properly prepared supports.

Figure 1626A shows sway bracing at the top as constructed between the upper parts of the main vertical trusses to make this part rigid and to transmit at least this much wind pressure without cantilever effect

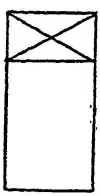


Fig. 1626 A

until a point is reached below the **sway bracing.** The sway bracing at the entrance of the span, as shown in Fig. 1625 J, is called **portal bracing.**

In a pony truss or plate girder bridge, there is no upper horizontal wind truss. Brackets are placed on sides of such bridges to secure lateral rigidity.

The clear headroom in truss bridges should be at least 15′ for highway and 22′ for railway bridges.

The truss is prevented from moving laterally by friction and by bolting it to the abutments. A strut connects the bases of the end posts resting on the same abutment and divides the wind pressure between the bed plates.

The ends of the girders rest on steel bedplates which have sufficient area to distribute the weight and reduce the pressure on the masonry within safe limits. In all plate girders and truss bridges if the friction is not sufficient to resist the lateral pressure from the wind, the girders

and truss are so fastened to the abutments that lateral movement is impossible. To prevent longitudinal movement, one end is anchored to the abutment; to allow longitudinal expansion, the other end is supported on a smooth plate if the span is short, and on steel rollers if the span is long.

Temperature Stresses.—Allowance is made in bridge design for a variation of temperature of 150°.

Arch Bridges.

1627. Arch bridges are constructed of masonry, of reinforced concrete, and of steel alone. The arch (paragraph 1120) is the type which is constructed whenever it is desired to secure a structure of fine architectural appearance: it is the only type that can be made wholly of masonry. The form of arch most generally used is segmental. The form of soffit is governed by the width of the span, the highest water level during freshets, the approaches to the bridge, and the architectural effect which may be produced by the structure, as it is more or less exposed to view at the intermediate stages between high and low water.

Oval and segmental arches are preferable to the full center arch, particularly for medium and wide bays, for the reasons that for the same level of roadway they afford a more ample waterway under them, and their heads and spandrels offer a smaller surface to the pressure of the water during freshets than the full center arch, under like circumstances. The full center arch, from the simplicity of its construction and its strength, is to be preferred to any other arch for bridges over water courses of a uniformly moderate current and not subjected to considerable changes in their water-level, particularly when its adoption does not demand expensive embankments for the approaches.

If the spans are to be of the same length, the curves of the arches should be the same throughout. If the spans are to be of unequal length, the longest should occupy the center of the structure, and those on each side of the center should either be of equal length or decrease uniformly from the center to each extremity of the bridge. In this case the curves of the arches should be similar, and the springing lines should be on the same level throughout the bridge. The level of the springing lines will depend upon the rise of the arches, and the height of their crowns above the water-level of the highest freshets. The crown of the arches should not, as a general rule, be less than 3′ above the highest known water-level, in order that a passageway may be left for floating bodies descending during freshets. Between this, the lowest position of the crown, and any other, the rise should be so chosen that the approaches, on one hand, may not be unnecessarily raised, nor, on the other, the

springing lines be placed so low as to mar the architectural effect of the structure during the ordinary stages of the water.

Masonry Arches.—The masonry arch has been employed in bridge construction since it was first developed by the Romans. It is practically indestructible if well founded, and requires little expense for maintenance. It is, however, more expensive than a metal bridge, and its employment is limited to shorter spans.

Masonry arches are made of stone, brick, or concrete. Stone arches have been constructed with spans of about 300′, brick arches with spans of about 150′, and concrete arches with spans of about 140′.

The centers used in construction should be strong, so as to settle as little as possible during the construction of the arch, and for wide spans, should be so constructed that they can be removed without causing extra strains on the arch. This is effected by removing the centering from the entire arch at the same time.

In wide spans, the centers are removed by means of an arrangement of wedge blocks. Another method of removing centers is by the use of sand, the centers resting upon cylinders filled with sand, so arranged that the sand can run out slowly near the bottom.

. The face walls of the spandrel may be continuous, or they may consist of transverse arches which rest on the arch and support the roadway. (Fig. 1627A.) The latter arch presents the more pleasing appearance,

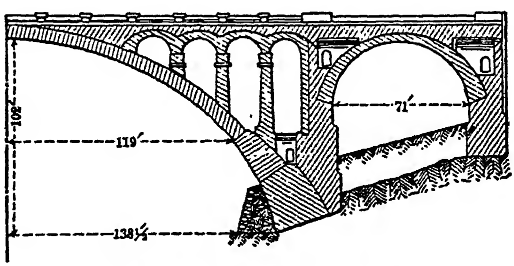


Fig. 1627 A · Spandrel Arches

and has a smaller load on the arch ring, so that the dimensions of the arch ring and the abutments may therefore be reduced.

To prevent the percolation of water through the arch ring, it is coated with an asphalt or other waterproof coating; the water runs into a cross drain at the lowest point of the extrados and is conveyed through the walls by a suitable conductor.

In ordinary masonry or concrete arches tensile stresses are not permissible; the ring must therefore be designed so that the line of pressure will not pass outside the middle third (paragraph 1108). In reinforced arches this limitation does not exist; the arch rib is a beam, and if

properly reinforced it may carry heavy bending moments involving tensile stresses in the steel. Besides this advantage, the reinforcement greatly aids in preventing cracks due to any slight settlement, and by reason of greater reliability makes possible the use of working stresses in the concrete considerably higher than are used in plain masonry.

Since a reinforced arch is a beam subject to either positive or negative bending moment, it is essential that it should be reinforced on both sides. The shearing stresses are relatively small, and little is needed in the way of web reinforcement.

In the Monier system, the reinforcement consists of wire netting, one net being placed near the extrados and one near the intrados.

In the Melan type, the steel is in the form of ribs of rolled I-sections, or of built-up lattice girders, placed near the intrados and extrados at intervals of 2′ to 3′.

In practice the amount of reinforcement along extrados and intrados thus varies from ½ per cent to 4 or 5 per cent of total area of cross-section.

Advantage is generally taken of the adaptability of reinforced concrete in the various details of the arch. The roadway is generally supported by spandrel arches, or by beam and slab construction resting on piers or cross walls, which in turn rest upon the main arch or arch ribs. The design of these parts is carried out as for other beam and column designs.

Steel Arches.—Steel arches are arched trusses and are divided into three general classes: those with hinges at the crown and springing lines, those with hinges at the springing lines only, and those without hinges. The hinge is a heavy steel pin against which the arch or its segments abut, and about which the arch can revolve slightly so as to accommodate itself to expansions and contractions caused by variations of temperature.

In the three-hinged arch the reactions at the crown and springing lines can be determined with the same accuracy as those of the simple truss, as explained in paragraphs 1126 and 1127. For this reason, it is preferred by many engineers to the other types; it is however less stiff than the other types. One of the most notable arches of this type is the railway bridge over the Viaur River in France (Fig. 1627B). The span of the arch is 721′, its rise 176′, and the height of the roadway above the river 380 feet.

In the two-hinged arch the reactions cannot be determined with as great accuracy as in the three-hinged one, but it is stiffer, and has many advocates. One of the most notable bridges of this type is the roadway over the Niagara River below the Falls. It has a two-hinged arch of the type shown in Fig. 1627C, which has a span of 840′, a rise of 137′, and carries a 46′ roadway 240′ above the water level.

The arch without hinges is not now employed for long spans; the

bridge over the Mississippi at St. Louis, constructed in 1874, is one of the most notable examples. This bridge has one span of 520′ and two spans of 502′ each.

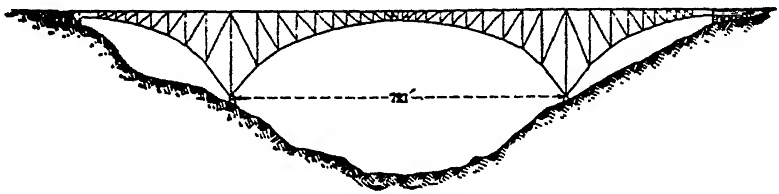


Fig. 1627 B Viaur River Bridge

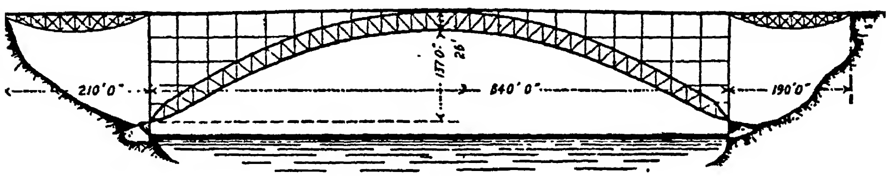


Fig. 1627 C. Niagara-Clifton Bridge over the Niagara River.

The roadway of a steel arch is supported by floor beams which are attached by steel members to the panel points of the arch, or are placed directly at the upper or lower panel points of the arch truss when these points are at the proper elevation. As steel arch bridges are almost invariably of the deck type, the floor beams are usually supported at the upper panel points. When the upper and lower chords of the arch are parallel, the loads of the floor beams are transmitted directly in short spans by short members to each panel point, but in long spans there is a roadway truss or girder which is supported by a series of vertical truss members which receive the weight of the roadway and transmit it to certain panel points of the arch truss. Even when the floor beams transmit the load directly to panel points of the arch truss, it is necessary to have a truss or plate girder to stiffen the roadway.

Cantilever Bridges.

1628. Cantilever bridges are suitable only for long spans. The economic limit of an ordinary truss span is 660′, beyond which the cantilever truss is cheaper. Short span cantilevers may be used over deep gorges where the cost of scaffolding for erection would be prohibitive, or over streams with swift current where false work would be washed out. Such structures may however be cantilevers only during construction and simple truss spans or arches when completed.

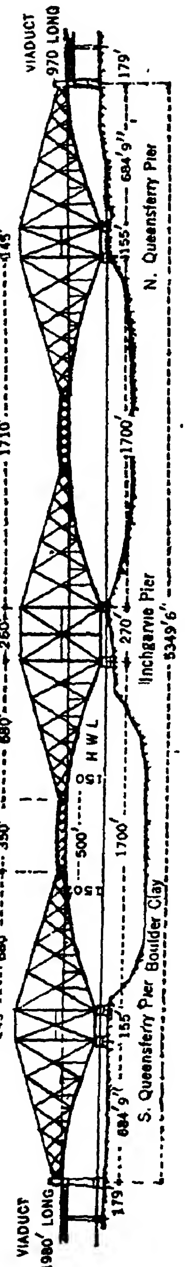


Fig 1628 A Bridge over the Firth of Forth at Queensferry, Scotland.

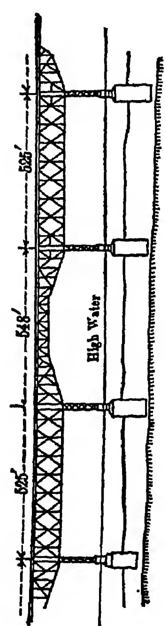


Fig 1628 B Pougkeepsie Bridge (cantilever)

The principal disadvantage of a cantilever bridge is its lack of rigidity as compared with a simple truss bridge of the same span.

The advantages of a cantilever for long spans are: (1) little false work required; (2) few piers required.

Cantilever Truss.—Cantilever bridges are of two general types, in each of which there is a cantilever truss with a suspended weight (truss) at its end. In the first type (Fig. 1628A), each pier supports a tower with long double cantilever arms, and the ends of the arms of adjacent towers are the supports for a simple or suspended truss which connects them. In the Forth bridge, Scotland (Fig. 1628A), there are three towers with cantilever arms. Each tower is about 330′ high. The central tower rests on a base 120′ by 260′, and the distance between the ends of its cantilever arms is 1620′. It is connected with the other towers by riveted trusses 350′ long. The shore ends of the other towers rest on the abutments. The suspended trusses are 150′ above high water level. Each tower rests on four masonry piers; the distance between adjacent piers of consecutive towers is 1710′ center to center. The St. Lawrence cantilever bridge near Quebec has a central span of 1800. between piers, center to center; it similarly supports a suspended truss′

In the second type, shown in Fig. 1628B, a simple truss resting on two points of support has attached to its ends cantilevers whose extremities form the supports for a simple suspended truss. Fig. 1628B shows the bridge over the Hudson River at Poughkeepsie; it has two simple truss spans of 525′ and three cantilever spans of 548′. The bridge over the Mississippi River at Memphis is of the same general type; it has a simple truss span of 621′, a single cantilever span of 621′, and a double cantilever span of 790′.

Suspension Bridges.

1629. A suspension bridge (Fig. 1629A) is one in which the floor is suspended from two or more cables or chains which are stretched between

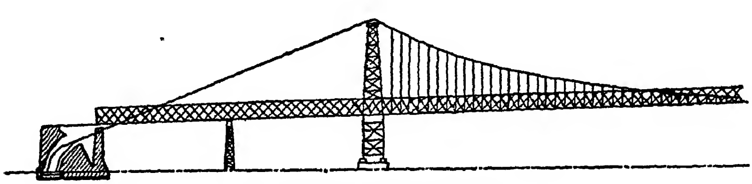


Fig 1629 A Supension Bridge

the piers or abutments. It is probably the only type which can be applied to spans exceeding 2000′; and it has been proposed to build suspension bridges of 3000′ span.

The essential parts of a suspension bridge are the main **cables** or **chains**, from which the floor is suspended; the **towers**, upon which these cables rest; the **anchorages**, which hold the ends of the main cables; the **suspenders**, by which the floor is attached to the main cables; the **trusses**, or other devices, by which the oscillations are checked; and the **floor system** itself.

Cables for small bridges are made of wire rope, and for large structures they are made by compacting a number of steel wires into cylindrical form and clamping them together by means of steel clamps. The Williamsburg Bridge in New York has four main cables, each composed of 7696 wires, each 0.19″ in diameter, clamped together and covered between clamps with cotton duck soaked in oxidized linseed oil and gum-varnish composition and with a sheet iron shell. The wire has a minimum tensile strength of 200,000 pounds per square inch.

Cable made of eye-bars is preferred by some engineers to the wire cable because by varying the size of the eye-bars the tensile strength at any point of the cable can be proportioned to the stress at that point. The allowable unit stress of steel in the form of bars is, however, much less than in the form of wire; and the weight of the cable must therefore be greatly increased. A notable bridge of this class is at Budapest, Hungary.

The curve of the unloaded suspension bridge cable due to its own weight is by definition a catenary. If its weight, which is small compared with the uniformly distributed load, is neglected, the curve of the cable due to the uniformly distributed load is a parabola.

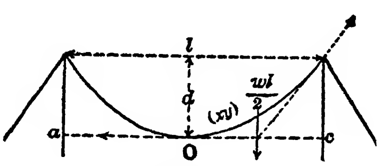


Fig. 1629 B

In Fig. 1629B, let the curve represent the cable as deflected by a load uniformly distributed along aOc.

Let l = span.
" D = deflection of cable at middle point.
" w = weight per unit length of roadway.
" S_t' = tension in cable at O.
" S_t = tension in cable at any other point.
" x and y = co-ordinates of any point.

If we take that portion of the cable from O to any point (x, y), and consider it as a "free body," it must be in equilibrium. Representing

the tension at (x, y) by S_t'', and remembering that the weight from O to (x, y), equals wx, we have by taking moments around (x, y)

$$S_t'y - (wx)\frac{x}{2} + S_t'' \times 0 = 0 \qquad (1629A)$$

$$x^2 = \frac{2\,S_t'y}{w} \qquad (1629B)$$

This is the equation of the curve of the cable. From the form of the equation, we know that it is the equation of a parabola referred to rectangular co-ordinate axes through its vertex.

To find the tension at O, we substitute for x and y in the equation of the curve, the co-ordinates of any known point as that at the top of the tower. The resulting equation is

$$\frac{l^2}{4} = \frac{2\,S_t'D}{w} \qquad (1629C)$$

$$S_t' = \frac{wl^2}{8\,D} \qquad (1629D)$$

Since there is only tension in the cable, the value of S_t at any point between O and the tower, as x, must be equal to the resultant of the other forces acting on the cable between O and x. These forces are S_t' and wx; hence $S_t = \sqrt{w^2x^2 + S_t'^2}$. Since S_t increases as x^2 increases, the tension in the cable will be a minimum at O and a maximum at the tower. By making $x = 0$, we find the minimum value of S_t is S_t'. By making $x = l/2$ and substituting for S_t' its value found above we have

$$S_t = \sqrt{\frac{w^2l^2}{4} + \frac{w^2l^4}{64\,D^2}} = \frac{wl}{2}\sqrt{1 + \frac{l^2}{16\,D^2}} \qquad (1629E)$$

From the values of S_t' and S_t we see that if the span and loading remain constant, the tension in the cable may be decreased by increasing its deflection.

Towers.—The towers are constructed of masonry or steel. The height of the towers is fixed by the clearance which must be left for navigation under the bridge, and by the maximum tensile stress which is to be allowed in the cables.

Each tower is subjected to (1) a vertical force which is equal to the sum of vertical components of the tensile stresses in the two branches of the cable at the top of the tower; (2) a horizontal force which is equal to the difference between the horizontal components of the tensile stress in the two branches at the same point. As the horizontal force tends to overturn the tower, it is desirable to reduce its intensity to a minimum. It reduces to zero whenever the two branches of the cable

make the same angle with the vertical through their point of intersection. This is effected by resting the cable on a saddle (Fig. 1629C) supported by rollers, so that the saddle may move longitudinally to restore the equality between the angles whenever this equality is disturbed by the

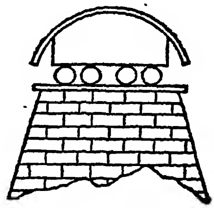


Fig. 1629 C
Saddle for Supension Bridge Cable

elongation or contraction of the cable due to changes of temperature or to the oscillations produced by a moving load. The same effect would be produced by rockers or a rocking tower.

Anchorages.—Anchorages for the cables are provided by attaching them to eye-bar chains and girders or castings which are set in masonry at each end of the bridge. The chain of eye-bars may be inclined in a straight line or in a series of broken lines, and they may be placed in an inspection tunnel as in the Williamsburg Bridge or may be completely

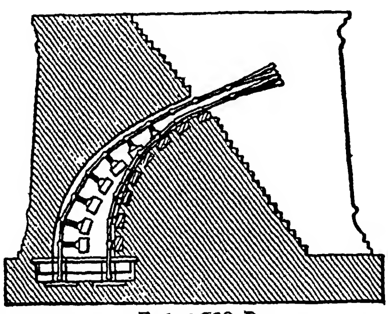


Fig. 1629 D
Anchorage for Supension Bridge Cable

embedded in concrete. The eye-bars of the anchor chains in the Point Bridge in Pittsburgh were embedded in a poor quality of masonry, and when examined after 27 years of service were found to be in good condition except that they were pitted considerably for 5 or 6 feet near the surface of the ground. Fig. 1629D shows an anchorage with double cables. The upper eye-bar chain is supported by plate girders extending across the tunnel, with their ends resting on large blocks of stone.

Suspenders.—The suspenders are the vertical cables which connect the floor system and the cables. When two cables support each end of the floor beams, compensating devices are frequently introduced, so that, whatever be the relative positions of the two cables vertically, each will bear one-half the total load.

Stiffening Devices.—If the floor beams are attached only to the suspenders and are not rigidly connected to each other, a moving load will cause each floor beam to oscillate in a vertical plane. Various methods have been devised for checking these oscillations. One of the most effective is the **stiffening truss,** on each side of the roadway. This is a vertical truss which is fastened to the floor beams. It is so designed that it can resist pressure acting vertically either upwards or downwards. When a concentrated load comes on a suspender the load is transferred from the cable to the truss as soon as the former yields at the point of juncture of suspender and cable; the truss distributes the load over several floor beams and suspenders and thus over a long section of the cables.

Another kind of stiffening truss is made by using the cable as one of the chords of a vertical truss and connecting by suitable web bracing to a second chord, designed to resist oscillation stresses only. This method has been applied to bridges with eye-bar chains. Other methods of checking the oscillations consist in guying the main cable by ropes tied to the towers above or below the main cable, and by ropes attached to suitable outside points.

Floor.—The floor system differs in no essential details from the floor system of truss bridges, but its camber is greater than that of a truss bridge.

1630. **Bear Mountain Bridge.**—The new suspension bridge across the Hudson River at Bear Mountain, 6 miles below West Point, N. Y. has a clear span of 1632′ (one of the longest spans in the world), and is founded upon solid rock. As the mountains are close to the river bank on both sides, ample clearance for navigation (153′ at center) is obtained without the necessity of constructing long approaches above the surrounding country. The bridge is a highway bridge only, carrying a reinforced concrete roadway 38′ wide and sidewalks on each side of the roadway. The shore spans are not suspended from the cables, but are carried on trusses from the towers to concrete piers on the banks; the west approach 300′ long is carried on two trusses, and the east approach 320′ long on one truss and two plate girder spans. Each of the two main cables consists of 7252 galvanized steel wires, 0.192″ diameter, laid parallel and formed into 37 strands of 196 wires each, giving a total cross-sectional area of 210 square inches. The maximum stress in each

cable due to live and dead loads is about 65,000 pounds per square inch.

Each cable was formed in place, two wires being drawn across at a time. The free end of the wire was taken from the wire reel and fastened to a grooved casting, called a shoe, fixed at the same end of the bridge. A wheel was placed in the loop of the wire between the reel and the shoe, and made to travel along the line of the suspension cable by means of a traveling wire rope just above this line. As the wheel moved across, the wire was unreeled and two wires were thus carried across. On the other side the wheel was transferred to a new loop and the process was repeated from the other end. Each two wires were adjusted to the proper sag before the next two were placed. 196 wires when laid were fastened together to form a strand, and when all strands were laid the whole was compressed, wrapped with wire, and painted with red lead.

The towers are of steel, 350′ high. They are supported on concrete footings resting on rock. The maximum load on each tower is 9217 tons. There is no provision for movement of the cables on saddles after the bridge is in use, hence there will be at times an unbalanced horizontal force at the top of each tower. The towers are designed to carry the resulting bending moment. The anchorages are inclined tunnels driven into solid rock, and are 100′ deep. The cable shoes are pin-connected to eye-bar chains at the ground level, the eye-bar chains reaching into the anchorage to the cast-steel bases at the bottom. The tunnels are filled with concrete. The stiffening trusses on the bridge are 30′ deep, continuous from tower to tower, and carry the roadway on their upper chords. The horizontal truss at the level of the lower chords forms the wind bracing, and the struts of this truss, extended beyond the stiffening trusses form the points of attachment for the 64 suspender cables from the main cables.

1631. **Problem.**—A plate girder railway bridge, with 2 girders, 50′ span, weighs 2 tons per linear foot. A train weighing $2\frac{1}{2}$ tons per linear foot, and longer than the bridge, passes over the bridge. Find the maximum shear in each girder 20′ from the end of the bridge.

Solution.—The dead load shear at the given point for one girder is

$$25 - 20 = 5.$$

The live load shear is maximum when the head of the train is at the given point, the train covering the 30′ segment of the bridge. Its value is

$$15/50 \times (30 \times 2.5/2) = 11.25$$

Adding live and dead load shears, maximum shear is

$$5 + 11.25 = 16.25 \text{ tons}$$

MOVABLE BRIDGES.

1632. A draw or movable bridge is one in which one or more spans can be temporarily raised or removed to allow the passage of vessels. Such bridges are required over navigable rivers with low banks.

The draw-span may be opened in several ways; it may be

a. Lifted vertically.
b. Moved longitudinally or laterally out of position.
c. Revolved around a horizontal axis.
d. Revolved around a vertical axis.

In the first type, or **lift bridge,** the span is lifted vertically until the clearance is sufficient for passing vessels. The span is attached to counterweights by cables which pass over towers; the force to lift the dead load of the span is practically reduced to friction of the apparatus, thus requiring special effort of the hoisting apparatus only for lifting the live load. In the second type, the draw-span is supported by a car which carries away the span, or it is supported on a boat or ponton which is floated out of position.

In the third type, or **bascule bridge** (Fig. 1632A), the draw-span rotates about either a fixed or movable horizontal axis and may consist

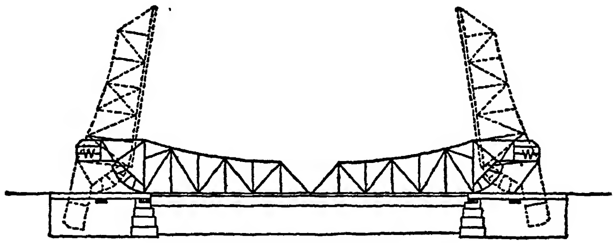


Fig. 1632 A Bascule Bridge

of one or two leaves. The leaf is rotated by means of counterweighted chains or cables attached to its outer end, which pass over pulleys in walls or towers above the abutments. Fig. 1632A represents a bascule bridge with a movable axis. The short arm supports a heavy counterweight *W*. This is the type commonly constructed in this country and has been applied to a span of 275′ in Chicago, Ill.

In the fourth type, or **swing bridge** (Fig. 1632B), the draw is in the form of a double cantilever with arms of equal or unequal length. If the span is short and a single opening is desired, the axis of rotation may be on the abutment and the shore cantilever may be short and heavy. Ordinarily, the axis of rotation is on a pier, the two arms are cantilevers of equal length, thus securing two equal openings when the bridge is swung open.

Swing bridges are preferred in wide rivers where the central pier is no serious obstruction to traffic: bascule bridges are preferred where the channel is narrow.

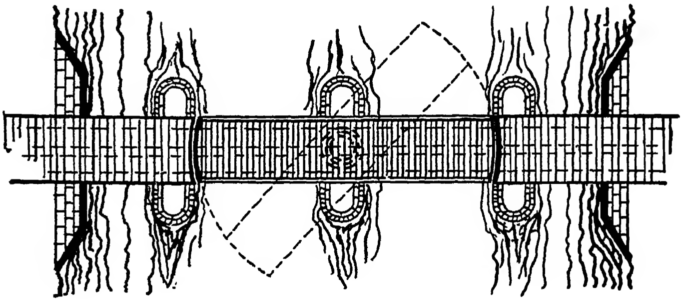


Fig. 1632 B. Swing Bridge

STRESSES IN BRIDGE TRUSSES.

1633. In the discussions on bridge trusses, the loads are taken as co-planar (acting in the plane containing the axes of the members) and so placed that the forces act only at the panel points. However, two trusses, one on each side of the highway, are normally used in each bridge. In large bridges, three or even four trusses are used. If two trusses support the bridge, each will carry one-half the total bridge loads; if there are three trusses, usually the middle one will carry one-half of the total loads and each of the other two, one-fourth. In general, it is necessary to make complete calculations for only one truss. The wind loads are considered in special calculations: and the wind trusses are made the subject of special design when necessary.

In all bridge trusses resting on end supports, the reactions are taken as acting along lines parallel to that of the resultant of the other external forces, unless the action lines of the reactions are fixed by special conditions. If the truss is designed to rest on a roller at either end, as is often done in order to allow for expansion and contraction of its members due to changes in temperatures, the reaction at that end is normal to the surface on which the roller rests. The same governs when there is no roller, as in short bridges, but where one end is left free to slide on a steel plate.

The reactions at the ends of a truss and all the other external forces therefore always form a system either of parallel or concurrent forces in equilibrium. The determination of the stresses is very similar to the

determination of the stresses in roof trusses, with the additional case of loads moving over the bridge, which does not occur in roof trusses.

The determination of stresses in bridge trusses is considered as follows:

a. Analytic methods by concurrent forces.
  (1) Actual stresses.
  (2) Index coefficients.
b. Analytic method by sections.
c. Stresses under moving loads.
d. Graphic method.

1634. **Theory of Beam, Girder, and Truss.**—An ordinary rectangular beam is subject to stresses of tension, compression, and shear: in a single beam, each fiber is subjected to both a longitudinal and a shearing stress, the longitudinal stress being greatest at the maximum distance from the neutral axis. In a plate girder, the number of the outer fibers is increased and the distance to them is made so great that it may be considered that the fibers farthest from the neutral axis take only longitudinal stresses and take all of such stresses: the web of the girder is often assumed to take all of the shear, though it is evident that some of the shear is taken by the fibers near the surface, which are assumed to take only longitudinal stresses. In a truss, the web is replaced by steel web members, and the members of the truss take longitudinal stresses only: if the truss has parallel chords, the chords take the longitudinal stresses of tension and compression, and the web members take the shear.

In making the transformation, the flanges of the girder become the chords of the truss, which, like the flanges, are designed to resist the entire bending moment of the forces acting on the truss. The stresses in both girder flanges and truss chords are similar longitudinal stresses; the only difference between them being that the girder flange stress changes at every web-rivet and is uniform between rivets, while the truss chord stress changes at every panel point, and is uniform between panel points. The web members perform in the truss the same function as the web in the girder; they are designed to resist the entire vertical and horizontal shear of the forces which act on the truss. The horizontal component in each web member must therefore be equal to the horizontal shear, and the vertical component equal to the vertical shear, at any particular section of the truss. In the girder, the loads and horizontal shears are transmitted from the rivets of the upper flange to the web plate and produce shearing stresses in it; in the truss the loads and horizontal shears are transmitted from the panel points of the chords to the web members, and produce longitudinal stresses in them. The connections at the panel points of the truss perform the same function

as the web rivets in the girder; that is they connect the chord and web members and, by transferring the excess longitudinal chord stress to the web members, allow the cross-sections of the different members of the chords to vary with the bending moment.

ANALYTIC METHOD BY CONCURRENT FORCES.

1635. **Actual Stresses.**—As there must be equilibrium at every point of the truss, there must be equilibrium at every panel point. Therefore, we may consider the whole truss as a "free body," or any part of it, or any panel point of it. In each case, by substituting for any member the force which must take its place in order to produce equilibrium, we have for the "free body" considered a system of forces which must produce equilibrium.

At each panel point, there is a system of concurrent forces: and the algebraic sums of the vertical and horizontal components must each be equal to zero.

1636. Let it be required to determine the stresses in the bridge truss with parallel chords and loads as shown in Fig. 1636A.

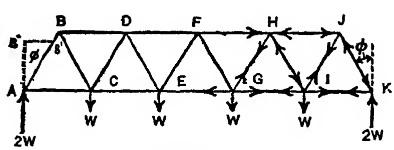


Fig. 1636 A

Panel Point A.—Considering the panel point as a "free body," we see that the vertical component of AB must be equal to $-2\,W$, this being necessary in order that the sum of the vertical components shall be equal to zero.

The total stress of AB is obtained by the equation

$$\frac{-2\,W}{AB'} = -\cos\phi \qquad (1636A)$$

$$AB' = \frac{2\,W}{\cos\phi} \qquad (1636B)$$

The horizontal component of AB is obtained by the equation

$$\frac{B'B''}{AB'} = \sin\phi \qquad (1636C)$$

$$B'B'' = AB'\sin\phi = \frac{-2\,W\sin\phi}{\cos\phi} = -2\,W\tan\phi$$

Bearing in mind the directions of the forces and stresses, we write the stresses with proper signs (see par. 109) in tabulated form as shown below:

PANEL POINT *A*

Force	Vertical Component	Horizontal Component	Stress	Character
$2\,W$	$-2\,W$	0	$2\,W$	Reaction
Stress *BA*	$+2\,W$	$-2\,W \tan\phi$	$\dfrac{2\,W}{\cos\phi}$	Compressive
" *AC*	0	$+2\,W \tan\phi$	$2\,W \tan\phi$	Tensile

Panel Point B.—Considering the panel point as a "free body," we see that *BC* and *BD* are both unknown. But we know the vertical component of *BC* must be equal and opposite to that of *AB*, to produce equilibrium: hence this vertical component must be equal to $2\,W$. Its total stress must be equal to that of *AB*, $2\,W/\cos\phi$, because it makes the same angle with the vertical and has the same vertical component: in direction, *BC* must act downward, to balance *AB*, hence it is tensile. *BD* must have a horizontal component equal to the sum of that of *AB*, and *BC*, or twice that of *AB*; hence it must equal $4\,W \tan\phi$.

Bearing in mind the directions of the forces and stresses, we write the stresses with proper signs in tabulated form as shown below:

PANEL POINT *B*

Force	Vertical Component	Horizontal Component	Stress	Character
Stress *AB*	$+2\,W$	$+2\,W \tan\phi$	$\dfrac{2\,W}{\cos\phi}$	Compressive
" *BC*	$-2\,W$	$+2\,W \tan\phi$	$\dfrac{2\,W}{\cos\phi}$	Tensile
" *DB*	0	$-4\,W \tan\phi$	$4\,W \tan\phi$	Compressive

Similarly, we find the stresses at panel points *C*, *D*, and *E*, to be as in tables below:

PANEL POINT *C*

Force	Vertical Component	Horizontal Component	Stress	Character
Stress *CB*	$+2\,W$	$-2\,W \tan \phi$	$\dfrac{2\,W}{\cos \phi}$	Tensile
" *CA*	0	$-2\,W \tan \phi$	$2\,W \tan \phi$	Tensile
" *W*	$-W$	0	W	Applied load
" *DC*	$-W$	$-\ W \tan \phi$	$\dfrac{W}{\cos \phi}$	Compressive
" *CE*	0	$+5\,W \tan \phi$	$5\,W \tan \phi$	Tensile

PANEL POINT *D*

Force	Vertical Component	Horizontal Component	Stress	Character
Stress *BD*	0	$+4\,W \tan \phi$	$4\,W \tan \phi$	Compressive
" *CD*	$+W$	$+\ W \tan \phi$	$\dfrac{W}{\cos \phi}$	Compressive
" *DE*	$-W$	$+\ W \tan \phi$	$\dfrac{W}{\cos \phi}$	Tensile
" *FD*	0	$-6\,W \tan \phi$	$6\,W \tan \phi$	Compressive

PANEL POINT *E*

Force	Vertical Component	Horizontal Component	Stress	Character
Stress *ED*	$+W$	$-\ W \tan \phi$	$\dfrac{W}{\cos \phi}$	Tensile
" *EC*	0	$-5\,W \tan \phi$	$5\,W \tan \phi$	Tensile
" *W*	$-W$	0	W	Applied load
" *EF*	0	0	0	....
" *EG*	0	$+6\,W \tan \phi$	$6\,W \tan \phi$	Tensile

1637. **Problem.**—Make up a table showing stresses in the various members of the Pratt truss shown in Fig. 1637A. Only one-half of the truss need be solved; the truss being symmetrical and symmetrically loaded the corresponding stresses in the two halves will be equal.

Solution.

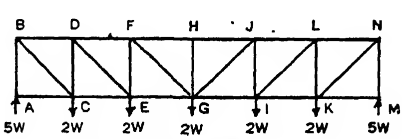


Fig. 1637 A **Pratt Truss**

Stress	Compression	Tension
	Upper Chord	
BD	$5\,W \tan \phi$	
DF	$8\,W \tan \phi$	
FH	$9\,W \tan \phi$	
	Lower Chord	
AC		0
CE		$5\,W \tan \phi$
EG		$8\,W \tan \phi$
	Web Members	
AB	$5\,W$	
BC		$5\,W/\cos \phi$
CD	$3\,W$	
DE		$3\,W/\cos \phi$
EF	$1\,W$	
FG		$1\,W/\cos \phi$
GH	0	

1638. **Index Coefficients.**—Inspection of the tables for each panel point in paragraph 1636 and of the table for all panel points in paragraph 1637 shows that each stress consists of a number, usually multiplied by some function of an angle. If we take each truss and write on the mem-

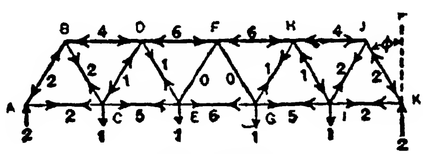


Fig. 1638 A

bers these numbers only, the trusses will be as shown in Figs. 1638A and 1638B.

If these diagrams are carefully examined, it will be seen that at every panel point the sum of the numerical coefficients of the stress which tend

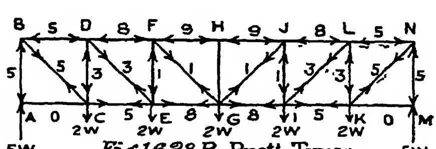


Fig 1638 B Pratt Truss

to move the point upward is exactly equal to the sum of the coefficients of the stresses which tend to move it downwards, and that the sum

of the numerical coefficients of the stresses which tend to move it to the left is exactly equal to the sum of the coefficients of the stresses which tend to move it to the right. This enables us to write out the index coefficients of the stresses of any truss with parallel chords and web members making the same angles with each other, provided we are careful to indicate by arrow heads the direction of the stress in each piece as soon as determined. The arrow heads will also indicate the character of the stress. A **compressive** stress must be indicated as acting **towards both extremities** of the member in which it is found, and a **tensile stress** as acting away from both **extremities** of the member in which it is found. As a matter of convenience, in solutions, the arrow heads are often omitted: then blue or minus indicates tension, red or plus indicates compression.

1639. Let us consider, for example, the truss in Fig. 1638A, and check the numerical coefficients as indicated.

Panel Point A.—The index coefficient (reaction) 2 is acting vertically: therefore AB must have an index coefficient of 2 as shown, acting downward.

Considering horizontal index coefficients: The member AB has an index coefficient of 2 acting to the left, therefore AC must have an index coefficient of 2 as shown acting to the right. The index coefficients are easily obtained thus until we reach panel point E.

Panel Point E.—The index coefficient of DE was found at panel point D to be 1 and acting downward and away from the panel point: therefore as regards the panel point E, it must act upward with an index coefficient of 1. As the vertical force (load) at the panel point has an index coefficient of 1, there will be no index coefficient for EF and consequently no stress in it.

Considering horizontal index coefficients: The member DE was found at panel point D to be 1 and acting away from the panel point; the member CE was found at panel point C to be 5 and acting away from the panel point; therefore, to balance these, the member EG must have an index coefficient of 6 and must act away from the panel point, that is to the right, and be in tension.

In considering and using index coefficients, it must always be remembered these are not actual stresses: they are numerical coefficients of some expression as $2\,W/\cos\phi$ or $2\,W\tan\phi$ or even $2\,W$ in some cases, and the true stresses are always obtained by multiplying the index coefficients by the remainder of the expression.

1640. **Problem.**—Write the index coefficients on the members of the truss in Fig. 1640A.

Solution.—See Fig. 1640B.

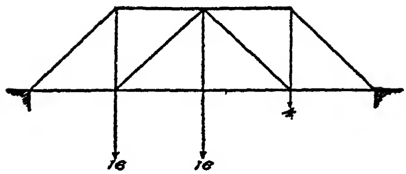


Fig. 1640 A

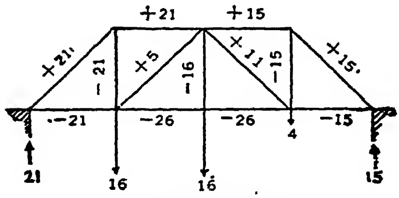


Fig. 1640 B

ANALYTIC METHOD BY SECTIONS.

1641. This method for determining stresses in bridge trusses is like that for determining stresses in roof trusses. A section is passed through the truss and cutting the member in which the stress is to be determined. Then all of the truss on one side of the plane is considered as a "free body," and forces known or unknown as the case may be are substituted for the stresses in the members cut. The stresses are then determined by equations of moments, the center of moments being preferably taken at the junction of unknown members so that their lever arms will be zero and the equation can be more quickly solved.

1642. **Problem.**—Let it be required to determine with the method by sections the stress in the member *FG* of Fig. 1642A.

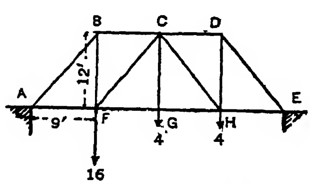


Fig. 1642 A

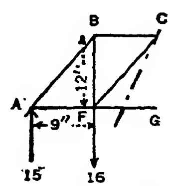


Fig. 1642 B

Solution.—The reaction at the left support is 15: that at the right support is 9.

Passing a plane through the members *FG*, *FC*, and *BC*, and taking the portion to the left as a "free body" we have it as shown in 1642B.

Taking the center of moments at *C*, we have

$$15 \times 18 - 16 \times 9 + FG \times 12 + FC \times 0 + BC \times 0 = 0$$

$$FG = \frac{-270 + 144}{12} = \frac{-126}{12} = -10.5$$

As the sign of the moment is negative, *FG* is in tension.

1643. In the method by sections, it is not at all necessary to take the center of moments at the panel points closest to the section. Any point in the plane will fulfill the requirements: but as a matter of convenience the center of moments is taken at one of the panel points and preferably so selected as to eliminate certain unknowns by having their lever arms equal to zero.

1644. **Problem.**—With the method by sections, find the stress in the member KL of Fig. 1644A.

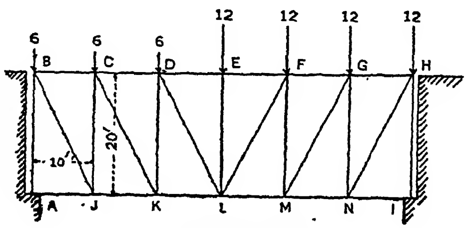


Fig. 1644 A

Solution.—The reaction at the left support is 27: that at the right support is 39. Passing a plane cutting KL, LD, and DE, and taking the portion to the left as a "free body," we have it as shown in Fig. 1644B.

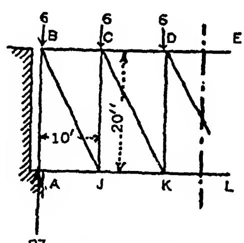


Fig. 1644 B

Taking the center of moments at D, we have

$$27 \times 20 - 6 \times 20 - 6 \times 10 + KL \times 20 + DL \times 0 + DE \times 0 = 0$$

$$KL = \frac{-540 + 120 + 60}{20} = \frac{-360}{20} = -18$$

As the sign of the moment is negative, KL is in tension.

If we had taken the center of moments at C, the equation would have contained the unknowns KL and DL, each with a definite coefficient. Other equations of moments would then have become necessary.

1645. **Problem.**—In the Baltimore truss (Fig. 1645A), assume each panel to have a length of 10′. Find the stress in the member EF.

Solution.—Cut out panel point $EFGH$ with a circle as shown.

As all members which are cut out by the circle meet at the point $GHIKL$ except members FG and EF, the point $GHIKL$ is taken as the center of moments. From the

conditions of equilibrium, we have

$$EH \times 0 + HG \times 0 + FG \times 10 + FE \times 20 \times \sin 45^\circ = 0$$

Remembering that $FG = 2$, we have

$$2 \times 10 + FE \times 20 \times \sin 45^\circ = 0$$

$$FE = -1/\sin 45^\circ = -1.414$$

As the sign of the moment is negative, FE is in tension.

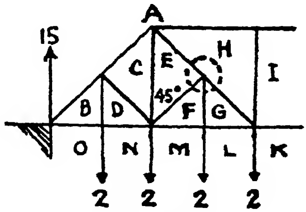


Fig. 1645 A

STRESSES IN BRIDGES UNDER MOVING LOADS.

1646. In paragraph 1634, it has been shown that, in a truss with parallel chords, the stresses in the chords correspond to the longitudinal (bending moment) stresses of a beam: that the stresses in the web members correspond to the shear stresses of a beam. With chords which are not parallel, the above is incorrect to an extent varying with the lack of parallelism of the chords: such trusses are not designed with the idea that the web members take all of the shear, as it is intended that some of it be taken by the inclined chords.

1647. In determining the stresses due to live (moving) loads, it is necessary to consider each single concentrated live load as one concentrated load placed successively at the different panel points; and to consider each uniformly distributed live load as a series of live loads of equal weight concentrated at a series of panel points.

Note: In the conventional method of determining the stresses due to a moving load, it is assumed that one panel point bears its maximum load, while the next is unloaded. This is actually not correct because the load on each panel point could only be a maximum when the load extended to the panel points on either side. If the maximum and minimum stresses in the web members are desired with great accuracy, the stresses due to the true loading of the truss must be determined instead of the stresses due to the conventional loading above described. However, as the conventional method produces slightly greater maximum and minimum stresses, its errors are on the side of safety, and, being simpler, it is the loading usually assumed.

In considering the moving loads on a truss with parallel chords, we follow the same general plan that was followed with the moving loads on a beam in Chapter IV. It is necessary to remember that:

a. As regards bending moments in a beam, the stress varies from point to point, whereas for similar longitudinal stress in the chord of a truss the stress is the same for the length of one panel at least.

b. As regards shear in a beam, the method of transmitting the shear to the supports is the same as in a truss: but while the shear in a beam with a uniformly distributed live load may vary from point to point, in a truss it is the same for the length of one member at least.

1648. **Stress in Chord Members.**—In the chapter on moving loads on beams, it is stated that, if a concentrated live load moves over a beam without weight, the live load bending moment at any section of the beam will be numerically greatest when the live load is at the section.

As applied to trusses with parallel chords, the above principle becomes: *If a concentrated load moves over a truss without weight, the stress in any chord member will be numerically greatest when the load is between the center of moments and the center of the truss and as near the center of moments as the method of loading will permit.* This is illustrated by Fig. 1648A. The stress in CD is greatest when the load is at panel point J

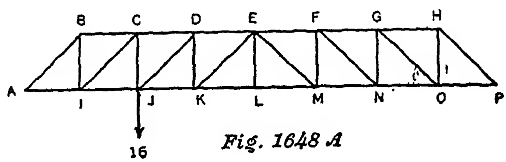


Fig. 1648 A

and has an index coefficient of 24. This stress is found by the method of sections, a plane being passed through panel point J and cutting CD. If the load were placed at any other panel point, the stress in CD would be less.

1649. **Stress in Web Members.**—In the chapter on moving loads on beams, it is stated that, if a concentrated live load moves over a beam without weight, the live load shear at any section will be numerically greatest when the live load is at the section.

As applied to trusses with parallel chords, the stress in any web member is at once evident when it is remembered that: (a) the reactions due to each load vary inversely with the distances from the load to the supports: (b) that this reaction is transmitted by web members with this numerical stress alternately in tension and compression. The above principle then becomes:

If a concentrated live load moves over a truss without weight, the stress in any web member will be numerically greatest when the live load is between the middle point of the truss and the member, and as near the member as the method of loading will permit. This is illustrated in Fig. 1648A. The stress in CJ is greatest when the load is at J. The load of 16 is transmitted, 12 to the left support, 4 to the right support. The 12 to left support is transmitted, alternately in tension and compression, from

J via *JC, CI, IB, BA*. It is not possible to put the load of 16 at any other panel point so that as much as 12 will be transmitted by *CJ*.

1650. **Application.**—By the use of the principles enunciated in paragraphs 1648 and 1649, it is easy to apply the principles in Chapter IV to the stresses in trusses due to live and dead loads. Some of the more important of these are considered in the problems below.

1651. **Problem.**—Let it be required to find the maximum index coefficients of the stresses in the web members of the deck truss in Fig. 1651A, with the dead loads as shown and a concentrated live load of 24 tons moving across the bridge.

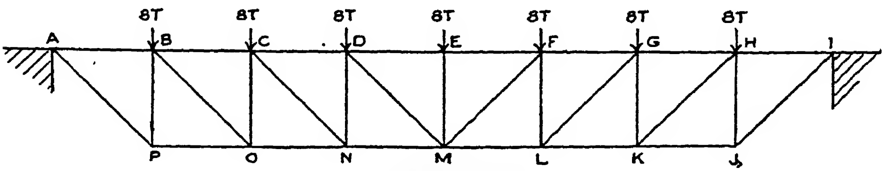


Fig 1651 A

Solution.—From the principle of paragraph 1649, and also from a knowledge of the method of transmission of the load from any panel point to the reaction, we see that the maximum stress in any of these web members is obtained by placing the live load at its panel point which is nearest the center.

For example, the maximum stress in *CN* will be obtained by placing the live load at *D*, whence $\frac{5}{8}$ of it, ($\frac{5}{8}$ of 24), will be transmitted to the left panel point; and $\frac{3}{8}$ of 24 to the right panel point. First writing on the truss the stresses in the web members due to the dead loads we have the figures in parenthesis in Fig. 1651B. Adding the 12 stress in *CN* due to the dead load to the 15 stress in *CN* due to the live load, we have a total stress in *CN* of 27. By placing the live load at each of the panel points we have the maximum live load web stresses as shown by figures not in parenthesis in Fig. 1651B. The sum of the two will be the index coefficients of the total maximum stresses.

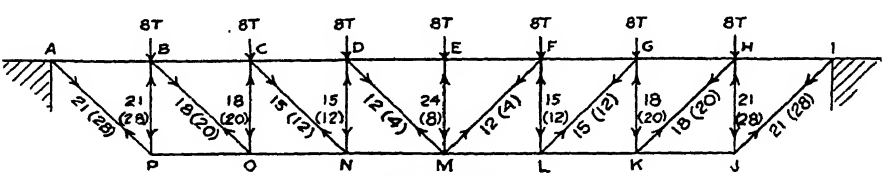


Fig 1651 B

1652. **Problem.**—Let it be required to find the maximum index coefficients of the stresses in the chord members of the deck truss in Fig. 1651A, with the dead loads as shown and with a concentrated live load of 24 tons moving across the bridge.

Solution.—From the principles of paragraph 1648, we see that the maximum stress in any of the chord members is obtained by placing the live load at the panel point which would be the center of moments in finding the stress in a chord member. For example the maximum stress in *ON*, will be obtained by placing the live load at *C*. First writing on the truss the stresses in the chord members due to the dead loads we have the figures in parenthesis in Fig. 1652A. Adding the 48 stress in *ON* due to the dead load to the 36 stress in *ON* due to the live load, we have a total stress in *ON* of

84. By placing the live load at other panel points, we have the maximum live load chord stresses as shown by figures not in parenthesis in Fig. 1652A. The sum of the two will be the index coefficients of the total maximum stresses.

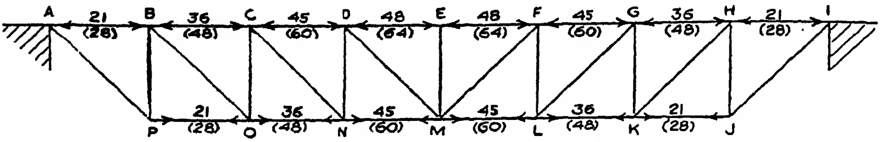


Fig 1652 A

1653. **Problem.**—A uniform live load of 20 tons per panel point moves over the ower chord of the truss shown in Fig. 1653A. Find the index coefficient of the maximum stress in DG.

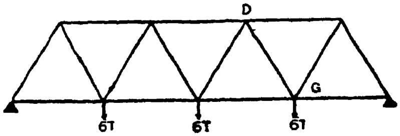


Fig. 1653 A

Solution.—The maximum stress is obtained in DG when the live load is at all the panel points except panel point G. (See Fig. 1653B.) With this loading we have index coefficients of

3 due to dead load alone
15 due to live load as placed

18 total

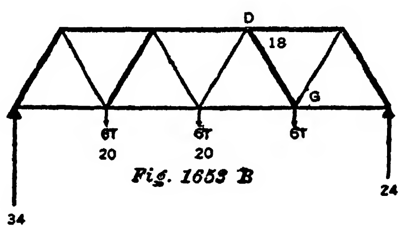


Fig. 1653 B

1654. By problems similar to the above, with concentrated live loads and uniformly distributed live loads, on a truss without dead loads and with dead loads, we find the following general principles to be true of trusses with parallel chords:

a. In a truss assumed without weight, resting on end supports and acted upon by a concentrated live load.*

(1) The maximum stress in any one web member is obtained by placing the live load between the middle of the truss and the member and as near the member as the method of loading will permit.

* Compare with beams, par. 404.

(2) The maximum stress in any of the web members is that in the web members next to the supports: this occurs when the live load is between the middle of the truss and the member and as near the member as the method of loading will permit.

(3) The maximum stress in any one chord member is obtained by placing the live load between the center of moments for that member and the middle of the truss and as near the center of moments as the method of loading will permit.

(4) The maximum stress in any of the chord members is that in the chord member at the middle of the truss: this occurs when the live load is between the center of moments for that member and the middle of the truss and as near the center of moments as the method of loading will permit.

b. In a truss assumed with a dead load, resting on end supports, and acted upon by a concentrated live load.*

(1) same as *a* (1) above.
(2) same as *a* (2) above.
(3) same as *a* (3) above.
(4) same as *a* (4) above.

c. In a truss without weight resting on end supports, and acted upon by a uniformly distributed live load sufficient to load all the panel points at one time.†

(1) The maximum stress in any one web member is obtained when the live load covers the greater segment and the end of the live load is so placed that it is between the middle of the truss and the middle of the member and as near the member as the method of loading will permit.

(2) The maximum stress in any of the web members is that in the web members next to the supports: this occurs when the live load covers the greater segment and the end of the live load is so placed that it is between the middle of the truss and the member and as near the member as the method of loading will permit.

(3) The maximum stress in any one chord member is obtained when the live load covers the entire truss.

(4) The maximum stress in any of the chord members is that in the chord member at the middle of the truss: this occurs when the live load covers the entire truss.

* Compare with beams, par. 406.

† Compare with beams, par. 412.

d. In a truss assumed with a dead load, resting on end supports, and acted upon by a uniformly distributed live load sufficiently long to load all panel points at one time:*

(1) The maximum stress in any one web member in the left half of the truss decreases as the head of the live load moves from the left support to the panel point between the middle of the truss and the member and as near the member as the method of loading will permit, increases while the head moves beyond this panel point, increases while the tail moves from left support to this point, decreases while the tail moves beyond this point.

(2) Same as *c* (2) above.

(3) Same as *c* (3) above.

(4) Same as *c* (4) above.

1655. **Counterbraces.**—In the preceding discussions of the effects of a concentrated or a uniformly distributed live load, we have considered only the maximum stresses which may be produced in a member. This is not sufficient to insure proper design of a truss. The effect of a live load in certain positions may be such as to change the character of the stress in a member from tension to compression or the reverse. This produces added complications of design, because members are designed to carry only one kind of stress.

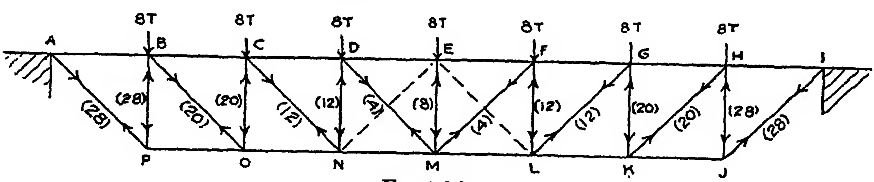


Fig 1655 A

For example let us consider the possible reversals of stress in Fig. 1655A, loaded as shown, and with a concentrated live load of 24 tons moving across the truss.

The index coefficients of stresses in the web members due to dead load above are shown by the figures in parenthesis: the arrows show the character of the stresses.

If, now, the concentrated load of 24 tons is at panel point *D*, 15 tons will be transmitted to the left support and 9 tons to the right support. The 15 tons to the left support are transmitted by compression in *DN*, tension in *NC*, compression in *CO*, tension in *OB*, compression in *BP*, and tension in *PA*, each of which is designed for this kind of stress and consequently there are no reversals of stress to the left of the concen-

* Compare with beams, par. 414.

trated load. The 9 tons to the right support are transmitted by compression in *DM*, tension in *MF*, compression in *FL*, tension in *LG*, compression in *GK*, tension in *KH*, compression in *HJ*, and tension in *JI*: each of which except *DM*, is designed for this kind of stress. *DM* now has a tension of 4 tons due to dead load and a compression of 5 tons as the result of both loads, which means that the stress in *DM* is reversed. But, in bridge design, except in riveted Warren trusses, no tension members are designed to carry compression: therefore, the bridge will fail unless *DM* is designed to carry both tension and compression or unless some change is made in the design.

The system of counterbracing is adopted to overcome these difficulties. The broken line *NE* (Fig. 1655A) is a counterbrace. With *NE* in the design, 4 tons of the 9 compression balance the 4 tons tension in *DM* so that there is 0 stress in *DM*; and the remaining 5 tons compression are transmitted to the right support by compression in *DN*, tension in *NE*, compression in *EM*, and thence via *MF*, *FL*, *LG*, *GK*, *KH*, *HJ*, and *JI*, with stresses for which originally designed.

If the concentrated load of 24 tons is placed at *C*, 6 tons will be transmitted to the right, which will reverse the stress in *DM*, but not in *CN*. If the load is placed at *F*, the 9 tons transmitted to the left support will reverse the stress in *FM*, making necessary a counterbrace *LE* as shown by the broken line.

For the concentrated load of 24 tons, no other counterbraces are needed than *NE*, and *LE*. For a larger concentrated load or with a uniformly distributed load, other members may be reversed, as *CN* and *GL*, near the middle of the truss.

Study of the possible positions of the live loads which may cause reversal of stresses and require counterbracing, shows that the stresses of the web members near the center are first reversed, and those near the supports are rarely if ever reversed.

1656. Problem.—In the truss (Fig. 1656A) with dead loads as shown, and with members designed to carry stresses as indicated by the arrows, find:

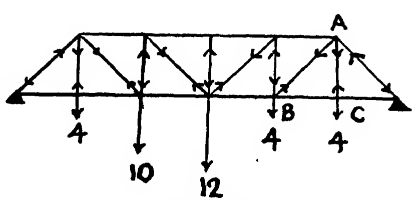


Fig. 1656 A

a. What load at *C* will make the index coefficient of the stress in *AB* equal to 2 in compression?

b. If the interior diagonals take tension only, put in counterbraces where needed for

this load, and write the index coefficients for each member with these counterbraces in place.

Solution.—*a*. With the dead load as shown, the stress in AB is 12 in tension. To make it 2 in compression, it must transmit 14 to the left support. Hence $\frac{W}{6} = 14.$ $W = 84$ = additional load required at C.

b. See Fig. 1656B.

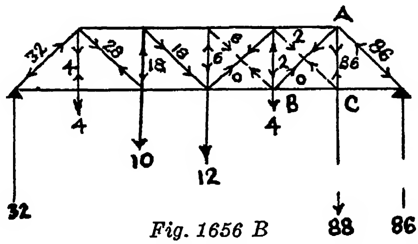


Fig. 1656 B

METHOD BY GRAPHIC STATICS.

1657. The **Graphic Method** is also used to determine the stresses in a bridge truss. As with the roof truss, it has certain advantages and disadvantages.

The advantages are:

a. It is often quicker.

b. It is much simpler, when the chords of the truss are not parallel.

c. It is simpler, when the reactions are not vertical.

d. An important error becomes evident in the graphical method whereas an error in numerical computations may not be detected.

The disadvantages are:

a. It is not as accurate: numerical computations are exact whereas drawings are not exact.

b. It is not as quick when the chords of the truss are parallel.

1658. **Problem.**—Let it be required to find graphically the stresses in the truss shown in Fig. 1658A.

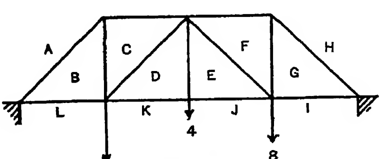


Fig. 1658 A

Solution.—The reactions are: 13 at the left support and 11 at the right support.

Draw the force polygon *ij–jk–kl–la–ai*, as shown in Fig. 1658B. Start with the reaction LA (*la*) which is known. Draw *ab* parallel to AB, and *lb* parallel to LB, meeting at *b*, and forming the stress diagram *lab*. Similarly, draw the stress diagram for each of the other panel points as shown in Fig. 1658B, forming the complete stress diagram. The characters (direction) of the stresses are as indicated in the diagram,

being obtained by following the perimeter in order as indicated by the known forces or known stresses.

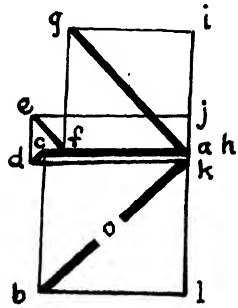


Fig. 1658 B

1659. **Problem.**—Find graphically the stress in each member of the cantilever (Fig. 1659A) and the reactions at the supports.

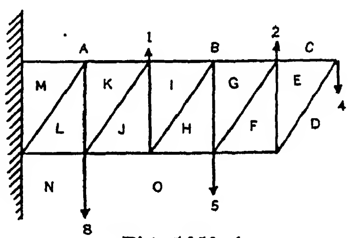


Fig. 1659 A

Solution.—In this problem, it is not necessary to determine the reactions at once. Beginning at the right, the stress diagram for each panel point may be drawn in succession, with as much of the force polygon as necessary. Each panel point is solved

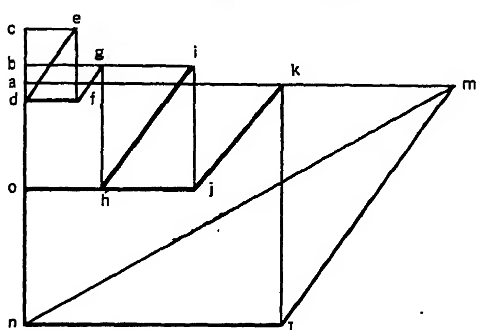


Fig. 1659 B

in succession. When the wall is reached, the reactions at the upper and lower chords in each case are the only unknowns, and the direction and intensity of each are obtained by simply closing the stress diagram in each case. The complete diagram is shown in Fig. 1659B.

1660. **Problem.**—Let it be required to design a bowstring truss with straight lower chords and eight panels loaded as shown in Fig. 1660A, the design to be graphical and the upper chord to have an inclination such that there will be no stress in the diagonal web members.

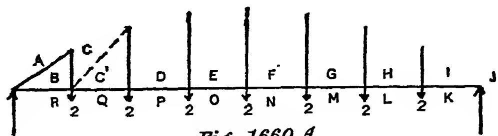


Fig. 1660 A

Solution.—Draw the force polygon *raklmnopqr* (Fig. 1660B). Assume the slope of the chord member AB and draw ab in the stress diagram parallel to it. Draw the stress diagram $bcc'qr$ representing equilibrium about panel point $BCC'QR$, in which c and c' coincide because there is to be no stress in CC'.

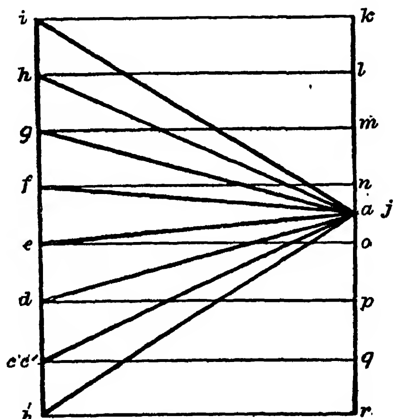


Fig. 1660 B

Draw the stress diagram bac representing equilibrium about the panel point BAC. The inclination of AC is determined by the fact that it must be parallel to ac.

Similarly, the inclination of the remaining members of the upper chord may be determined.

1661. The same result may be obtained analytically. Let it be required to determine the heights of the vertical web members of the bowstring truss, shown in Fig. 1661A, so that there will be no stress in the diagonal web members.

Let l = length of truss AI.
l' = length of panel.
h = height of truss EN.
h' = DO.
h'' = CP.
h''' = BQ.

Passing a plane cutting DE and ON and taking moments about E we have

$$7\,W \times 4\,l' - 2\,W \times 3\,l' - 2\,W \times 2\,l' - 2\,W \times l' + \text{stress}\,ON \times h = 0;$$

Thence $$h = -16 \frac{Wl'}{\text{stress } ON} \tag{1661A}$$

Passing a plane cutting CD and PO and taking moments about D of the upper chord,

$$7\,W \times 3\,l' - 2\,W \times 2\,l' - 2\,Wl' + \text{stress } PO \times h' = 0;$$

Thence $$h' = -15 \frac{Wl'}{\text{stress } PO} \tag{1661B}$$

Passing a plane cutting BC and QP and taking moments about C of the upper chord,

$$7\,W \times 2\,l' - 2\,W \times l' + \text{stress } QP \times h'' = 0;$$

Thence $$h'' = -12 \frac{Wl'}{\text{stress } QP} \tag{1661D}$$

Similarly, we find that

$$h''' = -7 \frac{Wl'}{\text{stress } AQ}$$

If there is no stress in the diagonal web members, the stresses in ON, PO, QP, and AQ must be equal. Hence to fulfill the conditions that there shall be no stress in the diagonal web members, the relative values of h, h', h'', and h''', in the truss shown in Fig. 1661A, must be as 16, 15, 12, and 7. Having assumed one of these lengths, the others must have the relative length deduced.

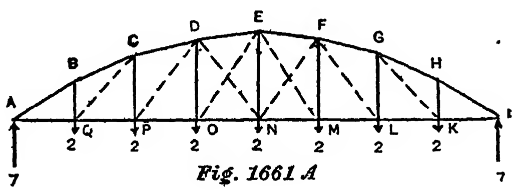


Fig. 1661 A

To support a moving load such a truss would require diagonal main braces and counterbraces. The bowstring truss is more easily solved by the graphic than the analytic method.

1662. **Problem.**—Let it be required to find graphically the reactions and the stress in each member of the three-hinged arch truss, with loads, as shown in Fig. 1662A.

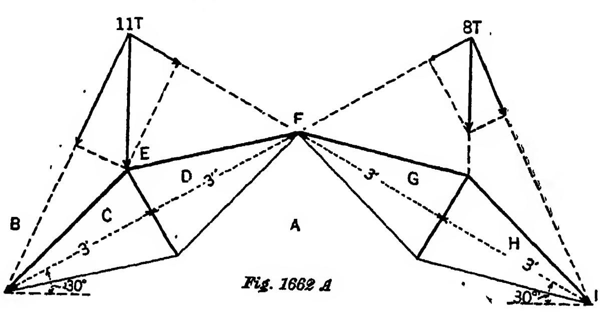


Fig. 1662 A

Solution.—It is known that the force of 11 tons causes reactions at the two panel points as shown by the resolution of the 11 tons in the figure.

Taking the panel point ABC as a "free body" and drawing the two known forces acting through this panel point with directions and intensities as shown by the resolution

of the 11 tons and 8 tons, we find (Fig. 1662B) the direction and intensity of the reaction *AB* at the panel point *ABC*.

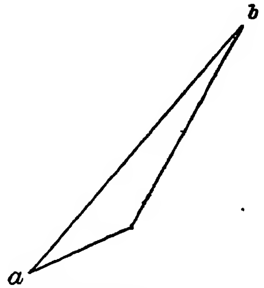


Fig. 1662 B

Now drawing the force polygon *ab–bf–fi*, (Fig. 1662C) and connecting *i* and *a* by the line *ia*, we have the reaction at the right panel point and also the complete force polygon. The stress diagram (Fig. 1662C) is completed in the usual manner.

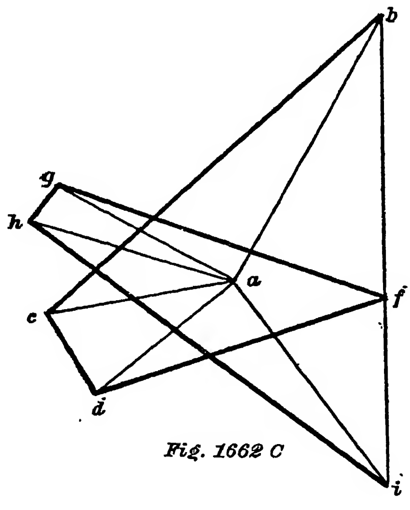


Fig. 1662 C

BRIDGE ERECTION.

1663. The term **plant** is applied to the derricks, travelers, and other machinery employed in erecting the bridge; the term **false work** to the trestling that supports the bridge during erection. A traveler is a car which runs on a track on the upper or lower chord platform and supports a longitudinal cantilever or crane at a level above that of the upper chord: the cantilever is provided with hoisting apparatus.

A bridge should have all its parts prepared before leaving the shop; no work should be done on a structure in place which can be done before

erection. Power and machinery should be employed in preference to labor.

Bridge superstructures are erected in general in one of the three following ways or combinations thereof:

(1) as a unit.
(2) on false work.
(3) cantilever method.

1664. **Erection as a Unit.**—Plate girders and trusses with a span not exceeding 100′ may be erected as a unit by using derrick cars, derricks, or gin poles. When small span girders or trusses are shipped in more than one piece, they are riveted on shore. If derrick cars are not available, a short span girder may be put into place by moving it as a whole on rollers which are supported on stringers resting on trestle bents: a large girder is hauled into position by a rope attached to a capstan. A small Howe truss like that shown in Fig. 1625I can be assembled on shore and placed in position in the same manner as a girder. Trestle bents if not too high are framed on the ground and then lifted into place. High bents and towers are raised into position by use of the ordinary hoisting apparatus. Small span steel arches are erected as a unit.

1665. **Erection on False Work.**—With spans up to about 400′ and where trusses cannot be erected as a unit, the trusses are erected on ordinary framed wooden false work, by means of one or two derrick cars on gantry travelers. A pin-connected bridge is erected on false work in the following order: floor system, lower chord, web members, top chord, lateral and sway bracing.

In erection of suspension bridges, the main and auxiliary cables are placed as soon as possible; and they serve for support of the rest of the bridge in the same manner as the false work serves in erection of truss bridges.

Masonry and reinforced concrete arches are constructed on false work called centers, as already described.

1666. **Cantilever Method of Erection.**—The cantilever method is employed in all cantilever bridge construction and for other bridge types where false work is impracticable because of ice, flood, swift current, navigation requirements, or expense. The cantilever method consists in constructing two separate cantilevers, from opposite sides of the span, and uniting them at the center by a suspended truss. Each cantilever which is being constructed from one side of the span is counterbalanced by a similar one constructed at the same time on the opposite side of its tower, or by a truss which has been previously constructed. The suspended truss which is to connect the two cantilevers coming from

opposite sides of the span is constructed as a part of the two different cantilevers; when the two cantilevers are united at the middle point of the span, the upper chord is severed between each cantilever and its part of the suspended truss. The stresses in the chords of the suspended truss during construction are therefore the reverse of their final stresses. This change must be provided for in the design. When truss bridges are erected by this method, special anchorages and counterweights are required or alternate spans must be erected in advance to serve as anchorages. Steel arch bridges are usually constructed by the cantilever method, the character of the site being such that the semi-arches can be held by cables until they are united at the crown. If the site does not admit of this method of construction, the arch is supported on a center.

BRIDGE DESIGN.

1667. The designing of a bridge consists in applying to the site and object of the bridge the principles as enunciated in the text. Each site will cause different problems; and the object for which the bridge is designed, whether for use in a road, railway, or viaduct, or combination of these, will determine which of certain standard designs will be required or whether a special design will be needed throughout.

Procedure in Design.

1668. Let us assume that it has been decided by competent authority that a certain bridge is desirable and that the requisite funds are available to provide it. It remains then for the engineer to design a suitable structure and to proceed to its construction. In some cases, complete working drawings may be prepared and sent to the shop. The entire structure may then be fabricated and shipped to the site independently of the erection forces in the field. In other cases, many features of the design can only be tentative and must be modified depending upon developments during construction. In these latter cases, design and construction proceed together, the completeness of the preliminary design depending upon the extent and accuracy of the original data.

The engineering departments of railways, bridge companies, and other large concerns, gradually accumulate sets of type plans for different classes of work and then the adjustment of such plans to varying sites and circumstances may be left to the engineer in charge of construction. In any case, however, the designing force keeps the details as much ahead of the construction as possible in order to relieve the man in the field of computing and drafting. The man in the field not only has no suitable facilities for such work but he also needs to bend all his energies and time to a different class of problems; that is, to questions of location, construction, or erection, and the handling of men, materials, and plant.

1669. The objects of design are, first, to provide a structure that will be safe under any probable circumstances, and, second, to do this as economically as possible. The site of the proposed structure should be surveyed and a map prepared.

The objects of this survey are

a. To locate the structure in the most economical position.
b. To make it possible to specify its details in advance.
c. To determine quantities and to estimate cost.

Property lines, buildings, contours, soundings, and borings, should be taken. The borings should extend to rock; and if there is any doubt as to the character of the rock, it should be examined by means of cores withdrawn by the diamond drill. A few rough measurements often show the impracticability of some designs. Every possible location should be considered carefully. It may be necessary to place the structure at a particular spot, but it is even then best to make the survey for purposes of location and to secure data for detailed plans.

Illustration.

1670. To indicate the necessity for thorough information, let us take the case of two towns on opposite sides of a valley which are to be connected by a railway. The cost of operation would be least if alignment and grade were straight from one town to the other. But first cost would probably be lessened by:

a. Going farther up the valley and using a shorter bridge.
b. Starting at a lower level at the towns, with a deeper cut, thus lessening the height of approaches and piers.
c. Lowering the grade in the valley.

Various modifications and combinations of these extremes should be worked out.

1671. **The Requirements of Economy.**—The alignment and grade being chosen, decision is next required as to whether it is best to use a viaduct, or a bridge with abutments, or to fill all the way across, leaving only a culvert. In the first two cases calculations must be made as to where it will be economical to stop the fill and begin the viaduct or bridge.

Roughly speaking it is advisable to stop the fill at a depth where cost of fill per lineal foot becomes greater than the cost of the bridge per lineal foot. However, the location of a bridge involving the smallest cost of construction is by no means the only desirable result to be obtained; economy of operation plays an important part and hence a very prominent engineer has defined Engineering as "The art of making a dollar earn the most interest."

It must be remembered, too, that maintenance charges are much lower for a properly-built fill than for a bridge and that no sinking fund would be required. These considerations operate in favor of the fill or tend to increase the depth to which the fill should be extended.

The sum of the following annual charges should be a minimum:

a. Interest on the first cost.

b. Maintenance.

c. Sinking fund. At the expiration of the life of the structure, this fund must be sufficient to rebuild it.

d. Operation.

The following simple example will show how the elements of cost enter into the problem. Suppose two alternative schemes for a drawbridge are as follows: No. 1 will cost $200,000.00; repairs, painting and so forth, $400.00 per year, bridge is estimated to last 30 years; it will be operated by nine men at a total cost of $20.00 per day. For No. 2, the corresponding quantities are: cost $150,000.00; repairs $300.00; duration 25 years; operation, 12 men, $27.00. Taking masonry alike in both cases and interest at 4 per cent, we may compare as follows:

Scheme	Interest	Maintenance	Sinking Fund	Operation	Total Annual Charge
1	$8000	$400	$3560	$7300	$19,260
2	6000	300	3600	9855	19,755

The above analysis shows a slight advantage in No. 1.

1672. **Materials.**—Location and ends of fill being decided, is it best to use timber, steel, reinforced concrete, or combinations of these? Timber is cheaper in its first cost; but maintenance and renewal charges are higher. Properly-built masonry bridges require neither maintenance nor sinking fund charges; but their first cost is high, they cannot be used for long spans, and they are ill-suited to foundations on compressible soils.

1673. **Span.**—The material having been selected, what shall be the spans? Shall they be uniform, making superstructures alike, or shall they increase as the valley becomes deeper? One very long span might be used or many very short ones; but the former would result in an exceedingly expensive superstructure, while the latter would require many abutments and piers, thus increasing the cost of the substructure. As a rough rule, the cost of the piers should equal that of the trusses.

1674. **Type.**—Next comes type of bridge. Assuming that a steel superstructure has been selected, it is now necessary to decide whether it shall be plate girder, suspension, cantilever, or simple truss; and also

if it shall be a through, pony truss, or deck structure. The answers to all these questions can be determined only after the site has been thoroughly examined and surveyed.

1675. **Structural Shapes.**—Type having been determined upon, what should be the individual pieces, what combinations of structural shapes will carry the stresses economically and make easy and efficient joints? Even in the details of the latter, there are various selections, each having its advantages.

Take the case of a plate girder of a given span and loading. The weight of the web and fittings is about constant per linear foot. Let D represent depth. We may then say:

$$\text{Total weight of web} = C_1D \qquad (1675\text{A})$$

The area of the flange and hence its weight will vary inversely as the depth, or

$$\text{Total weight of both flanges} = C_2/D \qquad (1675\text{B})$$

$$\text{Total weight of girder} = C_1D + C_2/D = W \qquad (1675\text{C})$$

For the minimum value of W,

$$dW/dD = C_1 - C_2/D^2 = 0$$

or

$$C_1D = C_2/D \qquad (1675\text{D})$$

Hence, depth of girder should be made such that weight of both flanges equals that of the web and its fittings. The above assumes, as often happens, that minimum thickness of web suffices. Where it does not, the depth should be increased as much as practicable or until minimum thickness of web is reached.

1676. In the above, are indicated only a few of the questions which are interwoven with the problem of design. Some, it will be noticed, are a little outside the province of the structural engineer. In all cases, the general rule to be followed is: **Choose the most economical design which gives the required degree of safety.**

Very often, considerations other than economical ones will govern. Such for instance, are architectural appearance or legal difficulties. All possible alternatives, however, should be very carefully investigated.

For doubtful or irregular cases, several designs should be made with estimates of quantities and costs. The greater the magnitude of the work, the more the need for careful design. To design a small or typical structure, it is best to follow current practice rather than spend a large amount of money in design: a saving of \$50.00 in material at a cost of \$100.00 in the drafting room is poor engineering. On the other hand,

it is poor economy to tolerate clumsy or expensive shop details in order to avoid the expense and work of drafting a suitable design.

1677. Summarizing the above illustrations, there are tabulated below the different steps in the work of designing a simple highway bridge, the steps being performed in the following order.

a. design of floor planks.
b. design the stringers.
c. check assumed weight of stringers.
d. design the floor beams.
e. check assumed weight of floor beams.
f. determine the number of rivets required to fasten stringers to floor beams.
g. determine stress coefficients of various truss members for uniform dead loads of unity.
h. determine maximum stress in each member due to the live load.
i. prepare stress sheet.
j. design all truss members of construction.
k. determine area of sole plate.
l. determine rivets required to fasten floor beams to truss verticals and to end gusset plate.
m. check assumed weight of truss.
n. prepare erection diagram.
o. prepare detail shop drawings.
p. bill of materials.

COSTS

1678. The following list gives a comparative idea of the costs of bridges, based on actual cost of bridges completed in 1923:

86′ Steel Pratt truss bridge with 27′ roadway and two 5′ sidewalks	$ 31,000
122′ Steel truss bridge	25,000
225′ Reinforced concrete bridge	70,000
640′ Double track cantilever bridge	3,000,000

For more detailed information, see:
Theory of Structures, Spofford.
Foundations of Bridges and Buildings, Jacoby and Davis.
Roofs and Bridges, Merriman and Jacoby.
Concrete Engineer's Handbook, Hool and Johnson.
Cyclopedia of Civil Engineering.
Civil Engineering, Fiebeger.
American Civil Engineer's Pocket Book, Merriman.

PROBLEMS.

P. 1601. Compute the impact in pounds for a bridge member subjected to the following conditions:

Maximum live stress	200,000 pounds
Dead stress	100,000 pounds
Loaded length when stress is a maximum	100′

P. 1602. A railroad freight car 40′ long, 10′ high, floor 4′ above the track, 8′ wide, weight 20 tons is exposed to a horizontal wind perpendicular to its side. Determine wind pressure per square foot necessary to tip the car.

P. 1603. A railway plate girder bridge, carrying a single track supported by stringers, floor beams, and two plate girders, composed of 7 panels of 10′ span each, has the following dead weight:

Stringers, 50 pounds per foot per stringer (2 stringers)
Track, (rails, ties, etc.), 400 pounds per foot of track
Floor Beams, 100 pounds per foot of beam (14′ long)
Girders, 300 pounds per foot of girder.

(*a*) Compute maximum dead load shear in the first and second panels from one end.

(*b*) Assuming the stringers placed 8′ apart, draw curve of dead load moments in first floor beam beyond the abutment.

P. 1604. On the bridge described in Problem 1603, compute maximum live load shear in first and second panels from one end when train with engine load of 8000 pounds per foot for 20 feet, followed by train of uniform load of 5000 pounds per foot, longer than the bridge, passes over the bridge from the direction opposite the abutment from which the panels are counted.

P. 1605. Design a steel I-beam stringer for the bridge and loads described in Problems 1603 and 1604, with 75 per cent impact.

P. 1606. A bridge pier, 8′ square at the top, 40′ high with a batter of 1/10 weighs 160 pounds per cubic foot. Determine unit pressure on base if load on the top (weight of bridge) is 600 tons, and water stands 20′ above the level of the foundation of the pier.

P. 1607. Compute the weight of steel in a 130′ highway bridge whose trusses are 16′ center to center, given $W = 34 + 22\,b + 0.16\,bl + 0.7\,l$.

P. 1608. Prove that the stress in a diagonal of a horizontal chord truss with a simple web system is $W \sec \phi$.

P. 1609. Prove that the chord stress of a horizontal chord truss is $\frac{M}{h}$ where M is the moment at the point, and h is the height of the truss.

P. 1610. Compute the maximum positive and negative live load shears in a 13 panel Howe truss, the live panel load being 40,000 pounds.

P. 1611. If the end shear of a plate girder is 394,500 pounds, design the web section, it being 108″ deep.

P. 1612. If the dead load moment is 8,489,000 inch-pounds and the live load moment is 30,610,000 inch-pounds, design the flange of a plate girder, if distance back to back of flange angles is 7′6½″, it being assumed that the web does not take any bending moment.

P. 1613. If in the girder in Problem P. 1612, the web were $90'' \times \frac{7}{16}''$, design the flange, considering ⅛ of the gross area of the web as effective flange area.

P. 1614. A masonry bridge pier of uniformly rectangular cross-section, 30′ long, 12′ wide and 50′ high. The weight of the bridge is 150 tons and acts at two points on the longer axis of the top of the pier at distances from the middle point of 10′. The maximum horizontal thrust of the bridge span is equally divided between these two points, is perpendicular to the long axis of the top of the pier and amounts to 10 tons. The weight of the masonry is 150 pounds per cubic foot. Find the position of the center of pressure on the base. Has the pier sufficient width of base? Assuming that there is no adhesion of base to its foundation, what is the maximum pressure in tons per square foot? Tons of 2000 pounds each.

P. 1615. In what positions will a loaded wagon of 8′ between axles produce the maximum shear and bending moment in the stringers of a bridge 15′ long? What will be the value of these stresses (shear and bending moment), assuming each front wheel to carry $\frac{1}{6}$ of the load of the wagon and each rear wheel to carry $\frac{1}{3}$ of the load.

P. 1616. A road-roller, 8′ between front and rear axles, passes over a bridge whose floor is supported by 4 steel I-beams 24′ long. The weight on front wheels is 4 tons, and on rear wheels is 12 tons. Select the I-beams, in order that the load may be safely borne. Assume that one I-beam takes half the weight of the roller. Neglect weight of floor and I-beams.

P. 1617. A crowd of people, weight 150 pounds per square foot moves over a bridge whose roadway is 40′ wide. The floor is supported by 4 I-beam stringers spaced 13′4″ center to center and each 18′ long. Select the proper stringers. Assume that the floor weighs 20 pounds per square foot.

P. 1618. A railway plate girder bridge 2 girders, 5′ deep, 40′ span, 10′ wide, carries a uniformly distributed load of 10 tons per square foot. Find the maximum shearing stress in one girder 12′ from one end when a train weighing $2\frac{1}{2}$ tons per linear foot crosses the bridge. Also find the minimum theoretical thickness of the web at the support, 4 tons per square inch being the safe shearing stress of the metal.

P. 1619. (Fig. P. 1619.) With the truss and loading given, write out the reactions at the supports.

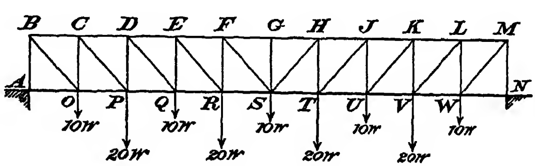


Fig. P 1619

P. 1620. (Fig. P. 1620.) With the truss and loading given, write out the reactions at the supports.

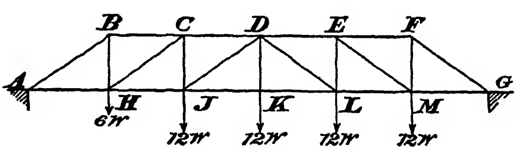


Fig. P 1620

P. 1621. (Fig. P. 1621.) With the truss and loading given, write out the reactions at the supports.

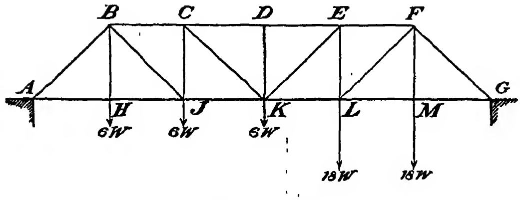


Fig. P 1621

P. 1622. (Fig. P. 1622.) By the analytical method of concurrent forces, find the stresses in the different members of the truss, with the loading indicated.

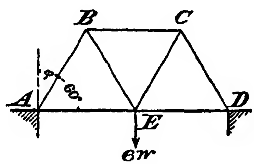


Fig. P 1622

P. 1623. (Fig. P. 1623.) By the method of sections, find the stresses in the different members of the truss with the loading as given.

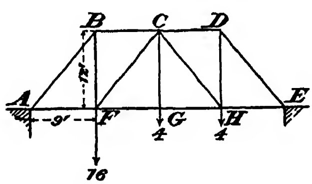


Fig. P 1623

P. 1624. (Fig. P. 1624.) By the method of sections, find the stresses in the different members of the truss with the loading as shown.

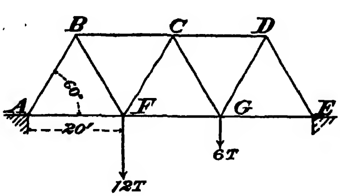


Fig. P 1624

P. 1625. (Fig. P. 1625.) By the method of sections, find the stresses in the different chord members of the truss with the loading as given.

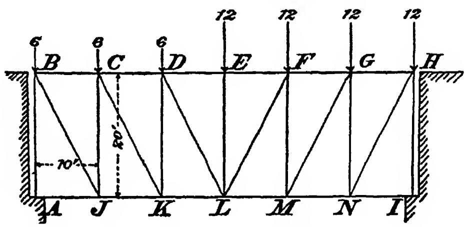


Fig. P 1625

P. 1626. (Fig. P. 1626.) The triangles in the truss are equilateral, the length of each side being 20′. By the method of sections, determine the stresses in the following members:—

(*a*) *AB*. (*b*) *CE*. (*c*) *CD*.

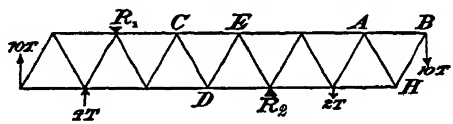


Fig. P 1626

P. 1627. (Fig. P. 1627.) Write on the members of the truss the numerical coefficients of the stresses in the members.

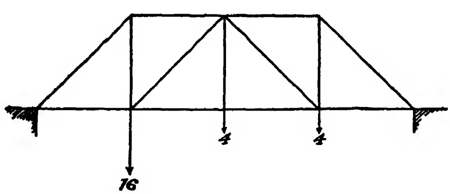


Fig. P 1627

P. 1628. (Fig. P. 1628.) Write on the members of the truss the numerical coefficients of the stresses in the members, when a concentrated load of 60 tons is at panel point *A*.

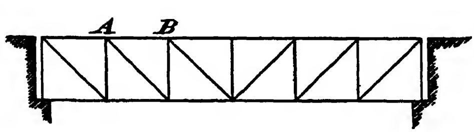


Fig. P 1628

P. 1629. (Fig. P. 1628.) Write on the members of the truss the numerical coefficients of the stresses in the members, when a concentrated load of 120 tons is at panel point B.

P. 1630. An I-beam bridge stringer 18′ long, carries a uniformly distributed dead load of 9000 pounds. Two wheels, 6′ apart, the one bearing 6000 pounds and the other 3000 pounds, roll over the joist. Find the maximum shear at the supports, and at 5′ from each end.

P. 1631. The base of a masonry bridge pier is 7 by 10′, and the center of pressure is one foot from the center of the base, measured parallel to the longer side. The weight of the bridge being 40,000 pounds, find the unit pressure at the corners.

P. 1632. At the base of an abutment 6′ wide, the pressure at one edge is 20 pounds per square inch; at the opposite edge it is 5 pounds per square inch. What is the mean pressure and where is the center of pressure?

P. 1633. A four panel Howe truss, resting on end supports, is loaded with a concentrated load of four tons at each panel point of the lower chord. Panel length $= l$ and panel depth $= d$. By the method of sections, determine the stresses in all the members of the third panel from the left.

P. 1634. A six panel Howe truss, resting on end supports, is loaded with a concentrated load of 4 tons at each panel point of the lower chord. The diagonal web members make an angle ϕ with the vertical. By the method of sections determine the stresses in all the members of the fourth panel from the left.

P. 1635. (Fig. P. 1635.) By the method of sections, determine the stresses in the chord members of the truss with the loading as shown.

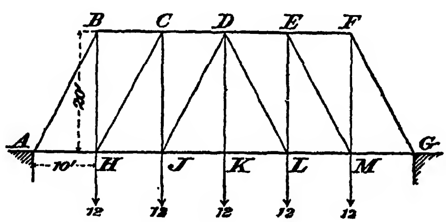


Fig. P 1635

P. 1636. (Fig. P. 1636.) By the method of sections determine the stresses in the members BC and FC.

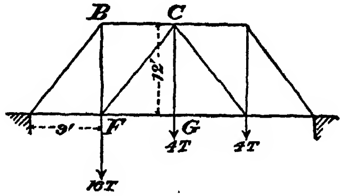


Fig. P 1636

P. 1637. (Fig. P. 1637.) By the method of sections, find the stresses in AB, BC, and CD. Write results in terms of ϕ.

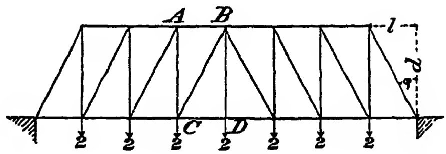


Fig. P 1637

P. 1638. In the Pratt truss (Fig. P. 1638), what is the least concentrated load which, placed at C, will reverse the stress in FG? What is the least concentrated load which, placed at E, will produce the same result?

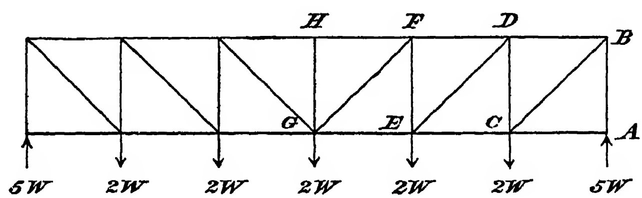


Fig. P 1638

P. 1639. What is the weight, per panel length, of the least uniform load which, moving over the Pratt truss, Fig. P. 1638, will reverse the stress in FG? Employ conventional loading.

P. 1640. A concentrated load of 4 W is moved over the lower chord of the Howe truss (Fig. P. 1640). Insert counterbraces where necessary in the truss, and find counterbrace stresses.

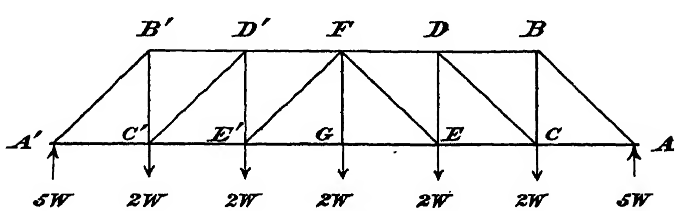


Fig. P 1640

P. 1641. A uniformly distributed live load, whose weight per panel length is 3 W, moves over the same truss (Fig. P. 1640); insert the necessary counterbraces and determine the stresses in them.

P. 1642. (Fig. P. 1642). Write on the members of the truss the numerical coefficients of the stresses in the members.

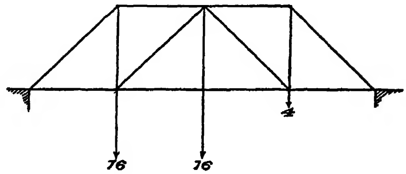


Fig. P 1642

P. 1643. (Fig. P. 1643.) Write on the members of the truss the numerical coefficients of the stresses in the members.

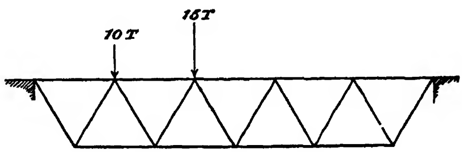


Fig. P 1643

P. 1644. (Fig. P. 1644.) The numerical coefficient of the dead load stress in the member CD is 12 compression. What concentrated live load on the lower chord will reduce the stress in this member to 0?

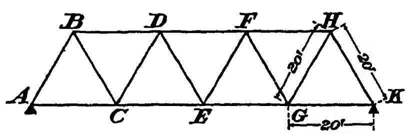


Fig. P 1644

P. 1645. (Fig. P. 1645.) There is a dead load of ten tons per panel point of the lower chord, and a concentrated live load of 40 tons moves over the top chord of the truss; find the greatest stress developed (*a*) in the chord member EF, (*b*) in the web member DT.

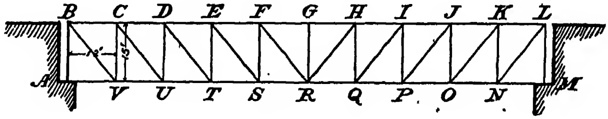


Fig. P 1645

P. 1646. In an eight panel Pratt truss insert the necessary counterbraces for a concentrated moving load of 8 and a dead load per panel point of 2 on the lower panel points.

P. 1647. (Fig. P. 1647.) The truss is to be constructed to support a concentrated live load of 90 tons applied at the upper panel points and the dead load as shown. Determine which panels must be counterbraced and the maximum stress which each counterbrace must be designed to carry.

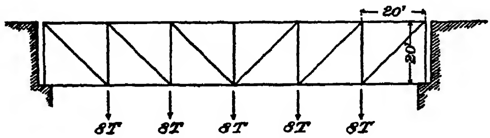


Fig. P 1647

P. 1648. A five panel Warren truss, resting on end supports, carries a dead load of 10 tons concentrated at each panel point of the lower chord. Determine the numerical coefficient of the stress for which each member of the truss must be designed in order to carry on the lower chord a concentrated live load of 50 tons.

P. 1649. (Fig. P. 1649.) The figure shows a truss and its dead load of 2 tons per panel point. The truss is to be built to carry in addition a concentrated live load of 24 tons on the upper chord.

(*a*) Determine what panels must be counterbraced and what stresses each counterbrace must be designed to carry.

(*b*) Determine what stress the member *DJ* must be designed to carry.

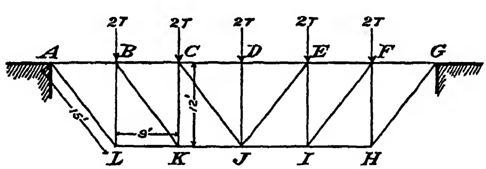


Fig. P 1649

P. 1650. (Fig. P. 1650.) The figure shows a truss and its dead load of 2 tons per panel point of the lower chord. A concentrated load of 32 tons may be applied at any panel point of the upper chord.

Determine what panels must be counterbraced and what stress each counterbrace must be designed to carry.

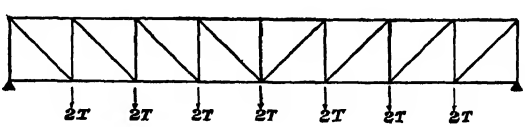


Fig. P 1650

P. 1651. A six panel Pratt truss, resting on end supports, carries a dead load of 2 tons concentrated at each panel point of the lower chord and a concentrated live load of 6 tons on the lower chord.

Determine the stresses for which each of the web members, including the counterbraces, must be designed.

P. 1652. In an eight panel Pratt truss with a concentrated moving load of 8 and a dead load of 2 per panel point of the lower chord, write the maximum numerical coefficients in each chord and web member and in each counterbrace.

P. 1653. (Fig. P. 1653.) The figure shows a truss whose dead load is 6 tons per panel point of the lower chord and whose live load is uniformly distributed and equal to 18 tons per panel point of the lower chord. The live load is longer than the truss.

(a) Find the greatest stress in the member DK.

(b) Find the greatest stress in the member JK.

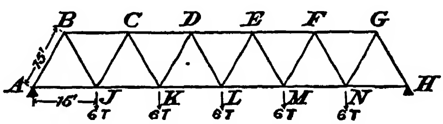


Fig. P 1653

P. 1654. (Fig. P. 1654.) The numerical coefficient of the dead load stress in the member PQ is 2 tension. What is the smallest uniformly distributed live load per panel point of the lower chord of length longer than the truss which will reverse the stress in member PQ?

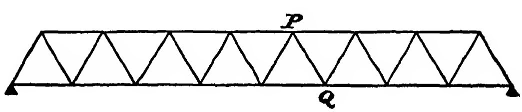


Fig. P 1654

P. 1655. (Fig. P. 1655.) The figure shows a truss and its dead load of 4 tons per panel point. The truss is designed to carry in addition a uniformly distributed live load of 35 tons per panel point of the lower chord.

(a) By the method of sections determine the stress in the member DN due to the dead load alone.

(b) Determine the stress which the member DE must be designed to carry.

(c) Determine the compressive and tensile stresses which the member DM must be designed to carry.

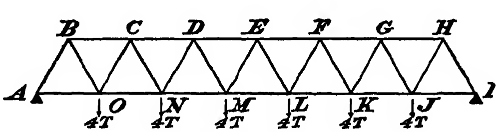


Fig. P 1655

P. 1656. (Fig. P. 1658.) The figure shows a truss whose dead load is 6 tons per panel point of the lower chord and whose live load is a uniformly distributed load on the lower chord of 24 tons per panel point. The life load is longer than the truss. Determine where tensile counterbraces are needed and the greatest stress which each will have to bear.

P. 1657. (Fig. P. 1657.) The figure shows a truss and its dead load.

(*a*) Write on the members of the truss the numerical coefficients of the dead load stresses and indicate the functions of ϕ to be used therewith.

(*b*) If the truss is to carry a uniformly distributed live load of 25 tons per panel point of the lower chord in addition to its dead load, determine the stresses for which the members *EF*, *CD* and *DF* must be designed.

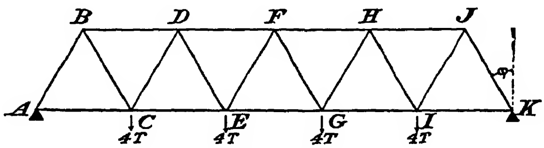


Fig. P 1657

P. 1658. (Fig. P. 1658.) The figure shows a truss whose dead load is 6 tons per panel point of the lower chord and whose live load is uniformly distributed and equal to 12 tons per panel point of the lower chord. The live load is longer than the truss.

(*a*) Which member of the upper chord will be subjected to the greatest stress? What will be the value of this stress?

(*b*) Which member of the lower chord will be subjected to the greatest stress? What will be the value of this stress?

(*c*) Which of the web members will be subjected to the greatest stress? What will be the value of this stress?

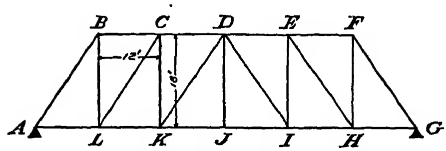


Fig. P 1658

P. 1659. (Fig. P. 1659.) The dead load per panel point of the lower chord is 3 tons and the live load, of length equal to the truss, is 18 tons per panel point of the lower chord. What is the greatest stress in each member of the truss, supposing that only one character of stress can be borne by each member? If any other members are needed, indicate them with the amount and character of the greatest stress developed.

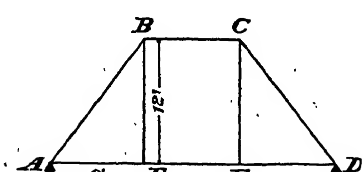


Fig. P 1659

P. 1660. (Fig. P. 1660.) The dead load is 12 tons per panel point of the lower chord and a live load, of length greater than the length of the truss, and of 60 tons per panel point, moves over the lower chord of the truss. In what members will the stress be reversed and how much?

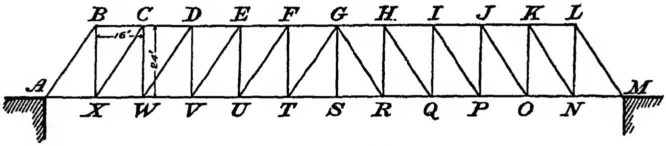


Fig. P 1660

P. 1661. A six panel Howe truss, resting on end supports, carries a dead load of 4 tons concentrated at each panel point of the lower chord, and a uniformly distributed live load of 24 tons per panel point of the lower chord. What counterbraces are necessary and what stresses must they be designed to carry?

P. 1662. A six panel Pratt truss, resting on end supports, carries a dead load of 4 tons concentrated at each panel point of the lower chord and a uniformly distributed load on the lower chord of 30 tons per panel point. What counterbraces are necessary, and what stresses must they be designed to carry?

P. 1663. In an eight panel Pratt truss, with a dead load of 2 per lower panel point and a uniformly distributed live load of 8 per lower panel point, write the maximum numerical coefficient in each chord and web member and in each counterbrace. (Diagonals take tension only.)

P. 1664. (Fig. P. 1664.) (*a*) What is the least concentrated load which when applied at *O* will reverse the stress in *DN*?

(*b*) What is the least concentrated load which when applied at *P* will reverse the stress in *DN*?

(*c*) What is the least concentrated load which when applied at *R* will reverse the stress in *DN*? (2) in *CO*? (3) in *BP*?

(*d*) What is the weight per panel point of the least uniform load which moving over this truss will reverse? (1) *DN*? (2) *CO*?

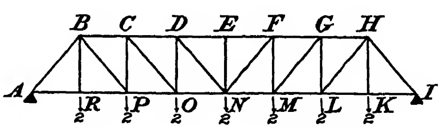


Fig. P 1664

P. 1665. (Fig. P. 1665.) The dead load per panel point of the lower chord is one ton. (Diagonals take tension only, except end diagonals which take compression).

(*a*) Determine the numerical coefficients of the dead load stresses in the members of the truss.

(*b*) Determine the coefficient of the stress for which each member of the truss (including the necessary counterbraces), must be designed if the truss is to carry, in addition

to the dead load, a uniformly distributed live load longer than the truss of ten tons per panel point of the lower chord.

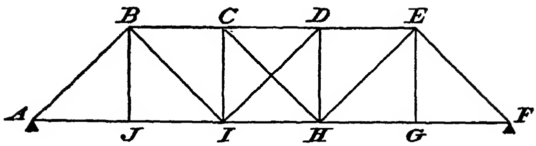


Fig. P 1665

P. 1666. (Fig. P. 1665.) The dead load per panel point of the lower chord is 2 tons. (Diagonals take compression only.)

(*a*) Determine the numerical coefficients of the dead load stresses in the members of the truss.

(*b*) Determine the numerical coefficient of the stress for which each member of the truss (including the necessary counterbraces), must be designed if the truss is to carry, in addition to its dead load, a uniformly distributed live load longer than the truss of 15 tons per panel point of the lower chord.

P. 1667. (Fig. P. 1667.) The figure represents a three-hinged arch loaded as shown. Pass an equilibrium polygon through the three hinges A, B and C.

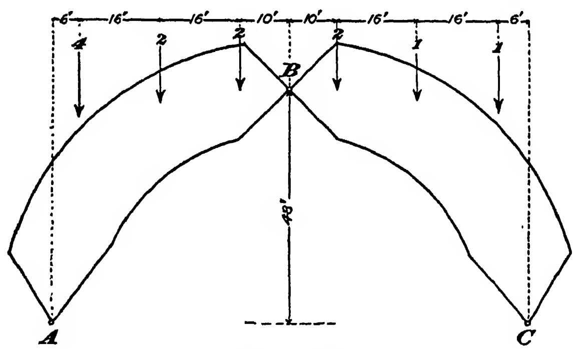


Fig. P 1667

P. 1668. (Fig. P. 1668.) The truss has a single concentrated load of 24 at panel point C. Write the numerical coefficient of the stress in each member of the truss.

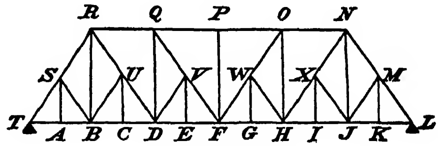


Fig. P 1668

P. 1669. (Fig. P. 1669.) The figure shows a three-hinged truss and its load. Find the reactions at the supports and the stress in the member AB.

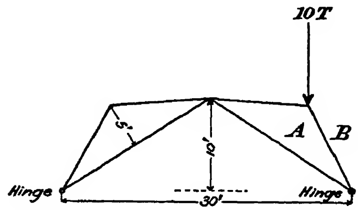


Fig. P 1669

P. 1670. (Fig. P. 1669.) The figure shows a three-hinged truss and its load. Connect the hinges at the points of support by a horizontal member. Find the reactions at the supports and the stress in the member AB.

P. 1671. (Fig. P. 1671.) The figure shows a three-hinged truss and its loads. Find graphically the reactions at the three hinges and the stresses in each member of the truss. The members CD and GH are 6′ long.

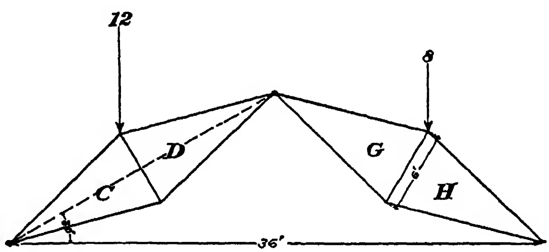


Fig. P 1671

P. 1672. (Fig. P. 1672.) Find graphically the stress in each member of the cantilever truss with the loading as shown.

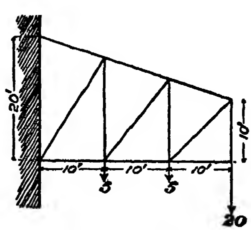


Fig. P 1672

P. 1673. (Fig. P. 1673.) Find graphically the stress in each member of the cantilever and the reactions at the supports.

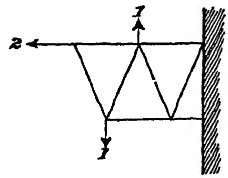


Fig. P 1673

P. 1674. (Fig. P. 1674.) Find graphically the stress in each member of the cantilever and the reactions at the supports.

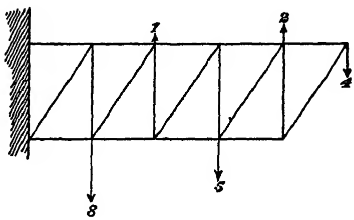


Fig. P 1674

P. 1675. (Fig. P. 1675.) Find graphically the stress in each member of the cantilever and the reactions at the supports.

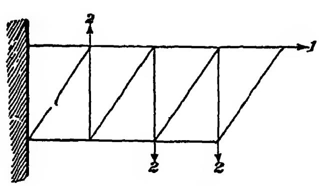


Fig. P 1675

P. 1676. (Fig. P. 1676.) Find graphically the stress in each member of the truss under the action of the forces shown.

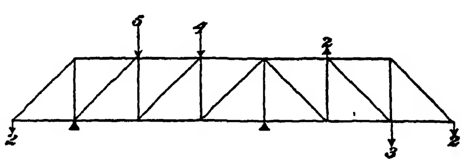


Fig. P 1676

P. 1677. (Fig. P. 1677.) (*a*) Find the reactions. (*b*) Find graphically the stress in the member *CE*.

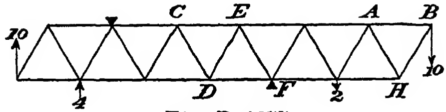


Fig. P 1677

P. 1678. (Fig. P. 1678.) The figure shows a combined simple and cantilever truss, with the loads and points of support as indicated. Write out analytically the numerical coefficient of the stress in each member of the truss.

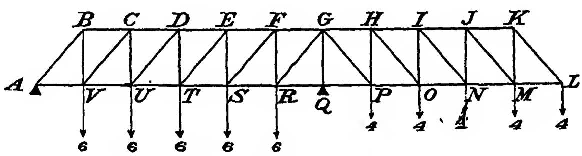


Fig. P 1678

P. 1679. (Fig. P. 1679.) The figure represents a continuous truss loaded as shown.

(*a*) Find the reactions at the supports.

(*b*) Write out analytically the numerical coefficient of the stress in each member of the truss.

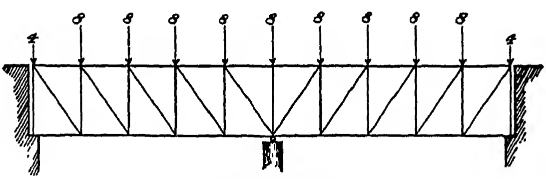


Fig. P 1679

P. 1680. (Fig. P. 1680.) The figure represents an arch loaded as shown. Pass an equilibrium polygon through the points *A*, *D* and *E*.

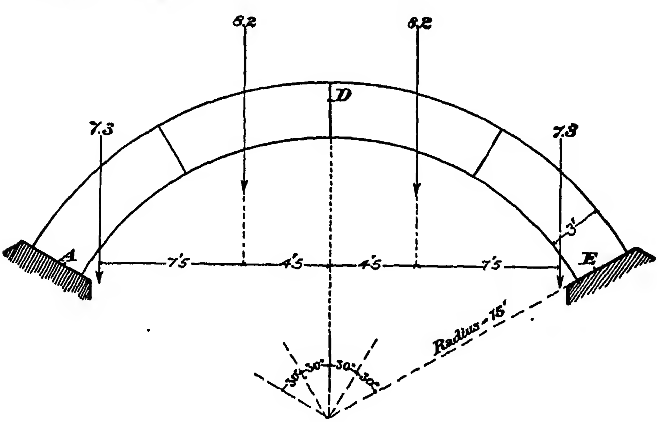


Fig. P 1680

CHAPTER XVII.

DAMS AND RETAINING WALLS.

1701. Dams and retaining walls are structures designed for the purpose of maintaining water or earth in positions other than those which they naturally take. The water or earth thus restrained exerts pressure upon the dam or wall in its effort to assume its natural position; and this pressure is counteracted by the weight of the dam or retaining wall, by friction, and by special design of the dam.

1702. The side exposed to the pressure is the **back,** the opposite side is the **face,** and the bottom surface is the **base.** The intersection of the back and the base is the **heel,** the intersection of the base and the face is the **toe,** and the intersection of the face and the top is the **crest** of the dam or wall. The profile is a vertical section normal to the face. The face and back may be either vertical or inclined, and if inclined, may be either plane or curved surfaces. The slope of the inclined face is called the **batter,** and is stated as the ratio of the horizontal distance to the vertical distance.*

1703. Dams and retaining walls may fail by

a. Overturning
b. Sliding
c. Crushing at the toe,
  (1) of material of the structure
  (2) of material of the foundation.

DAMS.

1704. Figure 1704A shows all of the forces ordinarily considered in designing a dam. The resultant is not shown. The forces are:

a. Water pressure on upstream side (F')
b. Water pressure on downstream side (F'')
c. Weight of dam itself (F''')
d. Weight of water acting so as to increase the weight of the dam (included in *a* and *b*)
e. Upward water pressure (F^{IV}).

* Road and railway center line slopes are stated in percentages of rise divided by horizontal distance, which is just the reverse of the batter of dams, retaining walls, and side slopes.

Each of these is discussed separately. In practice, a linear foot of the dam or wall is taken into consideration, its weight and the pressures on it being typical of the remainder of the structure.

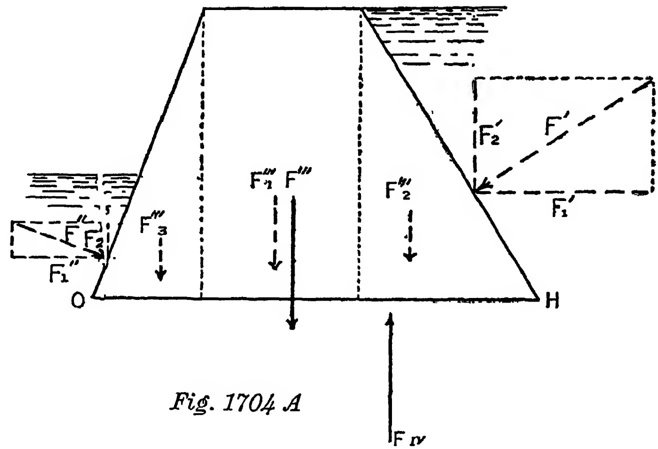


Fig. 1704 A

1705. **Water Pressure on Upstream Side.**—As shown in Chapter V, this pressure is normal to the side pressed: the resultant acts at a point two-thirds of the distance below the surface of the water, and is equal to $wh^2/2 \cos \alpha$, in which w is the weight of a cubic foot of water in pounds, h is the depth of the water in feet, and α is the angle which the side of the dam makes with the vertical.

1706. It is customary to resolve this water pressure, F', into its vertical and horizontal components, as shown in Fig. 1704A. Each is then considered separately. In Fig. 1706A, representing a dam whose back

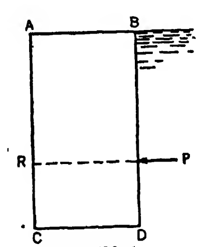


Fig. 1706 A

is vertical, the resultant pressure acts normal to the side BD and at a distance $\frac{2}{3}$ of BD from B, or $\frac{1}{3}$ of BD from D, and equals $wh^2/2$ pounds. In Fig. 1706B, representing a dam whose back is inclined, the resultant pressure also acts normal to the surface, and the pressure on a linear foot

of the side FH must be $(w \times FH \times h/2) = wh^2/2 \cos \alpha$ pounds. It acts at a distance of $\frac{2}{3}$ of FH from F or $\frac{2}{3}$ of h below the surface of the water.

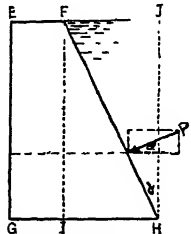


Fig. 1706 B

The horizontal component of this pressure equals (Eq. 519D):

$$\frac{wh^2}{2 \cos \alpha} \times \cos \alpha = \frac{wh^2}{2} \text{ pounds} \qquad (1706A)$$

Hence, whatever be the slope of the back of the wall, the horizontal component of the water pressure will be the same as the pressure on a wall of equal height with a vertical back, and will depend only upon the depth of the water.

The vertical component of the pressure equals (Eq. 519E):

$$\frac{wh^2 \sin \alpha}{2 \cos \alpha} = \frac{wh^2 \tan \alpha}{2} \text{ pounds} \qquad (1706B)$$

in which $h^2 \tan \alpha/2$ is the area of the triangle FJH, and $wh^2 \tan \alpha/2$ is the weight of a prism of water 1′ wide (perpendicular to the page) whose end is FJH. Hence the vertical component of the water pressure against the inclined back of a dam is equal to the weight of the prism of water vertically above the inclined back. The action line of this weight (or component) intersects GH at a distance $\frac{1}{3}$ of IH from H. As the action line of the vertical component acts within the base of the wall, it increases the total pressure upon the base.

1707. **Water Pressure on Downstream Side.**—This acts as shown by F'' in Fig. 1704A. It is not so deep as that on the upstream side. Quite often, it is not considered in the design of dams because it assists to counteract the pressure on the upstream side, and the omission of it is an error on the side of safety. When considered, it is resolved into its two components F_1'', and F_2'', which are considered separately as shown later.

1708. **Weight of Dam Itself.**—This is the main factor in the stability of the dam. The weight of the material of the dam is great per unit of volume and it is used to overcome the effect of the water pressure

acting against the dam. The force, F''', acting from the center of gravity of the dam may be taken as the force due to the weight of the dam; but it is usually more convenient to separate it into its three components, F_1''', F_2''', F_3''', as shown in the figure.

1709. **Weight of Water Acting so as to Increase the Weight of the Dam.**—If the face and back of the dam are vertical, there will be no such weight of water. However, in the general case covering all conditions, illustrated in Fig. 1704A, and explained in paragraph 1706, the vertical components of the pressures, F_2' and F_2'', are the weights of the prisms of water above the inclined face and back of the dam. From geometry it is apparent that the action lines of these weights, passing through the centers of gravity of the prisms, intersect the inclined face and back of the dam at the points where the total pressures are applied, $\frac{2}{3}$ of the distance from the water surface. As the weight of these prisms of water acts in the same direction as the weight of the masonry, they add to the stability of the dam; if considered, the amount of masonry may be reduced; if neglected, the error is on the side of safety.

1710. **Upward Water Pressure.**—With foundations on rock, the amount of upward pressure is not easily determined. It is evident, however, that there is such pressure. Experiments by the Corps of Engineers on dams already constructed show that water percolates through joints between the concrete and the rock foundation, and travels in small veins in these joints. Test holes were driven from the tops of dams to the foundations, and the height to which water rose in these holes was measured. It was seen that, if the water is given free exit, the upward pressure under the foundation decreases between upper and lower pools; but, if the exits are closed, the pressure in the veins becomes that of the upper pool as soon as enough water has passed through the veins to fill the test holes. The amount of space of these small veins, that is, the area of upward water pressure, varies from nearly zero in excellent granite foundations to 50 per cent or more in rotten shale. The final height of water in the test holes when free exit is provided below the dam varies roughly from that of the upper pool at the back to that of the lower pool at the face. If efficient drains are introduced, the upward water pressure below such drains is practically negligible in good rock. In this connection it is well to note that the drains must not be too small and must not be stopped. With rock foundations, the dams of the $60,000,000 canalization project on the Ohio River were designed in the general case in accordance with the following rule (Fig. 1710A):

Upward water pressure will be taken as varying at a constant ratio from upper pool at upstream edge to lower pool at drains; and as acting under 50 per cent of the base on firm, solid rock, and under 100 per cent of the base on weak rock.

While dams should always be founded on rock if possible, there are cases where the required site has no rock foundation within a practicable depth. This was often the case in the canalization of the Ohio River,

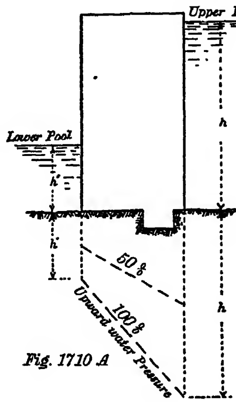


Fig. 1710 A

which project required that the dams be placed within certain limits of length of river, regardless of the suitability of the foundations available. As a result, many dams were built on piles driven in sand or gravel foundations. In these cases a row of sheet piles was driven at the toe of the dam to prevent washing away of the foundation and to reduce upward water pressure. In the project for the canalization of the Ohio River, such dams were designed to resist an upward water pressure varying in a constant ratio from upper pool at the heel to lower pool at the toe, and no allowance was made for reduction of water pressure by the use of steel sheet piles at the heel.

1711. Of the forces acting on a dam, it is possible for the designer to make accurate calculations for every one except the upward water pressure. Quite often, this is not considered at all; in most cases, it is estimated; and nearly all failures of dams due to poor design are found to be due to neglect of this upward water pressure. Fig. 1710A shows by dotted lines the maximum upward water pressure which can reasonably be expected. The percentage of this that is used in the design is a matter for the designer to decide: assumption of too little upward water pressure will result in the failure of the dam; assumption of too great upward water pressure will result in waste of money. The force F^{IV} in Fig. 1704A represents the upward pressure. It should always be considered in design of dams.

Safety Against Overturning.

1712. The dam shown in Fig. 1704A will not overturn if the forces acting to turn it clockwise around the toe, O, are greater than those

acting to turn it counter-clockwise. Considering all the forces, each with its proper lever arm to O, we have the equation:

$$\text{Moment to Resist Overturning} = F_2'l_2' + F_2''l_2'' + F_1'''l_1''' + F_2'''l_2''' + F_3'''l_3''' - F_1'l_1' + F_1''l_1'' - F^{IV}l^{IV} \qquad (1712A)$$

If the second member of the equation is negative, the dam will overturn.

1713. The resultant of all the forces pierces the base, or base extended, at some point. If we consider only the resultant, as R in Fig. 1713A, and resolve it into its two components where it pierces the base, at C, we have, for Fig. 1713A, center of moments being at O,

$$\text{Moment to Resist Overturning} = R_2l_2 \qquad (1713A)$$

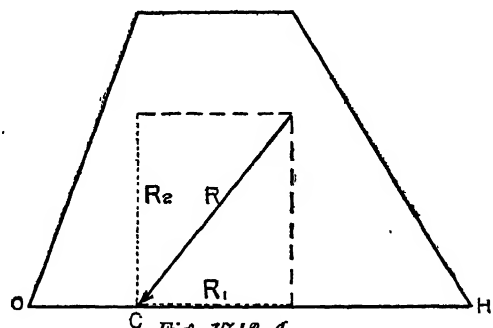


Fig. 1713 A

Comparing equations 1712A and 1713A, we see that the Moment to Resist Overturning equals the algebraic sum of all the vertical forces acting on the dam, multiplied by a lever arm equal to the distance from the toe to the point where the resultant pierces the base. If either the sum of the vertical forces or the lever arm is negative, the dam will overturn.

1714. **Middle Third.**—In the design of dams it is usual to require that OC be positive and at least one-third of OH; that is, that the action line of the resultant fall within the middle third. If this is not the case, there will be tension in the masonry near the heel, as explained in paragraph 1108. As the adhesion of masonry joints, or the adhesion of concrete to rock is not great, it is usually assumed that there is none, that tension will at once cause cracks to develop in the joint near the heel, and that at once upward water pressure in such cracks will become a distinct pressure equal in intensity to the maximum pressure due to the depth of water at the cracks. Such upward water pressure tends to shift the center of pressure still farther from the middle third, to cause more cracks, and to overturn the dam.

As this change of upward pressure continues, F^{IV} continues to increase until the overturning moment becomes negative, and the dam overturns. Therefore, for safety, the resultant must pierce the base within the middle third. The point where the resultant pierces the base may be determined by taking that point as the center of moments, its distance from O as X, and writing the equation of moments:

$$R_2 \times 0 + F_2'(l_2' - x) + F_2''(l_2'' - x) + \text{etc.} = 0 \qquad (1714A)$$

The unknowns are R_2 and x. But R_2 is eliminated from the equation, as its lever arm is zero. Therefore, x can be determined.

Experiments show that there is some adhesion of masonry to other masonry or to rock; but the strength of such adhesion is indefinite and unreliable; and it is proper practice to design the dam so that the center of pressure will fall within the middle third.

Safety Against Sliding.

1715. If the dam is constructed of horizontal layers of masonry, and depends only upon the friction between layers to prevent sliding, the friction at each joint must be greater than the resultant horizontal pressure upon the wall above the joint.

To illustrate this, let there be assumed a dam with a vertical back, and let

w' = weight of masonry per cubic foot = 144 pounds, for ordinary masonry
w = weight of water per cubic foot = 62.5 pounds
h = height of wall above the joint. Let it be assumed that the water rises to the level of the top of the dam. Then h = depth of water above the joint.
b = mean thickness of the wall
μ = coefficient of friction of masonry on masonry = $\frac{2}{3}$

Then, from the above condition for stability, we have

$$w'bh \times \mu > \frac{wh^2}{2}$$

Solving for b, we have: $$b > \frac{wh}{2\,w'\mu} > 0.33\,h \qquad (1714A)$$

If a dam is constructed on an earth foundation, the base must be enlarged, because the coefficient of friction between masonry and earth is less than $\frac{2}{3}$.

Masonry dams rarely fail because of weakness in horizontal planes, but sometimes fail because of insufficient frictional resistance at the foundation.

1716. In ordinary concrete construction it is not deemed advisable to rely on the friction between the concrete and the rock or between different monoliths of the concrete. Therefore, it is customary at all joints to place a key of concrete in a groove made in the underlying material, as shown in Fig. 1716A. The dam cannot then fail by sliding, unless the resulting horizontal force is sufficient to shear this key in addition to overcoming the frictional resistance as described in paragraph 1715.

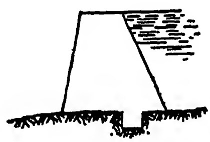


Fig. 1716 A

In dams constructed on pile foundations in earth, the tops of the piles extend about 12″ into the concrete; the dam cannot then fail by sliding, unless the piles are sheared off and the friction of the masonry on earth is overcome.

Safety Against Crushing.

1717. In Chapter XI, it was shown that:

a. If the center of pressure due to the applied forces, at any horizontal joint, is at the center of gravity, the pressure is uniformly distributed.

b. If the center of pressure is at the extremity of the middle third, the pressure at the near edge will be twice the mean pressure, and at the far edge will be zero.

c. If the center of pressure is on an edge, all of the joint will not be under pressure, and the pressure at the edge will be four times the mean pressure.

From these principles, it is seen that it will be best to have the center of pressure at the center of the base. If it is at the extremity of the middle third, the maximum unit pressure will be twice the mean unit pressure. The usual design against overturning requires that the resultant pierce the base within the middle third, thus making the maximum unit crushing pressure not greater than twice the mean unit crushing pressure. As dams are usually made of solid materials and well spread out, it is evident that the usual design for safety against overturning will almost invariably be satisfactory against crushing.

Where the foundation is on earth or piles, special consideration is given to the crushing value of the material; but with the resultant pierc-

ing the base within the middle third, the design is usually safe against failure by crushing.

1718. **Problem.**—Design a masonry dam, 30′ high, of trapezoidal cross-section, with vertical back, and water level with its top. Allow 6′ width on top for walkway, ice action, etc. Assume upward water pressure on base as 4/10 of the hydrostatic pressure at the heel and diminishing uniformly to zero at the toe. Allow no tension in the masonry. Assume the masonry weighs 150 pounds per cubic foot, the safe supporting power of the foundation bed is 4 tons per square foot, and the coefficient of friction of masonry on the material of the foundation bed is 0.65.

Solution.—Consider a section of the dam 1′ wide, and let thickness of the dam = x.

a. Against overturning:

In order that the dam may be stable against overturning and no masonry be in tension, the center of pressure (resultant) must pierce the base within the middle third. Assuming that the resultant (and the reaction) pierces the base at the edge of the middle third nearest to the toe of the dam, and taking this point as the center of moments, we have for the equation of moments (same as 1714A):

Moment of reaction + Moment of weight of rectangular section of dam + Moment of weight of triangular section of dam − Moment of direct water pressure − Moment of upward water pressure = 0.

Expressing this numerically for the dam in this problem, we have:

$$R \times 0 + (30)(6)(150)\left(x - \frac{x}{3} - 3\right) + \left[\frac{30\,(x-6)}{2}\right](150)\left[\frac{2\,(x-6)}{3} - \frac{x}{3}\right]$$
$$- \frac{(62.5)(30)(30)}{2}\left(\frac{30}{3}\right) - .4 \times \frac{(62.5)(30)(x)}{2}\left(\frac{2\,x}{3} - \frac{x}{3}\right) = 0$$

Solving, we obtain, $x = 19'$

b. Against sliding:

The tangent of the angle between the overturning forces and the stabilizing forces, with a width of dam of 19′, is

$$\frac{28{,}125}{49{,}125} = 0.57$$

which is less than the given coefficient of friction, 0.65. Therefore the width of 19′ is safe against sliding.

c. Against crushing:

As the center of pressure is at the extremity of the middle third, the maximum pressure is twice the average pressure, or with the width of 19′, equals

$$2\left[(30)(6)(150) + \frac{(30)(19-6)(150)}{2} - .4\,(62.5)\,\frac{(30)(19)}{2}\right]/19 = 5171 \text{ pounds}$$

which is less than the allowable load of 4 tons per square foot. Therefore the width of 19′ is safe against crushing. Even assuming that there is no upward water pressure, the calculated crushing load is well within the allowable limit.

1719. **Problem.**—A masonry dam of rectangular cross-section is 30′ high, 20′ thick, and has water standing on the back 3′ from the top. The specific gravity of the masonry is 2.3. Assume no upward water pressure. Find the center of pressure on the base.

Solution.—

Water pressure on back = $wh^2/2 = 62.5 \times 27 \times 27/2 = 22{,}800$ pounds.

Weight of masonry = $62.5 \times 2.3 \times 30 \times 20 = 86{,}300$ pounds.

Letting x = distance of center of pressure from the toe, and taking the center of moments around the center of pressure, we have the following equation of moments:

$$R \times 0 + 86{,}300\ (10 - x) - 22{,}800 \times 9 = 0$$

Solving, we obtain, $x = 7.6'$

Typical Cross-Section.

1720. In order that the action line of the weight shall intersect each horizontal section within its middle third when the reservoir is empty, the limiting forms of the profile must be a right-angled triangle with a vertical back, and a similar triangle with a vertical face, as in Figs. 1720A

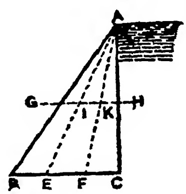


Fig. 1720 A

and C. In Fig. 1720A the action line of the weight alone passes through F; in Fig. 1720C, it passes through E. In the rectangular wall, Fig. 1720B, the action line of the weight passes through F, the middle point of BC.

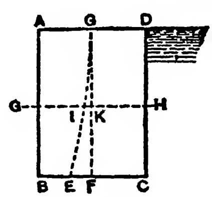


Fig. 1720 B

Let it be required to compare the amount of masonry in three walls of equal height, under the requirement that the resultant of the maximum water pressure and the weight shall not intersect any horizontal section outside the middle third.

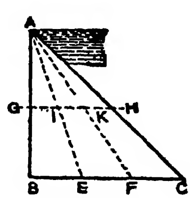


Fig. 1720 C

To determine the amount of masonry, it is necessary to ascertain the relation between the base and height in each case, under the extreme condition of the resultant piercing the base at the extremity of the middle third.

Let h = height of dam = depth of water
b = width of base
w' = weight of masonry per cubic foot = 144 pounds, assuming specific gravity as 2.3
w = weight of water per cubic foot = 62.5 pounds

The effect of the water pressure in Figs. 1720A and B is to cause the resultant to pierce the plane CB to the left of the point where it is intersected by the action line of the weight alone; the resultant may move from F, when the reservoir is empty, to E, when the reservoir is full. Equating the moments of the weight and the water pressure, we have:

Fig. 1720A: $$\frac{w'hb}{2} \times \frac{b}{3} = \frac{wh^2}{2} \times \frac{h}{3}$$

Thence, $$b = h\sqrt{\frac{w}{w'}} = 0.66\,h = \frac{2\,h}{3} \qquad (1720A)$$

Fig. 1720B; $$w'hb \times \frac{b}{6} = \frac{wh^2}{2} \times \frac{h}{3}$$

Thence, $$b = h\sqrt{\frac{w}{w'}} = 0.66\,h = \frac{2\,h}{3} \qquad (1720B)$$

In Fig. 1720C, when the reservoir is full, we have, by equating the moments of the vertical and horizontal components of the water pressure about the point E (moment of weight of masonry being equal to 0):

Fig. 1720C; $$\frac{whb}{2} \times \frac{b}{3} = \frac{wh^2}{2} \times \frac{h}{3}. \quad \text{Thence, } b = h \qquad (1720C)$$

The areas will be:

Fig. 1720A; $$\text{Area} = \frac{2\,h}{3} \times \frac{h}{2} = \frac{h^2}{3} \qquad (1720D)$$

Fig. 1720B; $$\text{Area} = \frac{2\,h}{3} \times h = \frac{2\,h^2}{3} \qquad (1720E)$$

Fig. 1720C; $$\text{Area} = h \times \frac{h}{2} = \frac{h^2}{2} \qquad (1720F)$$

Therefore, the masonry required by the three sections to fulfill the conditions stated will have areas in the proportions 2 : 4 : 3. Consequently, Fig. 1720A requires less masonry and less expense for the same degree of safety.

1721. To illustrate the above conclusions by a concrete example, let the height of each of the dams be assumed as 60′, and the widths of the bases be 40′, 40′, and 60′ in accordance with the relative widths determined in paragraph 1720. The weight of the dams per linear inch will be: Fig. 1720A, 14,400 pounds; Fig. 1720B, 28,800 pounds; Fig. 1720C, 21,600 pounds. When the reservoirs are empty, the mean pressures on the base per square inch will be: 30 pounds in Figs. 1720A and C, and 60 pounds in Fig. 1720B. The maximum pressure, due to the weight of the dam alone, will be 60 pounds, and will be the same in the three profiles.

When the reservoirs are full, these pressures are changed. In Fig. 1720A, the vertical component of the resultant pressure is the weight of the dam; it acts through E, the extremity of the middle third; the mean vertical pressure is therefore, as before, 30 pounds; the maximum pressure is at B and is 60 pounds; the minimum is at C and is zero. In Fig. 1720B, the vertical component of the resultant pressure is the weight of the dam; it acts through E, the extremity of the middle third; the mean pressure is therefore, as before, 60 pounds; the maximum is at B and is 120 pounds; the minimum is at C and is zero. In Fig. 1720C, the vertical component is the weight of the wall and the vertical component of the water pressure, both of which act through E; the mean pressure is $30 + 13 = 43$ pounds; the maximum pressure is at B and is 86 pounds; the minimum pressure is at C and is zero. Hence Fig. 1720A produces the least maximum pressure at the toe when the reservoir is full, and is the most economical profile for safety against crushing.

1722. **Modification of the Typical Cross-Section.**—Fig. 1720A has been shown in paragraphs 1720 and 1721 to be the most economical profile against overturning and crushing. It is not less economical against sliding. Therefore, it is the basis of design for such structures. However, certain modifications become necessary in actual construction, as follows:

a. The width is increased at the crest. The upper part of Fig. 1720A is designed to withstand static water pressure alone. Dams of large reservoirs are subject to wave action and ice pressure, and overflow dams are subject to the pressure of the current and the shock of floating bodies. Moreover, for communications it is desirable to have upon the top of all except overflow dams a footway at least 3′ wide, and upon the top of all long dams a driveway at least 10′ wide. Therefore, the crest of dams is made from 3′ to 20′ thick, and the upper part of the profile is approximately rectangular (Fig. 1722A).

b. The width is increased at the base. If a dam constructed as in Fig. 1720A were 120′ instead of 60′ high, then, since its weight increases with the square of the height and its base increases as the first power of

the height, the mean unit pressure at the base would be 60 pounds and the maximum unit pressure would be 120 pounds; if 180′ high, the mean unit pressure would be 90 pounds and the maximum 180 pounds. It is not considered desirable in dams of moderate height to have the maximum unit pressure exceed 100 pounds per square inch, or about 7 tons per square foot; nor in very high dams is it considered safe to have a maximum unit pressure exceeding 200 pounds per square inch or 14 tons per square foot. It is therefore best, in order to limit the maximum pressures to the intensities above given, to increase the width at the base of all high dams beyond that given in Fig. 1720A.

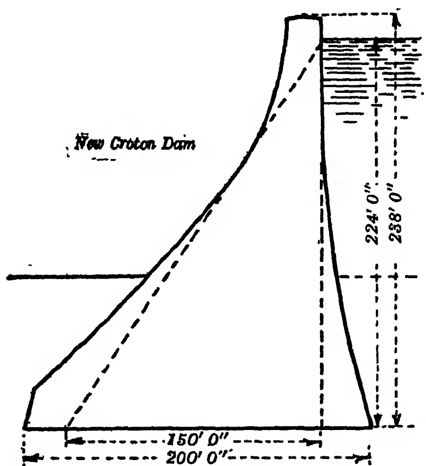


Fig. 1722 A

From this increase at the top and base of high dams, there results the form shown in Fig. 1722A, which is the profile of the Croton Dam. In determining the resultant pressure upon such a dam the vertical component of the water pressure is usually omitted; this is an error on the side of safety, because this component would tend to move the center of pressure towards the center of gravity when the reservoir is full.

1723. **Curve of Pressure.**—The curve of pressure is the curve which connects the centers of pressure of the horizontal joints. To find the curve of pressure of a dam, find the center of pressure at each of a number of horizontal sections, and join these points by a line. When the reservoir is empty, the center of pressure at each horizontal joint is the point obtained by dropping a perpendicular through the center of gravity of the section of the wall above it. When the reservoir is full, the center of pressure at each horizontal joint is the point where the resultant of the weight and the water pressure above the joint pierces that joint.

Types of Dams.

1724. **Masonry Gravity Dams.**—This is the type of dam most commonly used. The cross-section is similar to that shown in Fig. 1722A. The weight of the masonry in the dam, plus the weight of the water, are the forces to counteract the horizontal and upward pressure of the water.

Until recent years, stone and even brick were used in these dams: but monolithic concrete is now used in nearly every case, stone being principally used for ornamentation. In the Ohio River Canalization Project, involving the construction of 52 dams at a cost of about $60,000,000, every dam is of concrete.

1725. **Arch Dams.**—This type of dam is placed in a narrow gorge, and is so constructed that the pressure of the water acts to tighten the dam which transmits this pressure by arch construction to the sides of the gorge. The weight of the dam section is also a very important part of the design. Arch dams are advisable only in very high dams across very narrow gorges.

1726. **Reinforced Concrete Dams.**—Fig. 1726A shows the principle of construction. The dam consists of a reinforced concrete slab forming a sloping back, and supported at intervals by buttresses, which may take

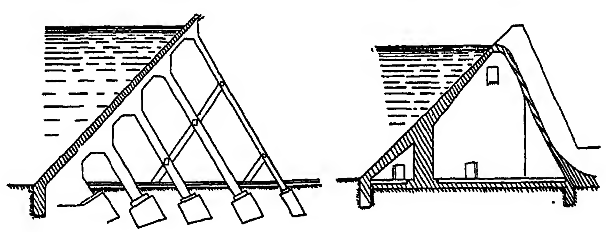


Fig. 1726 A Reinforced Concrete Dams

the form of solid walls, or rows of columns. The buttresses are tied together laterally. In case the dam is an overflow dam, the face of the dam is also a reinforced concrete slab supported by the buttresses, and connected by a curved surface to the back so as to form a smooth pathway for the water. The face and back are designed as are reinforced concrete floors, and the buttresses as walls and columns. The face and back are frequently tied together at the bottom by a floor to give greater stability. The dams may be hollow, or may be filled with earth to add weight, but in either case upward water pressure does not act as a force tending to overturn the dam or to reduce the friction on the base; the water escapes through the hollow part of the dams, or in channels left in the filling, thus causing no upward pressure.

1727. **Rock-Fill Dams.**—These are easily constructed, being simply a filling of rock in such shape as to form a dam, this filling being protected against leakage by a concrete or timber sheeting on the upstream side. It is not satisfactory, as pressure on the sheeting tends to move the rock-fill and break up the face. Upward water pressure is neglected, as the water leaks through the rock-fill. If the water level need be raised only a few feet, and there is a great volume of water, the sheeting may be omitted and the leakage allowed.

1728. **Timber Dams.**—A crib type of timber dam is shown in Fig. 1728A. It is similar to the rock-fill dam in that leakage is prevented by sheeting on the upstream side, and upward water pressure through the

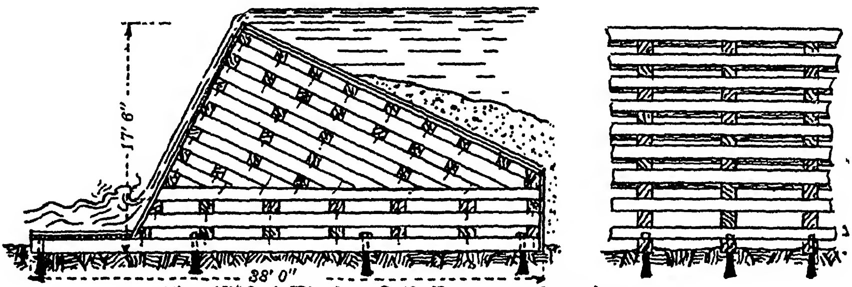


Fig. 1728 A Timber Crib Dam, anchored to rock

crib is allowed and expected. The sheeting is made of plank. A slight displacement of this sheeting is less disastrous than it would be in a rock-fill dam. The sheeting is sometimes omitted, when only a slight rise of water is needed and there is great volume of water.

1729. **Earth Dam.**—This type of dam is quite common, being easy to construct. The slopes are three or more horizontal to one vertical, so that it is very wide as compared to its height. However, there are many high earth dams. An earth dam will quickly wash away if the water runs over the top of the dam. Likewise, holes through the dam quickly enlarge and cause failure.

To make the dam more nearly water-tight and to prevent continuous holes through it, the center is often made of hydraulic fill. At Panama, the Gatun Dam consists of an hydraulic fill with a surfacing of rock.

In some earth dams, the center is a core made of concrete.

1730. **Movable Dams.**—These have the usual foundation of masonry gravity dams: but the upper portion is replaced by a movable structure. The usual movable portion of the dam consists of wooden wickets, each 4′ wide, so designed that they can be lowered flat on the top of the foundation or can be raised to form part of the retaining part of the dam.

Figure 1730A shows a cross-section of a movable dam. It is evident that the design of the concrete foundation must be such that it will alone

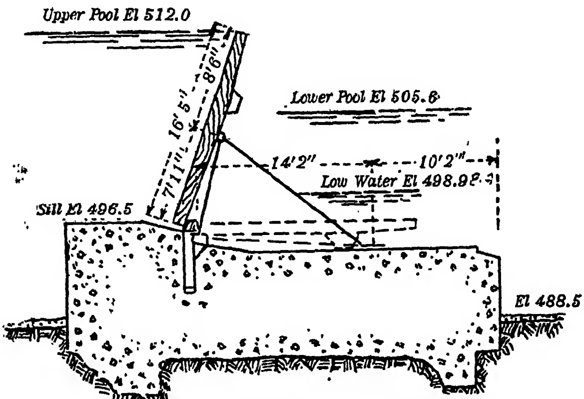


Fig. 1730 A Chanoine Wicket Dam

render the dam safe against the pressure of the water with the wickets up. This is necessary because the wickets themselves have practically no weight and do not at all assist in the safety of the dam.

RETAINING WALLS.

1731. Retaining walls differ from dams as follows:

a. There is no consideration of upward pressure under the wall.

b. The face of the wall is usually vertical: thus there is no weight of material on the face of the wall.

c. The earth is often higher than the top of the wall, increasing gradually from zero at the wall. This additional earth is called the *surcharge.*

d. The pressure and point of application of center of pressure of the earth are not definitely determined.

Pressure of Earth.

1732. The last of the above differences is the most important one. The pressure of earth against the back of the wall differs from that of water in that particles of earth cohere to each other with considerable tenacity, especially when in a moist condition and free movement of the mass is restricted by friction. Hence there is no exact method of determining the pressure of earth, as it varies from the tenacity of compact clay, which when dry will stand at any angle, to that of dry sand, which is practically without cohesion: it even varies in the same material with the degree of moisture and compactness.

1733. Various formulas for pressure of earth on retaining walls have been deduced or obtained by experiment. None of them is exact. All are based upon certain hypotheses, of which the three below are generally accepted:

a. The cohesion of the material is practically zero: and the earth consists of sandlike grains which are free to move, subject to the force of friction. It is evident such an hypothesis is on the side of safety.

If earth without cohesion is poured out of a vessel on a smooth surface, it will form a cone. The inclination of the surface to the horizontal is called the natural slope of the material; the angle of inclination is called the angle of repose. Since a particle is held in equilibrium on the surface by a force of friction developed by its normal component, equal to and counterbalancing the component of its weight parallel to the surface, the angle of repose is also called the **angle of friction**, and the tangent of the angle of repose is the coefficient of friction of the material. The angle of repose of wet earth is about 30°, of very dry earth 38°, of moist earth 45°. In formulas it is usually assumed to be about 34°, its natural tangent being $\frac{2}{3}$.

In Fig. 1733A, *ABCD* is a section of a straight retaining wall; *DF* is the natural slope of the earth. The angle ϕ is the angle of repose.

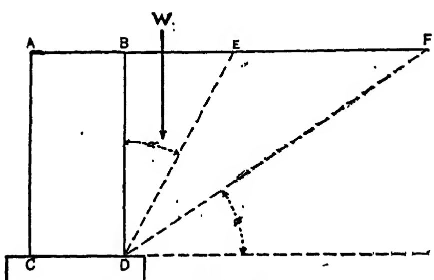


Fig. 1733 A

b. If the wall is suddenly removed, a portion of the earth will fall at once; the remainder will assume its natural slope gradually. The pressure upon the back of the wall is due to the volume which will at once fall: the plane along which this volume separates from the remainder of the earth is called the **plane of rupture**, *DE* (Fig. 1733A). The angle *BDE* is called the **angle of rupture.**

c. The pressure due to the volume *BDE* is distributed over the back of the wall *BD*, in the same manner as water pressure; the unit pressure at any point varies with its vertical distance from *B*, and the center of pressure on a rectangular area whose height is *BD* is $\frac{2}{3}$ *BD* from *B*.

1734. Coulomb's Formula.—Coulomb accepted the three hypotheses *a*, *b*, and *c*, above, and assumed also that

d. There is no friction on the surface BD; hence action line of the resultant pressure on the back of the wall is normal to the surface.

To deduce Coulomb's formula for the pressure against the surface BD, assume the retaining wall as shown in Fig. 1734A. Accepting the four hypotheses, taking the prism BDE as a "free body," and remembering that the forces must have signs in accordance with their action lines, we have the equation of equilibrium:

Weight of the Prism + Reaction of surface BD + Reaction of surface $DE = 0$ (1734A)

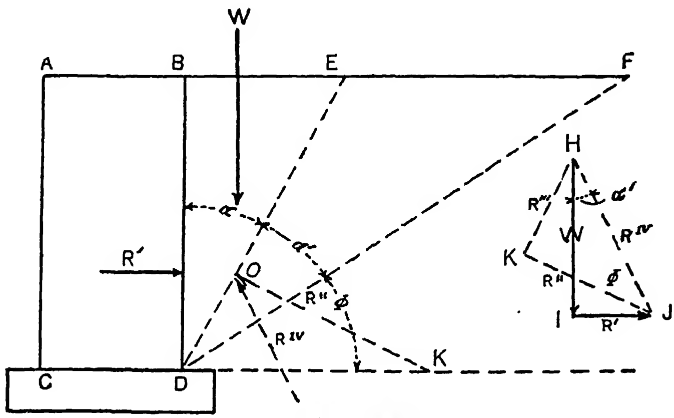


Fig. 1734 A

In the force polygon HIJ, we have

a. The weight (W) of the prism per unit of length $= \frac{1}{2}\, wh^2 \tan \alpha$: its action line is vertical and through its center of gravity.

b. The reaction of surface $DB(R')$ is horizontal and may be taken as one force R' at a point $\frac{2}{3}\, BD$ from B: its intensity is unknown.

c. The reaction of surface $DE(R^{IV})$ is unknown. However, we know that it is the resultant of the frictional resistance of $DE(R''')$ and the resultant normal pressure (R''). We do not know the intensities of R'' and R''' but we do know that they always retain a certain relationship, namely: frictional resistance of $DE(R''')$ = resultant normal pressure $(R'') \times$ coefficient of friction. Hence, $R''' = R'' \tan \phi$.

Therefore, in the force polygon HKJ, we have the direction of R^{IV} determined by drawing the line (R^{IV}) making an angle ϕ with the normal (R''):

We now know the intensity and action line of W: the direction of action lines of R' and R^{IV}. With them we construct the force polygon. $WR'R^{IV}$ (HIJ, Fig. 1734A). Also, with R^{IV} determined (HJ) and knowing the direction of R'' and R''', we draw R'' normal

to DE and R''' parallel to DE thus determining the intensities of R'' and R'''.

$$R' = W \tan IHJ = \tfrac{1}{2}\, wh^2 \tan \alpha \tan IHJ \qquad (1734C)$$
$$= \tfrac{1}{2}\, wh^2 \tan \alpha \tan \alpha' \qquad (1734D)$$

As shown in calculus, this expression has its maximum value when $\alpha = \alpha'$. Hence the maximum value of

$$R' = \text{Pressure on retaining wall} = \tfrac{1}{2}\, wh^2 \tan^2 \alpha \qquad (1734E)$$

Since $\alpha = \alpha' = \frac{1}{2}(90° - \phi)$, and $90° = \frac{\pi}{2}$, this may be also written

$$P = \tfrac{1}{2}\, wh^2 \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad (1734F)$$

which is the form in which it was first deduced by Coulomb..

1735. **Rankine's Formula.**—Rankine accepted the first three hypotheses above, and assumed also that the pressure on the back of the wall is distributed as in water pressure and that the earth is in layers parallel to its upper surface. The latter assumption makes the resultant pressure parallel to that surface.

He deduced the following formula:

$$P = \frac{wh^2}{2} \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \qquad (1735A)$$

in which ϕ is the angle of repose and β is the angle which the surface of the surcharge makes with the horizontal and lies between zero and ϕ.

If the surface of the ground is horizontal,

$$P = \frac{wh^2}{2}\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \qquad (1735B)$$

which is equivalent to Coulomb's formula.

Design of Retaining Wall.

1736. Having determined the earth pressure upon a retaining wall, the form of cross-section is designed upon the same principles as that of a dam. The horizontal width of the profile at every point should be sufficient to prevent the wall from shearing off in any horizontal plane or sliding on its foundation. The resultant of the weight and the earth pressure should pierce each horizontal section far enough within the outer edge to make the wall safe against overturning. The curve of pressure should always be within the middle third of the wall but this condition is not so vitally necessary as in dams. The maximum unit pressure at the toe and heel should not exceed the bearing value of founda-

tion, nor should the maximum unit pressure on any joint exceed the allowable compressive stress of masonry. As a retaining wall is usually constructed to support an embankment of considerable height, it is necessary in designing the profile to consider only the pressure of the earth when the embankment is in its final condition. Comparing the different profiles of retaining walls, made of materials of equal weights, the most stable wall to resist overturning is one in which the center of gravity is at the greatest distance from the toe.

1737. **Empirical Formulas.**—Although the theoretical formulas will give safe values for the thickness of retaining walls, their forms are very complex for surcharged walls and walls with inclined backs. For these reasons, and because they are based on hypotheses only approximately true, empirical formulas have largely replaced them. Merriman uses the following for walls with trapezoidal cross-section, vertical face:

For ordinary walls retaining level banks, $b = 0.45\,h$;
For railway walls, $b = 0.66\,h$;
For surcharged walls, $b = 0.70\,h$;

in which

b = breadth of wall
h = height of wall.

A width of the top of the wall must be assumed, usually about 2′.

1738. The Corps of Engineers uses the following as a standard:

The thickness of a retaining wall at any level should be at least $\frac{1}{2}$ the remaining height, and the average thickness not less than $\frac{1}{3}$ the total height.

In retaining walls, as in dams, the profile is modified when necessary, so as to make the base of such width that the resultant unit pressure is not greater than the pressure allowable for the material pressed.

1739. **Drainage.**—If the earth in rear of a retaining wall becomes saturated with water, it will greatly increase the pressure upon the back of the wall. To avoid such increased pressure a vertical layer of broken stone is placed against the back of the wall, and drainage-tubes, or weepers, to allow the escape of the water, are run through or under the wall. These drainage tubes must be regularly inspected to see that they are not stopped up. Stoppage of these tubes has often resulted in the failure of retaining walls. There are no definite data available as to increase of pressure because of stoppages of drains; but the designer often arbitrarily decreases the angle of repose so that there will be allowance for carelessness in allowing stoppage of drains.

1740. **Counterforts and Buttresses.**—A counterfort is a projection upon the back of the wall designed to strengthen the wall to resist the pressure of the earth; a buttress is a similar projection upon the face of the wall.

Counterforts of simple masonry are of doubtful efficiency in strengthening a wall either against shearing or overturning because of their liability to separate from the wall itself: reinforced concrete connections to the wall remove this objection. A better wall is usually secured by placing all the masonry in the wall itself. Counterforts are sometimes employed in long walls to divide them into panels, and have been largely employed in the masonry walls of fortifications to limit the field of destruction of an exploding shell or mine and to form the side walls of casemates.

Buttresses are more efficient than counterforts in strengthening a wall against rupture by shearing or overturning. They are not often employed for this purpose except for architectural effect, as it is usually desirable to have the face of the wall a plane surface.

1741. **Reinforced Concrete Retaining Walls.**—Figs. 1741A and 1741B show cross-sections of this type of wall. The walls consist of the wall

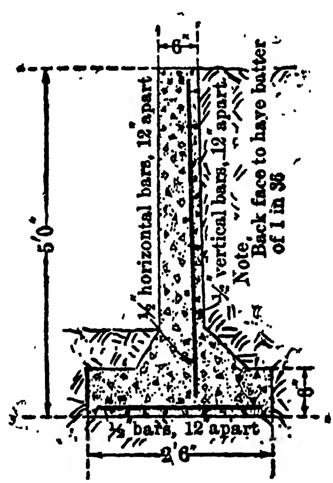


Fig. 1741 A **Reinforced-concrete Retaining-wall, 5 ft High**

proper, and the footing. The wall proper acts as a cantilever beam, the steel taking tensile stress. The footing takes the downward pressure of the fill and the upward pressure of the foundation bed; the steel in this case also takes tension and allows the arms of the footing to act as cantilevers. Counterforts of reinforced concrete can be effectively used with

reinforced concrete retaining walls. On account of the beam action of the wall, and the footing, reinforced concrete walls are usually much lighter in weight than masonry walls.

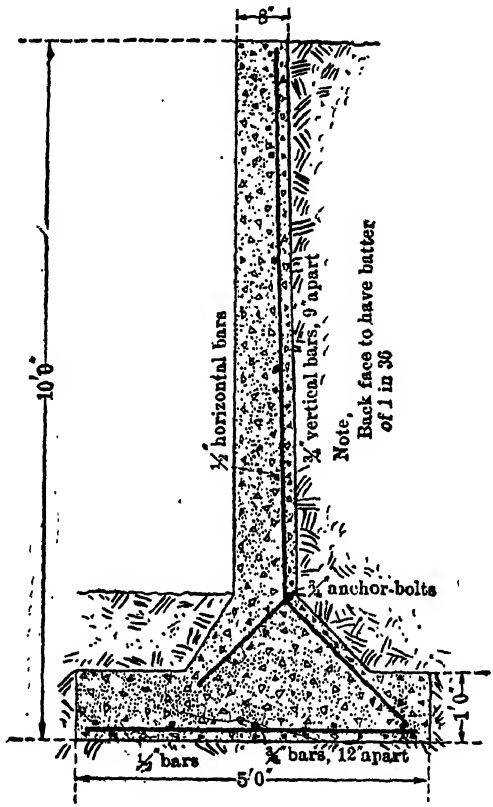


Fig. 1741 B Reinforced-concrete Retaining-wall, 10 ft High

1742. **Problem.**—Determine the total pressure per foot of length on the back of a retaining wall 24′ high, having a surcharge of 30 degrees. The back is vertical; the face has a slope of 10 vertical to 1 horizontal. Assume angle of repose 35°, weight of earth 100 pounds per cubic foot. Use Rankine's formula.

Solution.—
$$P = \frac{wh^2}{2}\cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$
$$\cos\phi = 0.819$$
$$\cos\beta = 0.866$$
$$\sqrt{\cos^2\beta - \cos^2\phi} = 0.281$$
$h = 24'$, in a vertical plane through heel of wall.

Substituting these values in the formula, we find

$P = 12{,}720$ pounds pressure on vertical plane through heel of wall.

1743. **Problem.**—By empirical formula for surcharged rectangular retaining wall, find thickness of a wall 10′ high, 3′ surcharge, $\phi = 34°$, masonry weight 150 pounds per cu. ft., earth weight 100 pounds per cu. ft. The coefficient of friction is 0.63. The foundation supports 6000 pounds per sq. ft.

Solution.— $b = 0.70\,h = 7'$.

This problem shows the inaccuracy of empirical formulas. The answer will be the same regardless of the height of surcharge, angle of repose, weight of masonry, weight of earth, coefficient of friction, and character of foundation. See Problem P. 1736.

For more detailed information, see:

Theory of Structures, Spofford
Concrete Engineers Handbook, Hool and Johnson
Waterworks Handbook, Flynn, Weston, Bogert
Cyclopedia of Civil Engineering
Civil Engineering, Fiebeger
American Civil Engineer's Pocketbook, Merriman.
Architects' and Builders' Pocket Book, Kidder.

PROBLEMS.

P. 1701. A masonry dam, with a vertical back, retains water to the depth of 30′. Find the total water pressure on one linear foot of the back of the dam and the distance of the point of application of the resultant water pressure from the heel of the dam.

P. 1702. A masonry dam of rectangular cross-section, 10′ high and 10′ thick, supports water level with its top. The masonry of the dam weighs 160 pounds per cubic foot.

(*a*) Determine analytically the center of pressure on the base.

(*b*) Assume that the bond between the base of the dam and its foundation is so defective that the water has access to the entire surface of the base with the full effect of its head. Determine analytically the center of pressure on the base.

P. 1703. A masonry dam, of rectangular cross-section, is 60′ high and 20′ thick. Assume that the masonry weighs 140 pounds per cu. ft. and that the coefficient of friction of the dam on the foundation is $\frac{2}{3}$.

(*a*) How high may the water rise on one side with a factor of safety of 4 against sliding on its foundation?

(*b*) How high may the water rise on one side under the condition that every horizontal section must be entirely under compression?

(*c*) Under the conditions of (*a*) and (*b*), how high may the water rise on one side?

(*d*) With the water at the height of 30′, what is the maximum unit pressure on the foundation?

P. 1704. A masonry dam, of rectangular cross-section, 100′ high and 70′ wide, retains water on one side level with the crest. Find the maximum pressure per sq. ft. on the base. Weight of masonry = 160 pounds per cu. ft.

P. 1705. A masonry dam, of rectangular cross-section, is 50′ high and 30′ wide and the water is level with the crest on the back of the dam. Find the factor of safety against sliding when the coefficient of friction against sliding is $\frac{2}{3}$. Weight of masonry = 160 pounds per cu. ft.

P. 1706. A masonry dam, of rectangular cross-section, is 30′ high. It supports water at a level of 2′ below its top. How thick should it be if the maximum pressure is not to exceed twice the mean pressure? Weight of masonry = 140 pounds per cu. ft.

P. 1707. In problem P. 1706, if the base is 18′ wide how high will the water stand when the maximum pressure on the base is 50 pounds per sq. in.?

P. 1708. In problem P. 1706, what should be the width of the base in order that the maximum pressure per sq. in. on the base shall be 50 pounds?

P. 1709. A dam, of trapezoidal cross-section, is 120′ high, 12′ thick at the top and has a vertical back. Water stands level with the top of the dam. What must be the thickness at the base if the resultant pressure pierces it at the extremity of the middle third? Weight of masonry = 160 pounds per cu. ft.

P. 1710. A dam, of trapezoidal cross-section, is 100′ high, 10′ wide at the top and has a vertical back. If the maximum height of the water is 96′, what must be the minimum width of the base in order that the center of pressure may not fall outside the middle third? Weight of masonry = 150 pounds per cu. ft.

P. 1711. A masonry dam, of trapezoidal cross-section, is 100′ high and has a vertical back. The width on top is 8′ and the water is level with the crest on the back of the dam. What should be the width of the base of the dam to satisfy the condition that the center of pressure shall be within the middle third? Weight of masonry = 160 pounds per cu. ft.

P. 1712. A masonry dam, of trapezoidal cross-section, is 100′ high, 8′ wide on top, 70′ wide on base and has a vertical back. The water stands level with the crest on the back and is zero depth on the face. The masonry weighs 160 pounds per cu. ft. The coefficient of friction on the base is $\frac{2}{3}$.

(*a*) Find the factor of safety against sliding on the base.

(*b*) Find the maximum and minimum pressures per sq. ft. on the base.

P. 1713. A masonry dam has its back inclined 10 vertical on 1 horizontal; the water is 100′ deep behind the dam. Find the total normal pressure on a linear foot of the back of the dam and the point of application of the resultant water pressure.

P. 1714. A masonry dam, of trapezoidal cross-section, is 60′ high with an inclined back and vertical face. The width on top is 12′. The maximum height of water on the back is 50′ and on the face is zero.

(*a*) Find the minimum width of the base in order that the center of pressure shall be in the middle third.

(*b*) Find the factor of safety against sliding on the base, the coefficient of friction being $\frac{3}{4}$.

(*c*) Find the maximum and minimum pressures per square foot on the base. Weight of masonry = 150 pounds per cu. ft.

P. 1715. A wall 30′ high is of triangular section *ABC*, the face *AB* being vertical, and water being retained on the side *AC* level with the top of the wall. The masonry weighs 125 pounds per cu. ft. Find the thickness of the base *BC*, under the condition that the center of pressure may safely be as far as $\frac{3}{16}$ the width of the base from the middle point.

P. 1716. A masonry dam, of rectangular cross-section, is 65′ high and 40′ thick. The surface of the water on its back is 5′ from the top of the dam, and the surface of the water on its face is 35′ from the top of the dam. How far from the toe of the dam does the resultant pressure pierce the base? Weight of masonry = 144 pounds per cu. ft.

P. 1717. A masonry dam, of rectangular cross-section, is 100′ high. The water on the back is 90′ deep and on the face the depth varies between 6′ and 15′. (*a*) What must be the thickness of the base if the resultant pressure pierces it at the extremity of

the middle third? The weight of masonry = 2.3 times the weight of water. (*b*) What must be the thickness of the base, if upward water pressure varying from full head on the back to full head on the face is considered?

P. 1718. A masonry dam, of rectangular cross-section, is 80′ high. The water on the back is level with the crest and on the face varies between the depths of 4′ and 10′. The factor of safety against sliding on the base must be at least 2 and the resultant pressure must pass through the middle third of the base. Find the thickness of the dam. Weight of masonry = 160 pounds per cu. ft.; coefficient of friction = $\frac{3}{4}$.

P. 1719. A masonry wall, of rectangular cross-section, sustains on one side a water level 60′ above its foundation and on the other side a level varying between 8′ and 20′ above its foundation. Its height is 66′. What breadth should be given it in order to insure that the maximum pressure should never exceed double the mean pressure? Water weighs 62.5 pounds and masonry 140 pounds per cubic ft.

P. 1720. The head of the water on the down-stream side of a set of lock gates at Gatun is 42′ 6″. On the up-stream side the head is 42′ 9″. How much additional resistance to opening would this difference of level produce on one leaf of the set? The gate is 65′ long.

P. 1721. Test the following dam for safety: A concrete dam, 140 pounds per cu. ft., resting on solid rock; height 30′, width 20′, rectangular cross-section. Upper pool stands level with top of dam, lower pool stands 20′ below top of dam; upward pressure according to Ohio River Board rules.

P. 1722. A concrete wall 30′ high is built so that its vertical back is 3′ from a natural rock wall. Test this concrete wall for overturning in case water should collect between the concrete wall and the natural rock wall to the full height of the wall, assuming the wall to be 10′ thick and weighs 140 pounds per cu. ft.

P. 1723. In the dam described in P. 1722, assume the full upward water pressure according to the Ohio River Board rules, and test for sliding.

P. 1724. A conduit 6′ wide and 20′ deep on the inside, is separated from a stream by a masonry wall; water level outside the conduit is 2′ from top of wall; full upward pressure under wall.
Assume masonry to weigh 150 pounds per cu. ft. Find thickness of wall. (Conduit to be tested full and empty.)

P. 1725. In the conduit described in P. 1724, the wall on the side away from the stream must support an earth bank level with the top of the wall, the earth having an angle of repose of 30°. Omit consideration of ground water and upward water pressure. Find proper thickness of wall. (Conduit to be tested full and empty.)

P. 1726. Determine, by Coulomb's formula, the thickness of a rectangular retaining wall 20′ high, to safely resist the pressure of an embankment of equal height. Specific gravity of earth, 1.5; of masonry, 2.3. Angle of repose of earth, 34°. Make the moment of the weight of the wall twice that of the earth pressure.

P. 1727. Solve problem P. 1726 by Rankine's formula.

P. 1728. A masonry retaining wall, with vertical face, is of trapezoidal cross-section and 16′ high. The earth is level with the top of the wall. Find the width of the base of the wall and the width on top, assuming the batter of the face to be 5 vertical to 1 horizontal.

P. 1729. A first-class masonry retaining wall, of rectangular cross-section, is 20′ high and the earth is level with the top. Find the thickness of the base applying Coulomb's formula. Weight of masonry = 150 pounds per cu. ft.; of earth, 100 pounds per cu. ft. Angle of repose = 34°. Center of pressure on base to be within middle third.

P. 1730. Solve problem P. 1729 by Rankine's formula, assuming a surcharge with an angle of repose of 34°.

P. 1731. A masonry retaining wall, of rectangular cross-section, is 20′ high and retains earth level with its top. Under the hypothesis that the center of pressure shall fall within the middle third of the base, find the necessary thickness of the retaining wall by the use of Rankine's formula. Weight of masonry = 150 pounds per cu. ft.; of earth, 100 pounds per cu. ft. Angle of repose of earth = 34°.

P. 1732. Solve problem P. 1731 with a surcharge 4′ high at a distance of 7′ back of the top of the wall, sloping to the top of the wall.

P. 1733. Find the pressure on a surcharged wall, angle of surcharge 28°, angle of repose of earth 33°, weight of earth 90 pounds per cu. ft., height of wall 20′, weight of masonry 140 pounds per cu. ft.

P. 1734. Solve problem P. 1731, under the assumption that the drains are stopped and water has risen to the top of the wall, thus making the earth pressure correspond to that of a fluid.

P. 1735. Solve the problem in paragraph 1743, using the formula in paragraph 1738.

P. 1736. Solve problem in paragraph 1743, using Rankine's formula.

CHAPTER XVIII.

WATER SUPPLY.

1801. A complete system of waterworks, designed to supply a town or city with water for domestic use, fire protection, and manufacturing purposes, consists of a collecting, a purifying, and a distributing system.

The **collecting system** comprises the intakes where the water is taken from the source of supply, the receiving reservoirs in which it is stored, the conduit through which it flows, and the pumping machinery which may be necessary to raise it from one level to another until it reaches the purifying system.

The **purifying system** comprises the settling basins in which it is clarified, and the filters in which it is rendered pure and fit to use. In some instances, where unusually clear water is available, there is no separate purifying system, the water being purified by the introduction of liquid chlorine into the distributing reservoir or watermains.

The **distributing system** comprises the distributing reservoirs in which the filtered water is received and the network of pipes by means of which it is conveyed to the points of consumption.

As will be shown later, not all these elements are found in every system of water supply.

1802. A **simple gravity system** of water supply is one in which the intake is the highest point in the system, and no other force than that of gravity is required to move the water to and through the distributing system. The elevation of the intake should preferably be such as to produce a pressure in all mains of the distributing system of about 50 pounds per square inch. This pressure will give about 20 pounds per square inch in an ordinary building of six stories and is not so great as to cause excessive leakage in the fixtures. In small towns, higher pressures are sometimes used to give fire protection without the aid of fire-engines; in such cases the pressure should be about 100 pounds per square inch in the mains.

Pumping machinery is introduced whenever the intake is not high enough to produce the required pressure in the pipes, or when it is necessary to raise the water over elevations where siphoning is not possible or economical. The pumps may be located at the intake, or at a reservoir supplied by gravity from the intake. A **pumping system** of water

supply is **direct** when the pumps act direct into the distributing system, and **indirect** when they pump into reservoirs from which the water is distributed by gravity.

The gravity system is the most economical to operate and the least likely to be interrupted by accidents. The direct pumping system is the most expensive to operate and the most likely to be interrupted by breakages. The indirect pumping system is more economical than the direct, especially for small plants, since the reservoir may be filled by day and the pumps stopped at night; it is also less likely to be interrupted, since the distributing system can draw on the reservoir whenever the pumps need repair.

1803. **Quality of Water.**—The requisite qualities of water to be used as a public supply can be stated as follows:

a. It should be free of disease-producing organisms and irritating or poisonous substances.
b. It should be clear, colorless, odorless and tasteless.
c. It should not be too hard for domestic or industrial use.
d. It should be free from iron and corrosive substances.
e. It should have a low and equable temperature.

These requisites differ in their relative importance; and since it is difficult to find all in any one source of supply, treatment of the water is generally necessary. First, and before anything else, a water supply must be safe from a sanitary standpoint. It must not, as a source of disease germs, be a menace to the health of the consumers.

Collecting System.

1804. **Source of Supply.**—The original source of water supply is the rain. Rainfall either flows as **surface water** until it reaches the sea, or sinks into the soil to form subterranean streams or reservoirs of **ground water,** which usually have their outlets in surface waters at a lower level. Much of the surface water is lost on its way to the sea by evaporation, or by sinking into the soil and becoming ground water. Some of the ground water is absorbed by vegetation.

The immediate sources of supply are the ground water of springs and wells, or the ground and surface waters mixed, of rivers, lakes, streams, and ponds.

Springs are the natural outlets of an inclined porous saturated stratum of soil which rests on or between impervious strata. Springs are found on hillsides where a porous stratum is exposed, and at points where the upper impervious layer is broken by a geological fault and the head is sufficient to force the water to the surface.

Where the ground water does not reach the surface naturally, **wells** may be sunk. A **shallow well** is a hole made in a porous surface layer to collect the water for removal by pumps or buckets. It is usually either a large masonry shaft or a small metal tube sunk below the surface of the ground water. To allow the water to enter freely, the bottom of the masonry shaft is open; the metal tube, which is 2″ to 12″ in diameter, terminates in a perforated section or strainer which allows the water to collect in the tube. The tube is usually sunk by fitting its base with a conical cap and then driving it like an ordinary pile; the well is therefore called a **driven well.**

Where the water bearing stratum is beneath an impervious layer, and at a considerable distance below the surface of the ground, the well must be driven through the impervious layer, and into the water bearing layer. When the head of water in the water bearing stratum is sufficient to force the water through the well to and above the surface of the ground, the well is called an **artesian** or flowing well; when the head is not sufficient, and pumps must be installed to bring the water to the surface, the well is called simply a **deep well.**

An **infiltration gallery** is a long underground chamber supported by timber or masonry, constructed across the line of flow of a water bearing stratum, or beneath a stream bed, in order to intercept a large amount of water.

1805. **Amount of Water Required.**—This quantity should be very carefully estimated as the first step in designing any water supply system. It is based on two primary considerations: the average daily per capita consumption of water, and the probable future population to be supplied. The great expense and time involved in water supply projects make it most economical to build for the future. Estimates of this kind can be based only on a study of other cities similarly situated, both as to growth over a period of years, say 30, and the kind of inhabitants, and also on waterworks data kept by these cities.

Where wastage of water is prevented as much as practicable, as in England, the daily per capita consumption varies from about 20 to 60 gallons, the average for English cities being 33. Where meters have been introduced, but no other restrictions enforced, as in most cities of the United States, 40 to 120 gallons per capita per day may be expected, the American average in such cases being 75. Where no control whatever is exercised, the entire capacity of the plant is used, no matter what the population, and the daily per capita consumption may exceed 300 gallons. For average American cities, with metered supply, 100 gallons per capita per day should be sufficient.

If 100 gallons per day per capita is taken as a basis it may be divided into its various uses as follows:

1. About 40 gallons will be used in households.
2. About 30 gallons will be used in manufacturing and commercial establishments.
3. About 10 gallons will be used by the municipality in public buildings, street sprinklings, etc.

This leaves about 20 gallons which must be accounted for as loss or waste due to leakage in mains and fixtures.

The daily, weekly, and monthly consumption of water varies considerably. In warm weather much water is consumed in watering gardens and lawns; in cold weather much is allowed to run to waste to prevent the freezing of the pipes in the buildings. The maximum monthly consumption may exceed the mean monthly consumption for the year by 33 per cent; the maximum weekly consumption may exceed the mean weekly consumption for the year by 50 per cent; the maximum daily consumption may exceed the mean daily consumption for the year by 100 per cent. The hourly consumption is also variable; the consumption for a single hour is sometimes 40 per cent more than the average hourly consumption for the day. The variations from the mean consumption are very much greater in small systems than in large ones; and larger in unmetered systems than in metered ones. The volume required by a steam fire engine during a conflagration depends upon the character of the engine; it usually varies between 400 and 1200 gallons per minute; the amount thus drawn may be a very heavy tax on a small system. The amount to be provided for fire service may be expressed:

$$Q = \frac{1000}{\sqrt{P}} \tag{1805A}$$

in which, Q = gallons per capita per day
P = population in thousands.

1806. **Flow of Streams.**—Because of the irregularity of the rainfall, the discharge or run-off of any drainage basin or catchment area is a variable and not a constant quantity. To prevent water famine in periods of extreme drought, the system of supply must be based on the minimum discharge of the stream which drains the basin.

The most reliable method of determining the minimum discharge is to measure the daily discharge for a long series of years. The approximate methods which are ordinarily resorted to are either to estimate the discharge from the measured discharge of another basin subject to similar climatic conditions, or to estimate the discharge from the rainfall, employing the ratio of discharge to rainfall determined in some basin having similar climatic conditions.

The Sudbury Watershed near Boston has been carefully observed since 1875, and the data so obtained are generally used as a basis for esti-

mating water supplies from watersheds east of the Mississippi River. This watershed has an area of about 75 square miles, of which $6\frac{1}{2}$ per cent is water surface; the terrain is generally hilly. The evaporation per unit area on the lakes, ponds and streams is about equal to the rainfall per unit of area on the whole surface. The records show that —

a. The run-off from the basin is about one-half the measured rainfall; or approximately 1,000,000 gallons per day from each square mile of the basin.

b. In a year of extreme drought the rainfall is two-thirds of its mean, and the discharge or run-off one-half of its mean.

c. For several years in succession the rainfall and run-off may be less than the mean.

d. The mean monthly rainfall is fairly uniform in this basin, being a minimum of 2.98″ in June, and a maximum of 4.57″ in March. The mean annual rainfall is 45.83″.

e. The average monthly run-off, on the contrary, is variable. It is equivalent to a depth of 5.17″ over the basin in March, and only 0.35″ in July. The run-off during February, March and April is 50 per cent of the total annual run-off, and that from November 1st to May 31st is 85 per cent of the annual run-off.

f. The minimum rainfall and the minimum run-off in a month may be only 10 per cent of their mean values.

From observations on other watersheds it appears that the ratio of the run-off to the rainfall decreases as the amount of annual rainfall decreases. Hence in a basin in which the annual rainfall is less than in the Sudbury Basin the run-off will be less than one-half the measured rainfall.

The difference between the run-off and the rainfall is due principally to evaporation and absorption by vegetation; and sometimes to escape through subterranean channels.

The annual evaporation from water surfaces may be either somewhat greater or somewhat less than the rainfall, depending on local conditions; if the rainfall is very great, the evaporation is relatively small, and vice versa. At Astoria, on the Pacific coast, the rainfall is 77″ and the evaporation is 25″. At Cheyenne, Wyoming, the rainfall is 13″ and the evaporation 76″.

In calculating the run-off of a watershed where the rainfall and evaporation are equal to each other, the area of all ponds, lakes, streams, etc., must be subtracted from the total area.

The amount of water absorbed by the vegetation will vary from 10″ to 15″ if fed by the rainfall; some irrigated fields can absorb ten times as much.

1807. **Flow of Springs and Wells.**—The irregularity of the rainfall, especially its inequality over long periods of time, as seasons and years, affects the underground streams and reservoirs in the same way that it affects the run-off of watersheds. The underground storage will be a maximum when the greater part of the annual rainfall occurs in the months when the soil is porous and the loss by evaporation is least. A mild wet winter is favorable to the underground flow and storage of water, and a dry cold winter is unfavorable to it.

The flow of a spring or well depends on the amount of rainfall on the drainage area from which it receives its supply, on the extent of the underground area drained by the spring or well, and on the velocity of flow toward the outlet.

Any difference of level between two points of a continuous water surface, whether above or below the surface of the ground, will cause a flow of water from the higher to the lower point; the velocity of the flow will vary directly with the difference of level between the two points, and will vary inversely with the resistance to the flow.

The flow of a spring or well may be increased by increasing the depth of its water-level below the general surface of the ground-water. This will increase both the area drained and the velocity of flow towards the outlet.

1808. **Measurement of Source of Supply.**—If the source of supply is a spring or well the volume delivered can be determined by noting the time required to fill a reservoir of known capacity, or the time required to regain its level after a known volume has been removed. Small streams may be measured by a weir, while in large streams the mean velocity can be determined by taking the velocity in several places with a velocity meter. This mean velocity, properly computed, will, with the area of cross-section, give quite accurate values for the discharge.

1809. **Selection of Drainage Areas.**—The suitability of a drainage area will depend on the geographical and geological nature of the country. It should, if possible, have an elevation considerably greater than the area to be supplied. Limestone formations should be avoided due to the hardness they impart to the water. The area should be free from pollution, uncultivated land with steep slopes being advantageous.

1810. **Receiving Reservoirs** are artificial basins in which is stored the water collected in the collecting system. **Distributing Reservoirs** are similar basins for storage of water to be fed into the distributing mains. In either case the function of the reservoir is to equalize the water flow; in the receiving system an irregular supply is collected, and water removed at a uniform rate; in the distributing system, water is received at a uniform rate, and is fed out irregularly according to the demands of the consumers. Pumping systems, with pumps acting intermittently,

must have reservoirs to store water so as to supply the system when the pumps are not operating. Where a large river or lake, with capacity many times the maximum daily consumption of the system, is used as the source of water, this river or lake itself becomes the receiving reservoir.

If the source of supply is a drainage area along the Atlantic coast, the average daily supply which can be collected from a square mile of area is about 1,000,000 gallons. In a year of extreme drought, however, the average daily yield may be reduced one-half, or be only 500,000 gallons. It is evident that if the distributing system requires more than 500,000 gallons daily from each square mile of the basin, there will be a water famine during this year of drought, unless sufficient water has been stored in reservoirs to provide for this deficiency. It is also apparent that the nearer the daily consumption is to 1,000,000 gallons the greater must be the volume of water stored. If this year of extreme drought is preceded by one or more years in which the average daily yield is also less than the average daily consumption, an additional supply will be needed for these years.

As several years of drought may follow each other, the maximum daily consumption is usually limited to 60 or 75 per cent of the average daily capacity. Reservoirs having a capacity of 200 to 250 days' supply will then be sufficient to supply the deficiency of three successive years of drought.

Even during a year of extreme drought, it will still be necessary to have receiving reservoirs, if the maximum daily consumption is more than the minimum daily flow which has been recorded or may be expected in any summer or autumn month. From the Sudbury basin the average daily flow in July is 200,000 gallons per square mile, which is only about one-fifth of the average daily flow for the year. In a year of extreme drought this may be reduced to 20,000 gallons daily. If, therefore, the daily consumption exceeds 20,000 gallons per square mile of drainage area, some provision must be made for storage. To provide for an average daily consumption approaching 500,000 gallons, the total excess yield of the winter and spring months must be stored for use in the summer and autumn. The minimum storage required to equalize the flow for a single year is about 75 days' supply.

If pumping machinery is employed to raise the water from a constant supply to a receiving reservoir, the latter need hold only a few days' supply to secure a constant flow in the distributing pipe while the engines are shut down over night or for repairs.

When the streams emptying into a reservoir carry considerable silt, due allowance should be made in computing the size of the reservoir to allow for the loss of capacity due to the silting of the reservoir.

1811. **Dams.**—The dams for receiving reservoirs are generally made of earth or masonry, of the types described in Chapter XVII.

To prevent leakage along the discharge pipe which passes through the dam, this pipe is laid on a bed of concrete resting on the impervious stratum beneath the dam. The pipe itself is then imbedded in concrete, and this covering of concrete has projecting rings to increase the resistance to leakage along its surface. The pipe may also be laid in a small culvert which is constructed on the impervious layer, passes underneath the dam, and terminates in a vertical masonry tower, inside the reservoir, called a **valve tower** or **gate house.** A pipe thus laid is subject to constant inspection. (Fig 1811A.)

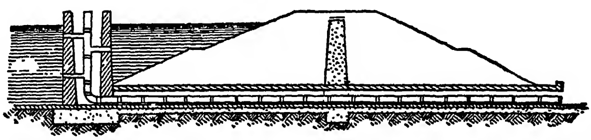


Fig. 1811 A Earthen Dam with core wall showing valve tower and outlet through dam

One of the most important features of an impounding reservoir having an earthen dam is the **spillway** through which the surplus water is discharged when the reservoir is full. If the outlet is too small, in time of flood the water will rise above the crest of the dam, and flowing over it will soon cause its total destruction. If placed in the dam itself and not properly constructed, the spillway itself may be washed out and thus destroy the dam.

The length of the crest of the spillway may be determined by the weir formula:

$$Q = 3.33\, b\, H^{\frac{3}{2}} \qquad (1811A)$$

where Q = discharge in cubic feet per second
b = length of spillway crest in feet
H = height of water in reservoir above crest of spillway

It should be such that it can discharge the maximum flood without causing the water in the reservoir to rise to a plane above the top of the core-wall or 3′ below the crest of the dam.

Various empirical formulas are in use for the maximum flood discharge; the most common is Fanning's formula,

$$Q = 200\, A^{\frac{5}{6}} \qquad (1811B)$$

in which Q = the discharge in cubic feet per second
A = the area of the basin in square miles

In this formula no factors are introduced which depend on the shape, slope, or character of the surface of the basin, although all these must affect the maximum discharge. It is thought to give too small a discharge for rapidly discharging basins in which the area is less than ten square miles.

The spillway should, if possible, be constructed in the solid ground beyond one end of the dam, and the water carried in a separate channel so far away from the dam that backwater will not come in contact with the earth embankment. Unless this channel passes over a ledge of rock it should be paved to prevent scouring. If the spillway must be made in the dam itself the part of the dam containing the spillway should be of masonry; the channel for the water should be so enclosed that the water cannot reach the earthen dam. Spillways, in unimportant dams, are made of cribwork filled with stones and covered by a plank sluiceway through which the water flows. The length of the weir of the spillway may also be determined less accurately by the Gould formula

$$l = 20\sqrt{A} \tag{1811C}$$

in which l = length in feet
A = area of basin in square miles

1812. **Earth and Rock-fill Dams** for reservoirs of earth or rock dumped in place, the faces of the dam taking the natural slope of the material, may be used without precaution against leakage, or they may be protected by concrete or reinforced concrete aprons, or by masonry core-walls, both of which should reach into rock or an impervious stratum. Rock-fill dams may take the form of timber cribs filled with rock; this permits a facing of plank on the upstream side. (See Chapter XVII.)

1813. **Masonry Dams** for reservoirs may be of the gravity type or of the hollow reinforced concrete type. (See Chapter XVII.) The entire wall or a portion thereof may be used as a spillway. If the water wastes

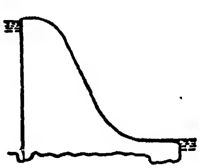


Fig. 1813 A Ogee-faced dam

over the dam itself, the dam is usually made of the form shown in Fig. 1813A; this is called an **ogee-faced dam.** The water follows the contour of the face and is discharged at the base parallel to the river bed. This reduces to a minimum the tendency to erode the bed.

1814. **Reservoir Outlets** are usually put in the natural soil outside the dam, as they tend to weaken the dam if constructed in the dam itself. The pipes are often placed in conduits or tunnels, to permit ready inspection. Reservoir outlets usually pass through masonry **gate houses** or **valve towers** placed in the dam or placed as separate intake towers inside the reservoir (Fig 1811A). The outlets are arranged to take water from different levels, each outlet being provided with a valve and a screen, the valve and sometimes the screen also being inside the gate house. A separate waste pipe at the bottom can be used to drain the reservoir. A **by-pass** is a pipe connecting the inlet and outlet of some reservoirs, permitting the water to be passed around the reservoir when it is emptied for any purpose.

1815. **Intake.**—If the water is taken from a stream, a dam is usually constructed at the intake so that the level of the water will be constant, and the inlet may be placed well below the level of the ice; the entrance of the conduit is covered by a screen to keep out materials which might obstruct the flow through it. If the pool at the intake is a large one so as to be only slightly affected by the suction of the conduit, there will be little danger of the obstruction of the conduit by the needles of ice called anchor-ice.

In the Great Lakes the intakes are located in strong cribwork shafts which are far enough from the shore to secure water unaffected by the city sewage.

Conduits.—The main conduits may connect the intake directly with the purifying or distributing system, or the impounding reservoirs with either of the above systems. Conduits are constructed of masonry, steel, cast iron, or wood. Large conduits not exposed to internal pressure are usually made of masonry; they are simply covered canals, and are constructed with a uniform grade of slight inclination. In cross-section they are usually either circular, elliptical, or horseshoe shaped; the latter being the most generally used type. The cut and cover conduits of the Catskill water supply of New York City have a cross-sectional area of 241 square feet. Large conduits which are exposed to considerable internal pressure are made in the form of riveted steel pipes; conduits from 4′ to 9′ in diameter have been so constructed. Cast-iron pipes are more commonly employed than any other form, and are manufactured in sizes varying from 4″ to 5′ in diameter. Conduits made of wooden staves with steel bands are often constructed where suitable lumber is cheap and the pressure in the conduit is not excessive; conduits 6′ in diameter have been made of wood.

Pumping Machinery.—Pumps are of a great variety of forms, depending upon the volume of water to be raised in a unit of time, the head or back pressure against which they must work and the depth of the water

level below the surface of the ground. The back pressure is constant if the water is pumped into a reservoir, and variable if pumped directly into the distributing pipes. One or more reserve pumps are usually installed to provide for emergencies; these are absolutely essential in a direct pumping system.

1816. **Problem.**—Determine thickness of plate required for the bottom course of a steel standpipe 60′ high and 18′ in diameter; allowable unit tension in steel, 12,000 pounds per square inch; efficiency of joints, $66\frac{2}{3}$ per cent.

Solution.—Using the formula for pressure in pipes (527B)

$$t = \frac{pD}{2\,a_t}$$

in which t = thickness of steel in inches = unknown
D = diameter of standpipe in inches = 18×12
a_t = allowable unit tension in steel = 12,000 pounds per square inch
p = pressure of water in pounds per square inch = $\frac{62.5}{144} \times 60$

Substituting and multiplying allowable unit tension by given efficiency, we have

$$t = \frac{62.5 \times 60 \times 18 \times 12}{144 \times 2 \times 12{,}000 \times .667} = .35'' \text{ or } \tfrac{3}{8}''$$

1817. **Problem.**—A drainage basin of 150 square miles feeds a reservoir. Determine (*a*) approximate discharge of basin and (*b*) length of spillway of dam.

Solution.—Using the formulas in paragraph 1811, and substituting proper values, we have

$$\begin{aligned}
(a)\ Q &= 200\,A^{\frac{5}{6}} = 200 \times 150^{\frac{5}{6}} \\
\log 200 &= 2.30103 \\
\tfrac{5}{6}\log 150 &= 1.81341 \\
\log Q &= 4.11444 \\
Q &= 13{,}015 \text{ cubic feet per second} \\
(b)\ l &= 20\sqrt{A} = 20\sqrt{150} \\
\log 20 &= 1.30103 \\
\tfrac{1}{2}\log 150 &= 1.08805 \\
\log l &= 2.38908 \\
l &= 245'
\end{aligned}$$

Purifying System.

1818. **Impurities.**—A purifying system has for its object the removal from the water of:

a. Matter in suspension which colors the water or makes it turbid
b. Mineral matter in solution which is undesirable
c. Pathogenic bacteria (disease organisms)
d. Organic matter which is food for these bacteria

The mud and organic coloring matter are removed by allowing the water to settle in settling basins for a long time, or by the use of coagulants for a short time. The mineral matter in solution is undesirable principally

on account of giving hardness to the water. This is due to the presence of the carbonates or sulphates of calcium and magnesium. Hardness due to carbonates is termed temporary since it may be more easily removed than that due to the sulphates, which is termed permanent. Hard water will not dissolve soap easily and forms boiler scale when used in steam boilers.

All water contains bacteria and other organic substances. The larger number of the bacteria present in any supply are harmless or beneficial but some forms mostly derived from sewage are harmful. The pathogenic bacteria usually found in water are those producing intestinal diseases such as typhoid fever, cholera, dysentery, and diarrhœa.

The organic substances which furnish food for pathogenic bacteria and the bacteria themselves are to a great extent removed in a settling reservoir if the water is allowed to remain in it long enough, perhaps for a month or more; as a rule, however, to remove bacteria the process of sedimentation is followed by that of filtration. As it is impossible at present to destroy the pathogenic bacteria without at the same time destroying the harmless varieties, the purity of the water is measured by the total number of bacteria in it. If the number of bacteria in a cubic centimeter does not exceed one hundred, the water, while not absolutely pure, is considered healthful within reasonable limits. This result is obtained in ordinary river water only by the destruction of over 99 per cent of the bacteria in it.

1819. **Selection of Sources of Supply.**—As it is impossible to remove all bacteria from water, a source of supply should be selected which is not liable to contain pathogenic bacteria. Sources of supply are classified as follows:

a. Springs, deep wells, and the surface drainage of uncultivated and unpopulated lands, as wholesome.
b. Surface drainage of cultivated and sparsely populated land, and rain-water drained from roofs, as suspicious.
c. Rivers into which sewers empty, shallow wells, and densely populated drainage basins, as dangerous.

Ground waters, except those from rivers, are usually wholesome as far as bacteria are concerned, but are liable to contain inorganic substances which may be objectionable or even dangerous. Ground waters are also more liable than surface waters to nourish vegetable growths which give the water an offensive odor or an objectionable taste. These growths may be prevented by keeping the water under cover from the time it reaches the surface of the ground until it passes into the distributing mains.

A source of supply having been selected, proper police supervision should be maintained over the drainage area to prevent its pollution.

1820. **Settling Basins.**—The term settling basin is ordinarily applied to a reservoir which contains from one to four days' supply of water; in such a reservoir the water derived from surface drainage is cleared so that it will not clog the filter beds. The bed of the basin is of concrete, so that it can be thoroughly cleaned when necessary, and in restricted areas the basin is usually walled and roofed with concrete. The depth of the water in the basin is usually from 8′ to 15′. A lesser depth does not prevent the growth of vegetation, and a greater depth makes the time required for sedimentation so great as to require a very large settling basin. However sedimentation covering such a short period has no important effect in reducing the number of bacteria.

The flow of the water through the basins may be either continuous or intermittent. In a continuous system, if there is but a single small basin, the water enters near the bottom at one end, moves across the basin with a very slight velocity, and leaves the basin near the top at the opposite end. The reservoir can be cleaned only by conducting the water through a by-pass or pipe which connects the conduit above with the conduit below. The water may be compelled to pursue a serpentine course through the reservoir by inserting vertical plank partitions called baffles. If there is a large receiving reservoir in the system, it will perform all the functions of a settling basin better than would be done by a special basin, except with waters which require the use of a coagulant, as the water remains in it a longer time and its depth is greater than that of a basin.

In an intermittent system two or more settling basins are used, one being filled while another is being emptied, and the others, if any, are settling.

When the material in suspension in the water is in very small particles, such as finely divided clay, it is necessary, especially in rapid filtration, to aid these materials in settling out by the use of a coagulant. The most commonly used coagulant is sulphate of aluminum. Others employed are potash, alum, and hypochlorite of lime. The chemical to be used is prepared as a solution which is introduced in known quantity, either into the water before it reaches the settling basin, or while moving through the latter. The reaction of the chemical in the water produces gelatinous precipitates, which in settling carry down the suspended inorganic substances, thus greatly reducing the work of the filters.

The turbidity of the water is tested by noting the distance below the surface at which some standard object can be seen. This object is usually a fine platinum wire fastened at right angles to the axis of a rod graduated to millimeters. Another method is to compare the

water with standard samples containing a known amount of suspended matter.

1821. **Storage** in a reservoir gives opportunity for settling and clarifying the water and reducing the number of bacteria and other organisms by natural death and by the action of sunlight. However organisms which cause objectionable odors may grow in stored water, and organic matter in the water may decompose. **Stagnation** of water takes place in the lower portions of deep reservoirs due to this cause. While water is fed into and drawn from a reservoir frequently, all of the water in a reservoir is not changed except at the periods of **overturning,** once in the spring and once in the fall, at which times a thorough mixing takes place due to the change in temperature of the air. Stagnation has the effect of exhausting the oxygen from the water, replacing it with other gases, and of encouraging organic growths. In the period of overturning some of these gases tend to coagulate suspended matter in the water. Stagnant water, however, cannot without special treatment be drawn for distribution in the water supply system.

1822. **Filters.**—The object of filtration is to complete the purification of the water begun in the settling basin or the receiving reservoir. Filters are often divided into two classes: one is the slow sand filter and the other is the rapid, or mechanical, filter.

Slow Filters.—The ordinary slow filter is a basin into which the water flows through an inlet above the filter-bed, and from which it escapes through an outlet below the filter-bed. The basin may be open or covered. In a cold climate it is usually considered advisable, though not absolutely necessary, to cover the basin to prevent the water freezing, as the filter does not operate satisfactorily with very cold water. In warm climates covered filters prevent the growth of vegetation in the water, but open filters have the advantage of the bleaching and bacteria-destroying effect of the sun, although, as sunlight does not penetrate water readily, this effect is not very great.

Fig 1822A shows a typical sand filter, the walls and bed being of concrete. The filter bed consists of $3\frac{1}{2}'$ of fine sand; around the outlet, or collecting pipe, is a bed of gravel, grading into the sand, so that none of the sand will be washed away with the filtered water. The filter is in two or more divisions, so that any one can be cut off for cleaning. In each division, below the bottom of the filter beds, is a large collecting drain, into which empty the collecting pipes from each section of the filter; these are all tile pipes, laid with open joints. The water so collected is run into a clear water basin. Fig 1822B shows the layout of the drains: the heavy line represents the main drain; the lighter lines, the small collecting pipes.

The filter bed must be protected from being disturbed by currents of

water; the inlet is in a separate chamber, the water coming at very low velocity over the partition wall between this chamber and the filter chamber. In starting the filter, water is first admitted through the outlet pipe, until the sand is thoroughly flooded; then water is admitted through the inlet.

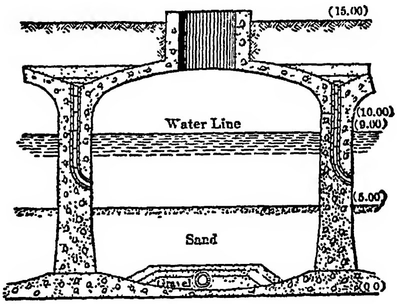


Fig. 1822 A Slow sand filter

The operation of the filter causes the formation of a gelatinous coating, formed from the matter strained from the water; this is an important part of the filter — in fact the filter will not operate properly until the coat has been formed. The filter operates both physically and chemically: physically as a screen or strainer, taking out the matter in the water; chemically at the gelatinous coat, where the bacteria cause

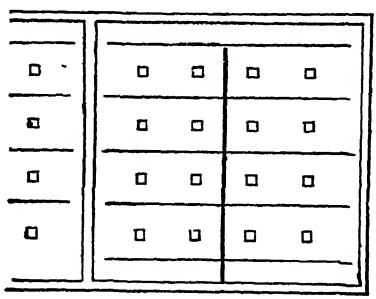


Fig. 1822 B Drains for filter

organic matter to break up into its elements, and with the disappearance of this matter, the bacteria also disappear. These bacteria are dependent on oxygen, hence if the water and air do not supply enough, the filter must be operated intermittently. The gelatinous coat must not be ruptured by a too rapid flow of water. The water passes through at the rate of about 4″ per hour, but as the gelatinous coat gradually builds up,

either the rate slows down, or the head must be increased. In due time, it becomes necessary to empty the filter and scrape off the gelatinous coat. Slow sand filters handle from 3 to 6 million gallons per acre per day.

1823. *Mechanical Filters.*—The system described above is very slow, and requires a large area of filter beds. To remedy these defects, mechanical filters (Fig 1823A) have been devised. Mechanical filters differ from slow filters in the use of a coagulant to remove inorganic matter, and the aëration or washing of the sand by mechanical means.

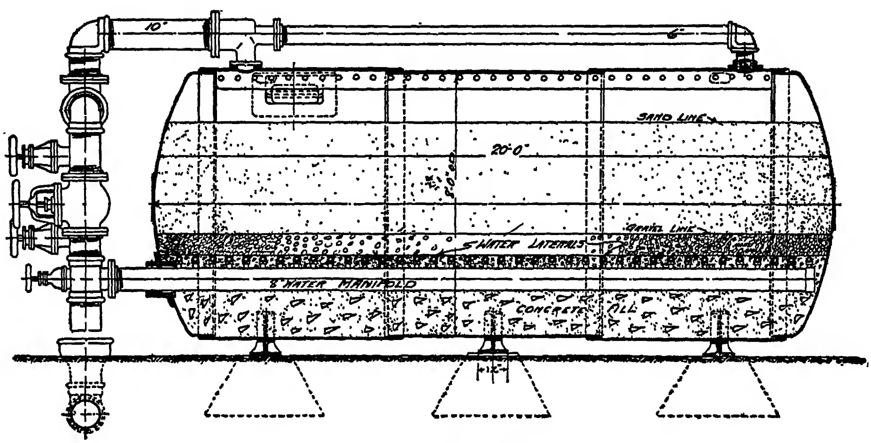


Fig. 1823 A. Section of a Mechanical Filter

The coagulant causes the formation of the gelatinous coat, and the filter operates in the same way as the slow filter. When the rate of flow becomes too slow, the sand is washed by reversing the current, which carries off some of the gelatinous coating. The mechanical filter is thus intermittent. Its rate of flow is high, and it will handle about 125 million gallons per acre per day. The usual form of filter is a wooden or steel vat comparatively small in size. Hence where it is necessary to filter a large quantity of water, several filters, usually called a battery, must be used. The coagulant is introduced before the water enters the filter bed. Alum is the usual coagulant; lime is sometimes added to precipitate any free sulphuric acid.

Iron may be removed from the water by first oxidizing it by aëration and then passing it through a sand filter. This process is employed to purify water from wells. If the water does not contain organic matter, the process may be a rapid one.

Temporary hardness in water is due to the presence of calcium carbo-

nate in water which contains free carbonic acid. The carbonate is soluble only when the carbonic acid is also present, and may therefore be precipitated by any process that removes the acid. The carbonic acid may be removed from small quantities by boiling the water; from large quantities, by adding lime-water, with which the carbonic acid can unite and form additional carbonate and thus remove the free carbonic acid. The carbonate will then be deposited. Filters have been designed for expediting this process, which is usually a very slow one. A *simple filter* for field use is shown in Fig 1823B.

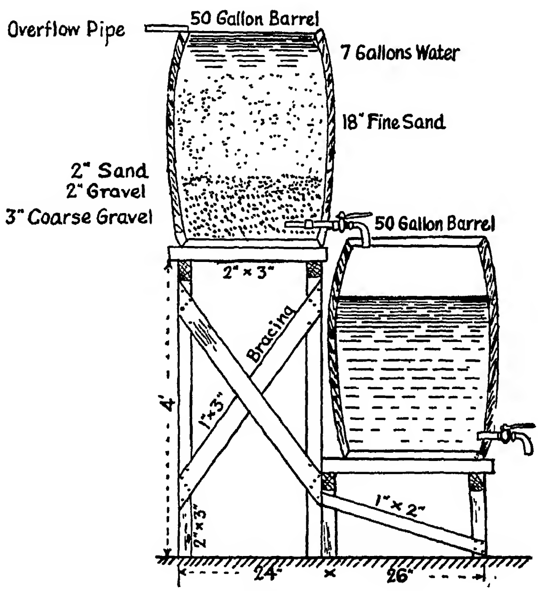


Fig. 1823 B
Intermittent Barrel Filter

Sterilization.—After passing through the filters, water from questionable sources is usually *sterilized* to remove those harmful bacteria which may have passed the filter. Liquid chlorine, sodium hypochlorite, hypochlorite of lime, or ultra-violet rays are used for this purpose. The last named agent destroys the bacteria by the liberation of oxygen, as free oxygen kills bacteria. Liquid chlorine is now very generally used due to its effectiveness and the ease with which it can be handled. To determine the presence of pathogenic bacteria tests are usually made to determine the number of colon bacilli present. This germ is easy to detect and if it is found in any quantity the water is generally regarded as unsafe.

Distributing System.

1824. **Distributing Reservoir.**—A distributing reservoir is intended for one or more of the following purposes:

a. As a source of supply for the distributing system to prevent a water famine in case of accident to any part of the receiving system; three or four days' supply is usually considered sufficient for this purpose.

b. As a basin for the storage of the filtered water which flows from the purifying system. A reservoir of this type is also called a storage reservoir. To reduce the probability of contamination of the water in a storage reservoir to a minimum, the reservoir is usually limited to a day's supply or less, and the reservoir is wholly inclosed and covered.

c. To allow the hourly consumption to be variable, although the hourly supply which flows from the receiving reservoir or the purifying system is constant. This hourly consumption may be 40 per cent in excess of the mean hourly consumption; for this purpose the capacity of the reservoir is based on the hourly variation on the day of maximum daily consumption. In small plants the maximum hourly consumption is at the time of a large fire, and the reservoir is constructed to meet the emergency.

In pumping systems the distributing reservoir may be a small tank placed on a skeleton steel **tower,** or it may be a vertical cylinder of riveted steel plate, called a **standpipe.** The object of the former is simply to regulate the back pressure, while the latter is in addition a small reservoir. If the water supply is wholesome and the receiving reservoir a large one, settling basins and filters are often omitted and the receiving reservoir also serves as a distributing reservoir.

1825. **Water mains** distribute the water from the distributing reservoir to the houses of the users. The system of water mains should fulfill the following requirements:

a. Every part of the system should be strong enough to resist the static head of the distributing reservoir.

b. The hydraulic head at every point of the system should at all times be sufficient to raise the water to the desired height in adjoining buildings.

c. The water in every part of the system should be in constant motion, and should not be allowed to become stagnant.

d. If a break or leak occurs in any main, it should be possible to stop the flow through a short section in the immediate vicinity of the break without interfering with the supply in other parts of the system.

e. It should furnish an abundant supply to fire-plugs placed at intervals along the mains.

These requirements are fulfilled in general by laying, in the streets of a city, a network of connecting mains, sometimes called a gridiron system, thus forming an underground reservoir of considerable capacity, Fig 1825A. The smallest of these mains has a diameter of at least 4″. The aggregate area of cross-section decreases with the decrease of the area to be supplied.

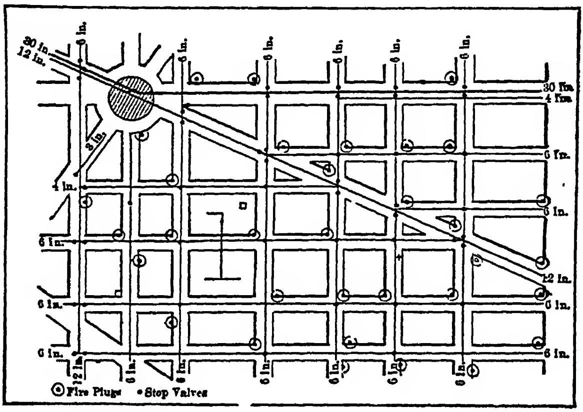


Fig. 1825 A Gridiron system of distributing pipes

If in the low part of the city the hydraulic head is too great, it is decreased by partially closing a valve in the main which supplies it, or a special valve may be introduced to cut off the supply when the hydrostatic head reaches a certain amount.

Where a considerable district is elevated, a special **high service system** with separate reservoirs, piping, and usually special pumps, is installed. Water for high service is usually taken from the mains fed by gravity from the principal reservoirs, which must have a capacity sufficient for the high service water in addition to their regular gravity or **low service** supply. Special pumps and piping are also installed for high pressure fire service in some cities.

The water mains of cities are usually cast iron pipes dipped in coal tar, with bell and spigot joints which are closed with hemp and lead. Valves are inserted at intervals so that any part of a main can be temporarily closed at both ends in order to make necessary repairs. The valves are vertical leaves which are lowered by means of a screw; this prevents their being closed rapidly. If the flow of water through a pipe is suddenly checked, the pipe is subjected to a severe blow from the **water-hammer** thus formed. The intensity of the blow varies directly with the volume and velocity of the flowing water.

Brackett's formula for cast iron pipe is

$$t = \frac{(p + p')r}{3300} + 0.25 \qquad (1825A)$$

in which t = thickness of pipe in inches
p = static pressure in pounds per square inch
p' = allowance made for water-hammer
= 70 for pipes over 36″ in diameter;
75 for 36″; 80 for 30″; 90 for 20″;
110 for 12″; 120 for smaller pipes
r = radius of pipe in inches

Small house connections are generally lead or galvanized iron pipes, usually about $\frac{3}{4}$″ in diameter, which can, by a special device, be attached to the main without stopping the flow of water. Fire-plugs are attached to short branches connected with the street mains.

A **blow-off** is a short branch with a suitable valve, through which the water in the main can be discharged. Blow-offs are placed at low points and at the dead ends of conduits and mains to force out deposits which may collect there.

Air-escape valves are placed at the high points of conduits to remove the air which may collect there and interfere with the flow.

Water-meters are employed for measuring the house consumption; they are attached to the service pipes. The Venturi meter is one which may be employed for measuring the flow under pressure through a main or conduit of any size.

1826. **Electrolysis** in cast iron pipes is caused by stray return currents of electricity from various sources, especially from electric railways. These currents flow along the pipe and leave the pipe near the power stations. Where the current leaves the pipe, either at the power stations or when skipping over the joints, which have high electrical resistance, it gradually decomposes the material of the pipe, causing leaks. Electrolysis can be much reduced by connecting the pipe at intervals to the street car tracks, and by use of insulated joints which increase its resistance. The current is carried to the pipe and around joints through moist soil; it does not pass readily through dry soil. Where the joints are welded, they offer no resistance to the passage of current, so electrolysis occurs only near the power stations.

1827. **Catskill Water Supply for New York City.**—This system is of the gravity type. The drainage areas lie in the center of the Catskill Mountains about 100 miles north of the city, and have a combined area of 771 square miles.

A series of dams across the valley of the Esopus Creek form the Ashokan reservoir, which has an available capacity of 128,000,000,000 gallons

at an elevation of 590′ above mean tide in New York Harbor. Water is taken from this impounding reservoir by an aqueduct to the Kensico reservoir about 30 miles north of New York, which has an available capacity of 29,000,000,000 gallons, sufficient for several months, and acts as a storage reservoir to make possible a continuous supply if repair of the system above this point were to become necessary. To equalize the difference between the use of the water, as it varies from hour to hour, and the steady flow in the aqueduct, the Hill View reservoir was constructed just north of the city limits with a capacity of 900,000,000 gallons.

Four general types of aqueduct were used: cut and cover, grade tunnel, pressure tunnel, and steel pipe siphon. The cut and cover aqueduct is of the horseshoe type of simple concrete. (Fig 1827A.) The grade

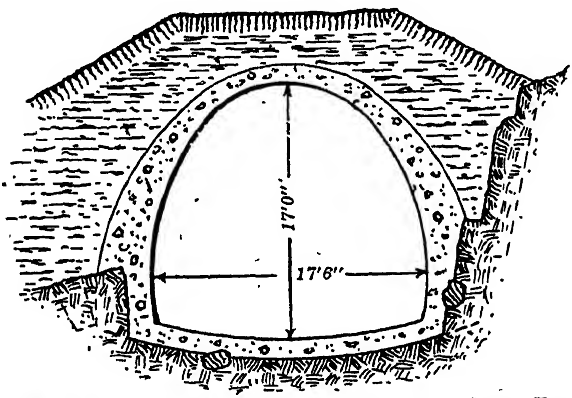


Fig. 1827 A Catskill Aqueduct - Cut and Cover Type

tunnels are of the same shape but smaller and with a steeper grade for economy in excavation. The pressure tunnels are circular in cross-section and lined with concrete. The Hudson River is crossed by means of a tunnel about 14′ in diameter wholly in granite rock (Fig 1827B), at a depth of 1114′ below sea level, between a shaft at Storm King Mountain on the west bank and another at Breakneck Mountain on the east bank. Steel pipe siphons are used in valleys where the rock is not sound or where for other reasons pressure tunnels would be impractical. They are covered inside and out with a protective coating of concrete.

To improve the quality of the water the following treatments are used: aëration, coagulation, and chlorination. Aërators are installed at both the Ashokan and Kensico reservoirs, each with about 1600 nozzles designed to produce a fine spray to provide as thorough oxidation as possible. Near Kensico reservoir is the coagulating plant which is arranged to introduce a coagulant, when necessary, into the water in the

aqueduct so that a complete mixture will be obtained before reaching the settling reservoir. As the water leaves the Kensico reservoir it passes through a screen chamber so arranged as to admit of chlorination. The filtration plant designed for the aqueduct below the Kensico reservoir is of the rapid sand type.

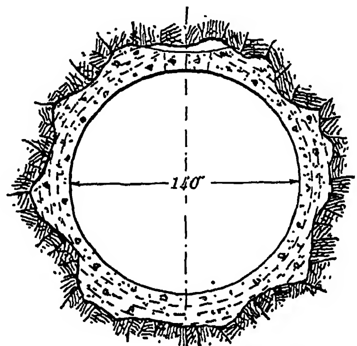


Fig. 1827 B.
Catskill Aqueduct-pressure tunnel

The bacteria content is reduced by each of these treatments: by aëration due to the action of the oxygen, by the coagulant through the settling of the suspended matter which carries some bacteria down with it, and by the chlorine which kills the greater part of those remaining.

From Hill View reservoir, Catskill water is delivered into the five boroughs by a circular tunnel in solid rock reducing in diameter from 15′ to 14′, 13′, 12′, and 11′. The total length of the tunnel is 18 miles. From two terminal shafts in Brooklyn, steel and cast iron pipe lines extend into Queens and Richmond Boroughs. A 36″ flexible-jointed, cast iron pipe buried in a trench in the harbor bottom has been laid across the Narrows to the Staten Island shore, whence a 48″ cast iron pipe extends to the Silver Lake reservoir, holding 435,000,000 gallons. The total length of this delivery system is over 34 miles. The tunnel, under the city, is at depths of 200′ to 750′ below street surface, thus avoiding interference with streets, buildings, subways, sewers, and pipes. The waterway of the tunnel is lined throughout with concrete.

1828. **Problem.**—A cast iron water main is 20″ in diameter, and 0.8″ thick. What is the maximum pressure head that it will carry under Brackett's formula?

Solution.—

$$t = \frac{(p + p')r}{3300} + 0.25$$

$$0.8 = \frac{(p + 90)10}{3300} + 0.25$$

$$p = 91.5$$

$$h = \frac{144 \times 91.5}{62.5} = 210.8'$$

1829. **Problem.**—A reservoir, elevation 120′, delivers water at a point 5000′ distant, elevation 20′, under the condition that the velocity in the main shall not exceed 6′ per second.

(*a*) What size pipe should be used? (Friction coefficient 0.01.)

(*b*) What amount of water will this pipe supply?

Solution.—(*a*) Using equation 542A, and transposing for H, we have

$$H_t = \frac{4\ \mu l V^2}{2\ gD} = 0.0621 \frac{\mu l V^2}{D}$$

$$100 = \frac{.0621 \times .01 \times 5000 \times 36}{D}$$

$$D = 1.12' = 13.4''$$

(*b*) Using equation 542B and substituting above value of H, we have

$$Q = 3.152 \sqrt{\frac{H_t D^5}{\mu l}}$$

$$= 3.152 \sqrt{\frac{100 \times (1.12)^5}{.01 \times 5000}}$$

$$= 5.9 \text{ cubic feet per second}$$

For more detailed information see

Waterworks Handbook, Flynn, Weston, and Bogert
Hydraulics, Merriman
Civil Engineering, Fiebeger
Water Supply Engineering, Folwell
American Civil Engineer's Pocket Book, Merriman

PROBLEMS.

P. 1801. What is the maximum flood discharge under Fanning's formula for a basin averaging 4 miles wide along 10 miles of a stream? How long would the spillway of a dam closing this area have to be?

P. 1802. How long would it require to fill a reservoir having a capacity of 900,000 gallons from the flood discharge of a drainage area of 20 square miles?

P. 1803. Determine thickness of cast iron outlet pipe for a steel standpipe 70′ high, 20′ in diameter, the lowest point of the pipe being 10′ below ground under the standpipe.

P. 1804. A cylindrical steel standpipe 40′ high, 10′ in diameter, receives water at a pressure of 90 pounds per square foot. Determine thickness of plates at bottom of standpipe.

P. 1805. A cylindrical steel water tank without a roof is 20′ high, and 10′ in diameter. Determine weight of tank on tower.

P. 1806. A reservoir, elevation 235′, supplies water through a steel main 20 miles long. If the main at its terminal is 20″ in diameter, at elevation +15′, what should be its diameter at its lowest point, elevation −15′?

P. 1807. A reservoir has a catchment area of 37,000 square miles. The average run-off is 0.05 c.f.s. per square mile, and the average silt content is 0.25% by volume. Compute the volume of silt deposited in 10 years on these assumptions.

P 1808. A reservoir has a water surface of 2000 acres and a catchment area of 50,000 acres. Starting with the reservoir full to crest of wasteway, water enters at the rate of $\frac{1}{12}''$ per hour over the entire catchment area for 24 hours, followed by a 24-hour period during which the inflow is equivalent to $\frac{1}{4}''$ per hour. Find the length of waste weir so that the water will not rise more than 1.5′.

P. 1809. A pipe line is to be 42,800′ long and to supply an average demand of 3,200,000 gallons per day. The difference in elevation between intake and outlet is 146′. The maximum demand is 175% of the daily average. What size of cast iron pipe is required?

P. 1810. A broad shallow stream has a depth of 3′ and a slope of 5′ per mile. If a dam 8′ high be built across this stream how much will the water surface be raised one mile upstream using Chezy $C = 75$.

P. 1811. What would be the pressure in pounds per square inch at the bottom of the tunnel under the Hudson River at Storm King, elevation −1114′, reservoir elevation 590′, if the flow should be stopped?

P. 1812. Determine thickness of cast iron water main 20″ diameter, under a head of 200′.

P. 1813. A 16″ cast iron water main .60″ thick carries 100 pounds per square foot pressure. It is desired to increase this pressure to 250 pounds per square foot for fire purposes. An 8″ pipe would supply enough water for fire purposes alone. If cast iron pipe costs $40 a ton new, and can be sold as scrap at $8 per ton, is it cheaper to lay a separate 8″ fire main or replace the water main with heavier 16″ pipe?

P. 1814. A town of 2000 is in need of a water supply system; water can be obtained by damming a small stream at an elevation of 290′ above the town, distance one mile. Determine the size of drainage area needed, the capacity of the reservoir for 100 days' supply, and the diameter and thickness of cast iron water pipe for the supply main.

P. 1815. What should be the thickness of a rectangular masonry dam, whose height is 30′, if the level of the reservoir is 2′ below the crest of the dam? Specific gravity of masonry is 2.3. Maximum pressure must not exceed twice mean pressure.

P. 1816. What will be the maximum and mean pressure upon the base of the dam in preceding problem?

P. 1817. What will be the discharge, in gallons per second, from an orifice 2″ square, whose center is 4′ below the surface?

P. 1818. What is the discharge in cubic feet per second of a weir 3′ long under a head of 6″, if there is no end contraction? If it is contracted at the ends?

P. 1819. A long pipe, 1′ in diameter, has a hydraulic gradient of 1/100. What is the velocity of flow and discharge per second if $f = 0.012$?

P. 1820. The velocity of flow in a long pipe, 2′ in diameter, is 3′ per second. What is its hydraulic gradient and discharge per second if $f = 0.010$?

P. 1821. A long pipe, 6″ in diameter, delivers one cubic foot of water per second. What is the velocity of flow and the hydraulic gradient if $f = 0.012$?

P. 1822. The hydraulic gradient of a long pipe is 1/100 and the velocity of flow 4′ per second. What is its diameter and its discharge per second if $f = 0.010$?

P. 1823. The hydraulic gradient of a long pipe is 1/100 and its discharge is 3 cubic feet per second. What is the velocity of flow and its diameter if $f = 0.012$?

P. 1824. A pipe, 2000′ long, is made of two equal lengths; one has a diameter of 8″ and the other of 6″. What is the discharge per second under a head of 20′? $f = 0.01$.

CHAPTER XIX.

SEWERAGE AND SEWAGE DISPOSAL.

1901. **Definition of Terms.**—**Sewage** is the term applied to the liquid and soft solid waste products of a community. The means of removing the sewage, through an arrangement of pipes or conduits, by aid of the community's water supply, is called **sewerage.** A sewerage system, then, comprises all the conduits or **sewers,** pumping stations, treatment plants, or other works, which are necessary to collect, purify, and dispose of the sewage.

Surface run-off after rainfalls, and household and manufacturing wastes, are the main sources of sewage. Sewerage systems are spoken of as **combined** or **separate** according as the waste products and surface run-off are carried in one system of sewers, or in two.

SEWERAGE SYSTEMS.

1902. **Separate System.**—The principal advantages of the separate system are:

(*a*) The first cost of the system is reduced since sewers to take care of the household and factory wastes can be built while the surface drainage is allowed to run off in surface drains. If it becomes necessary, later on, to remove the surface run-off in sewers, these storm-water sewers can be constructed as a separate project.

(*b*) Where sewage must be purified before it can be discharged into a body of water the separate system is the more economical, since the cost of treatment is a function of the amount of sewage. In the combined system it is necessary to purify all sewage, until the normal dry-weather flow has been diluted sufficiently by storm water to permit the discharge of the sewage without creating a nuisance. In the separate system, however, the household and factory wastes can be treated and the storm water discharged directly into the lake or stream.

(*c*) The separate system is more economical to construct, from the point of view of the amount of excavation necessary. Since the basements of the houses must be drained, in the separate system it is necessary only to place the house sewers at a depth to take this drainage while the storm-water sewers are placed nearer the surface of the ground. In the combined system the sewer, which is larger, must be constructed deep enough to drain the basements.

1903. **Combined System.**—The principal advantages of the combined system are:

(*a*) With the combined system the chances of error in connecting the houses with the sewer are reduced, since there is only a single system of conduits in each street.

(*b*) It is more economical to maintain a combined system. Flushing, for example, is a maintenance item which is provided for naturally in the combined system, due to the cleaning action of the storm-flow. In a separate system, however, flushing has to be done by the use of automatic flushing tanks or the uncertain action obtained by connecting small drainage areas, as roofs and yards, with the house sewers.

1904. In general the combined system is used in large cities so located that the sewage can be discharged without treatment into a large body of water. Where the sewage must be treated it is usually best to use the separate system, due to the reduced cost of purification. For small communities, where the surface run-off can be carried in open drains, the separate system is generally used, due to its economy. It may also be found advantageous to use the separate system when it is necessary to pump sewage before it can be discharged into the natural drainage system of the country.

1905. **Parts of Sewerage System.**—A sewerage system consists of the following principal parts, some or all of which are used in every system: conduits or sewers, manholes, house connections, inlets, pumps, and treatment plants.

The sewers, or conduits, are generally classified as:

1. Lateral sewers, which collect the sewage from the house connections, but do not receive sewage from any other sewer.
2. Branch sewers, which receive the flow from the laterals.
3. Main or trunk sewers, which collect the flow from two or more branch sewers.
4. Outfall sewers, which receive the sewage from the trunk sewers, and discharge it either into some body of water or at some treatment plant.

Manholes are placed at intervals, not greater than 350′, along each conduit, at all points of change of grade or direction, and at all junction points, to give access to the sewer for cleaning and inspection. A standard type of manhole is shown in Fig 1905A.

The term inlet is applied to the opening by which the storm water enters the sewer.

Storm water entering a sewer usually passes through a catch-basin, a boxlike shallow well designed to catch material in suspension before

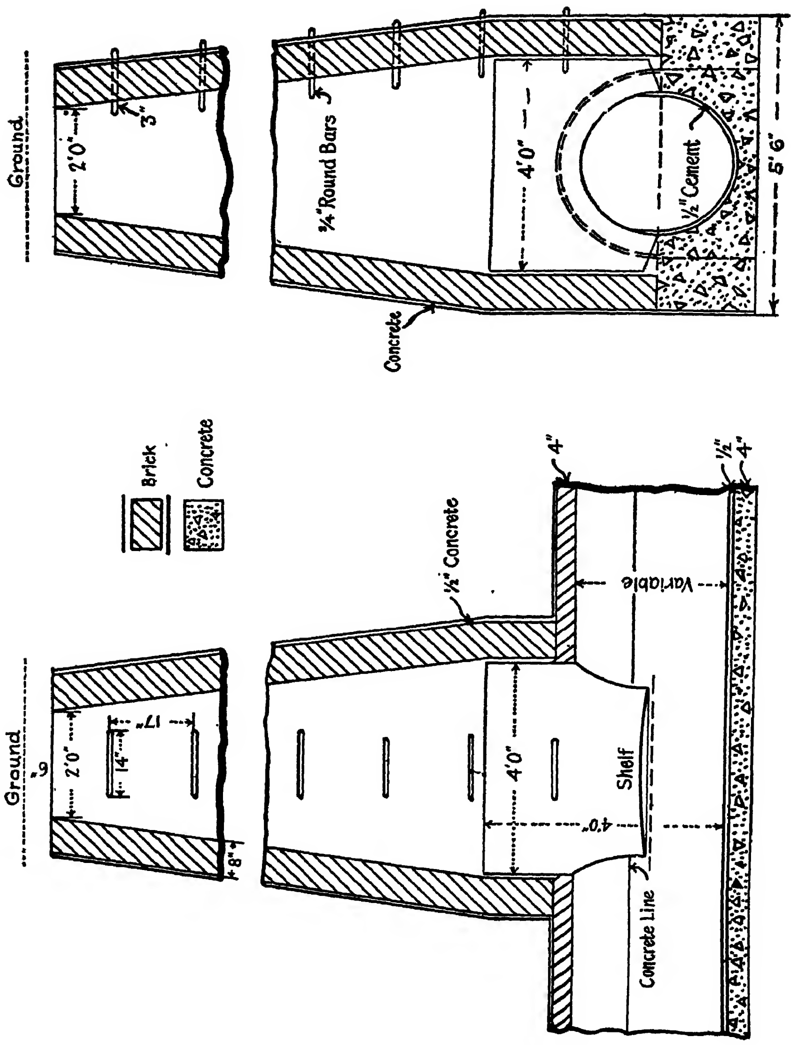


Fig. 1905 A. Standard Manhole

discharging the water into the sewer. A vertical section of a catch-basin is shown in Fig 1905B. The water enters at C and passes into the sewer at A, which is the inlet. At B is a manhole so that the material caught can be removed.

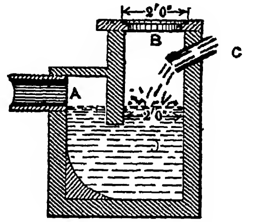


Fig. 1905 B

House connections are made with the sewer through either a T or a Y branch. Where the topography makes it necessary, pumps are used to raise the sewage to a higher level.

Classes of Sewage.

1906. **Domestic Sewage.**—Household waste, or domestic sewage, is the water which has been used in the residences. It has a pollution of about one-tenth of one per cent, consisting of various organic matter. In appearance it is slightly turbid, with floating matter, and has hardly any odor when fresh.

1907. **Industrial Sewage.**—The factory waste, or industrial sewage, varies greatly in amount and character. It may contain both acids and alkalies, organic matter, tar, and many other substances, all of which have been found to amount to about 7 or 8 per cent of the sewage. Gasoline, due to its explosive nature, should never be allowed to enter a sewer. In amount industrial sewage is about $\frac{1}{4}$ to $\frac{1}{2}$ of the domestic sewage.

1908. **Surface Drainage.**—Storm sewage, or surface drainage, is the rain water running off the roofs and streets. It carries such dirt as its amount and velocity make possible, consisting of sticks, leaves, paper etc.

1909. The aim of a sewerage system is to remove, in closed conduits, all sewage from the limits of the community before the polluting substances begin to decompose, and dispose of it in a manner which will not produce a nuisance, or a menace to public health.

Design of a Sewerage System.

1910. **Preliminary Estimates.**—Before the structural details of the system can be designed it is necessary to know the amount and kind of

sewage to be cared for, how it shall be disposed of, and whether a separate or combined system of sewers is best suited to the needs of the community. These matters are so related that a careful study should be made before a decision is reached.

The amount of domestic sewage will depend on the future population of the area, it being customary to design for an estimated population about 30 years in advance, and the probable consumption of water per capita. The amount of storm-flow is a function of the rainfall and the topography. It is greater in amount in a given area in a congested district than in a residential district. The estimation of the amount of industrial sewage is very difficult since it is hard to foretell changes in the types of manufacturing, but the general commercial characteristics of the city are considered in the light of possible changes. Where the *combined system is used, the storm-flow,* due to its large amount though for a limited time, *is the principal factor governing the size of the system.*

The method of disposal is decided on, after consideration of the available points of discharge and effect of the sewage upon the body of water which receives it. At many places, laws govern the discharge of sewage into the natural drainage systems thus fixing certain limits to the design. If it is physically possible to discharge the raw sewage directly, the effect upon the body of water may make this method impossible, since the dilution may not be great enough to prevent the creation of a nuisance either from the standpoint of public health or commerce. Where some method of treatment is necessary, the amount and character of the sewage, the cost of construction of the plant, and the cost of operation must be considered. The various methods of treatment, to be discussed later, depend for their effectiveness upon the condition of the sewage. This makes necessary an investigation of what chemicals will be present in the industrial sewage.

The principles bearing on the choice between the separate or combined system have been previously discussed.

1911. **Field Work.**—Not only must the total future population be estimated, but the area must be divided into districts based on the density of population so that larger sewers may be placed where necessary to serve the more congested parts of the city. These districts must be then considered as to their topography in order to arrange the system so that a gravity flow will be obtained whenever possible. This requires that a carefully prepared contour map be available. A tentative system is then laid out on a map. In doing this, consideration must be given to the location of street railways and sewers or other pipes then in place in the streets and to the depths of the cellars, if these are to be drained. Lines of levels are then run so that profiles of all the proposed lines can be plotted and tied in with the elevation at which the sewage

must be discharged. Where it is impossible to discharge the sewage by gravity, as at New Orleans, it must be delivered at a pumping plant where it is raised to the elevation necessary to discharge.

The cost of excavation is dependent on the character of the soil, hence the presence of ground water or rock should be determined by borings or test pits. The amount of sewage that is expected in each part of the system is then computed. Having determined the quantity of sewage, and the limits as to grade, the sizes of the conduits can be determined.

1912. **Flow in Sewers.**—Having determined the size of pipe to be used, it is necessary to consider the grade at which it is to be laid, with a view to reducing the amount of excavation to a minimum, and to securing the proper velocity. The earthwork table below shows the variation of the amount of excavation with the size of pipe and the depth.

EARTHWORK TABLE.

Depth of Trench	Cubic Yards of Excavation Required per Linear Foot of Trench							
	6″ pipe 1.6′ trench	8″ pipe 1.8′ trench	10″ pipe 2.0′ trench	12″ pipe 2.2′ trench	14″ pipe 2.4′ trench	16″ pipe 2.5′ trench	18″ pipe 2.7′ trench	20″ pipe 2.9′ trench
4′.0	.24	.27	.30	.33	.36	.37	.40	.43
4′.5	.27	.30	.33	.37	.40	.42	.45	.48
5′.0	.30	.33	.37	.41	.44	.46	.50	.54
5′.5	.33	.37	.41	.45	.49	.51	.55	.59
6′.0	.36	.40	.44	.49	.53	.56	.60	.65
6′.5	.39	.43	.48	.53	.58	.60	.65	.70
7′.0	.41	.47	.52	.57	.62	.65	.70	.75
7′.5	.44	.50	.56	.61	.67	.69	.75	.81
8′.0	.47	.53	.59	.65	.71	.74	.80	.86
8′.5	.50	.57	.63	.69	.76	.79	.85	.91
9′.0	.53	.60	.67	.73	.80	.83	.90	.97
9′.5	.56	.63	.70	.77	.84	.88	.95	1.02
10′.0	.59	.66	.74	.81	.89	.93	1.00	1.07
10′.5	.62	.70	.78	.86	.93	.97	1.05	1.13
11′.0	.65	.73	.82	.90	.98	1.02	1.10	1.18
11′.5	.68	.77	.85	.94	1.02	1.06	1.16	1.23
12′.0	.71	.80	.89	.98	1.07	1.11	1.20	1.29
12′.5	.74	.83	.93	1.02	1.11	1.16	1.25	1.34
13′.0	.77	.87	.96	1.06	1.15	1.20	1.30	1.40
13′.5	.80	.90	1.00	1.10	1.20	1.25	1.35	1.45
14′.0	.83	.93	1.04	1.14	1.24	1.30	1.40	1.50
14′.5	.86	.97	1.07	1.18	1.29	1.34	1.45	1.56
15′.0	.89	1.00	1.11	1.22	1.33	1.39	1.50	1.61
15′.5	.92	1.03	1.15	1.26	1.38	1.44	1.55	1.66
16′.0	.95	1.07	1.19	1.30	1.42	1.48	1.60	1.72
16′.5	.98	1.10	1.22	1.34	1.47	1.53	1.65	1.77
17′.0	1.01	1.13	1.26	1.39	1.51	1.57	1.70	1.83
17′.5	1.04	1.17	1.30	1.43	1.56	1.62	1.75	1.88
18′.0	1.07	1.20	1.33	1.47	1.60	1.67	1.80	1.93
18′.5	1.10	1.23	1.37	1.51	1.64	1.71	1.85	1.99

The velocity of flow in a sewer should not be so small that deposits will be formed in the sewer, nor should the velocity be so great that stones will be swept along swiftly and chip the bottom of the sewer. Even if the velocity is so great that the sewer is worn, it will be some time before it is necessary to replace it; but if it is too small, allowing the forming of deposits, a serious menace to health is soon produced which will require the reconstruction of the entire sewer. To prevent deposits from domestic sewage, a minimum velocity of 1.5′ per second is required, while for storm sewage, due to the greater weight of the material carried, the velocity should be not less than 3′ per second. In sewer systems where the flow in any branch does not maintain the required velocity, it is necessary to flush out the sewer by pumping water through it or by water from a flush tank. A flush tank is a tank with a siphon whose highest point is below the top of the tank. A small stream of water continually flows into the tank and the tank is discharged by means of the siphon when the level of the water reaches the top of the siphon.

To prevent the clogging of sewers, no pipes smaller than 6″ or 8″ are used in practise. All pipes less than 18″ in diameter are ordinarily designed to flow half full and all greater than 18″ to flow three-quarters full. This allows for ventilation of the sewer while at the same time introducing a factor of safety to provide for unforeseen future increases in the volume of sewage. A velocity of from 2′ to $2\frac{1}{2}$′ per second has been found to give satisfactory results. With this velocity the following minimum grades have been computed:

Pipe	Grade	Pipe	Grade
8″	0.040 per cent	18″	0.014 per cent
12″	0.024 "	20″	0.012 "
16″	0.016 "	24″	0.010 "

Sewer Construction.

1913. **Materials of Construction.**—Since sewage does not ordinarily run under a pressure head, the sewer is subjected to external rather than internal forces. Hence, the conduit is designed only to withstand these external forces. In order to offer little resistance to the flow of the sewage it is necessary that the inner surface of the sewer be smooth. Since acids are present in the sewage, it is also important to use a material which they do not attack readily. The number of changes of direction should be reduced as much as possible in order that the resistance to flow caused by each change of direction may be reduced to a minimum.

The materials which have come into general use in sewer construction because they best fulfill the above conditions are: concrete pipes, brick conduits, iron pipes, and glazed stoneware pipes. Iron pipe finds its greatest application in very compressive soils, where stoneware pipe would be likely to break, and in all cases where the sewage flows under a head, as is the case in an inverted siphon connecting the sewerage systems on opposite banks of a stream.

1914. **Vitrified or Stoneware Pipes.**—Stoneware pipe can be obtained in standard sizes varying from 2″ to 24″ in diameter. Manufacturers make larger sizes to fill special orders. The smallest pipe used in standard sewerage work is the 6″, although 4″ pipe is sometimes used for house connections. Where the diameter of the conduit exceeds 24″ brick or concrete is generally employed instead of the special sizes of vitrified pipe.

Fig. 1914 A

There are two general types of stoneware sewer pipe: the ring type, Fig 1914A, and the spigot-and-bell type, Fig 1914B. The ring pipe is simply a cylinder which is provided with rings or bands to cover the joints between sections. A semi-cylindrical pipe is also made, called split or channel pipe. This pipe is used as a lining for the bottom of brick or concrete conduits.

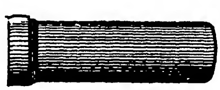


Fig. 1914 B

1915. **Pipe Joints.**—Joints with the spigot-and-bell type are made by inserting the spigot of one pipe into the socket of another and closing the joint with a gasket of waste covered with well compressed cement mortar. It has been found by experiment that the tightest joints can be made with a neat Portland cement. A 1 : 1 Portland cement mortar is, however, quite generally used. The object of placing the waste in the joint is to prevent the mortar from protruding into the pipe and forming a rough surface which would impede the flow. Ring pipe is usually laid in a bed of concrete, the joint bands being completely covered with the concrete. The method used in Washington, D.C., in laying ring pipe is

shown in Figs 1915A, B, C. Fig 1915A is at a joint and Fig 1915C between joints. Where a wider base is necessary the method shown in Fig 1915D can be employed.

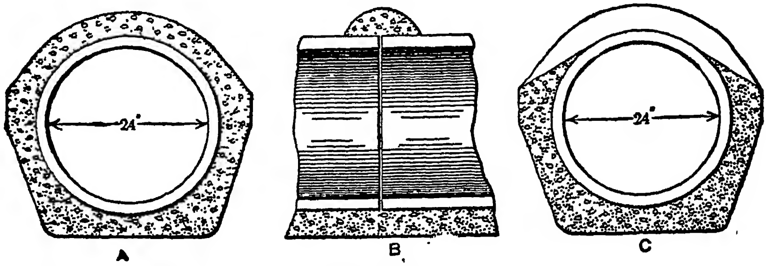


Fig. 1915.

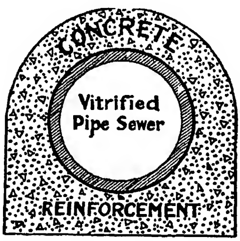


Fig. 1519 D

1916. **Brick and Concrete Sewers.**—Where the required area of cross-section of a sewer exceeds that of a 24″ pipe, conduits are generally constructed of brick or concrete. For these larger sized sewers three forms of cross-section are commonly employed: the circular, the egg-shaped, and the horseshoe. The circular section is the cheapest, since it gives the greatest area of cross-section for a given amount of material. The egg-shaped and horseshoe sections, however, have characteristics which make them advantageous under certain circumstances. In a combined sewer where the dry weather flow is small as compared with the storm flow the egg-shaped section maintains a higher velocity of flow, which prevents deposits. This is due to the fact that this section has a greater hydraulic mean depth and hence a greater velocity than the circular section when only partly full. When it is desired to secure a large area of cross-section with a reduction in the height of the sewer the horseshoe section is used. Fig 1916A shows a horseshoe shaped sewer as built in Washington, D. C.

The lower part of a brick sewer is often lined with vitrified stoneware blocks to reduce the amount of friction. These liners are called vitri-

fied sewer or invert blocks. Where very steep slopes have to be constructed the invert can be lined with paving brick to reduce the resulting excessive wear.

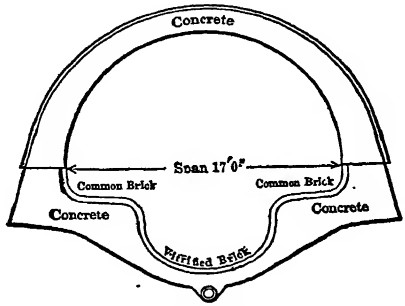


Fig. 1916 A

1917. **Size of Pipe.**—For determining the size of pipe required, Kutter's formula has been almost universally adopted in connection with sanitary sewers. Engineers have used different values for n, but 0.013 for vitrified pipe, and 0.015 for concrete and brick, have been found to give very good results. Certain experience has led to the belief that this formula gives a result which is a trifle low, but on taking into consideration a safety factor of $1\frac{1}{2}$ to 2, the figure is still within safe limits. While it is assumed that the student is familiar with this formula, he will find it discussed at some length in paragraph 559.

The hydraulic elements of a circular pipe as computed by Kutter's formula are shown in Fig 1917A. By referring to the formula, which is $V = C\sqrt{RS}$, it is seen that the velocity varies directly with the $\sqrt{R}$. The figure shows this variation graphically. If it is assumed, for instance, that a pipe is running 0.40 full, then by starting at 0.40 on the left edge of the diagram and following the horizontal line we find, from the curve intersections, the discharge to be 0.34 of the discharge when the pipe is running full, the area of cross-section of the water to be 0.38 of the total area, the hydraulic mean radius to be 0.84 of the value when full, and the velocity to be 0.9 of the velocity when the pipe is full. It will be noted that the maximum velocity occurs when R, or the hydraulic radius, is a maximum, which is when the pipe is 0.8 full. The figure also shows the variation of the area of cross-section of the water and since the discharge is a function of the area and the velocity, it is seen that the maximum discharge occurs when the pipe is about 0.9 full.

1918. **Pipe Diagram.**—It is impractical in practise to attempt to figure the diameter of each pipe individually, and, while tables may be

used, some form of diagram will be found to give sufficiently accurate results and is a much quicker method. The diagram shown in Fig. 1918A is plotted on logarithmic paper, the lines on which are arranged on the same principle as on the slide rule, permitting practically straight lines to be used. The method of compilation is simply to calculate for the two extremes in grade, plot the points in their proper location and draw in the lines between them.

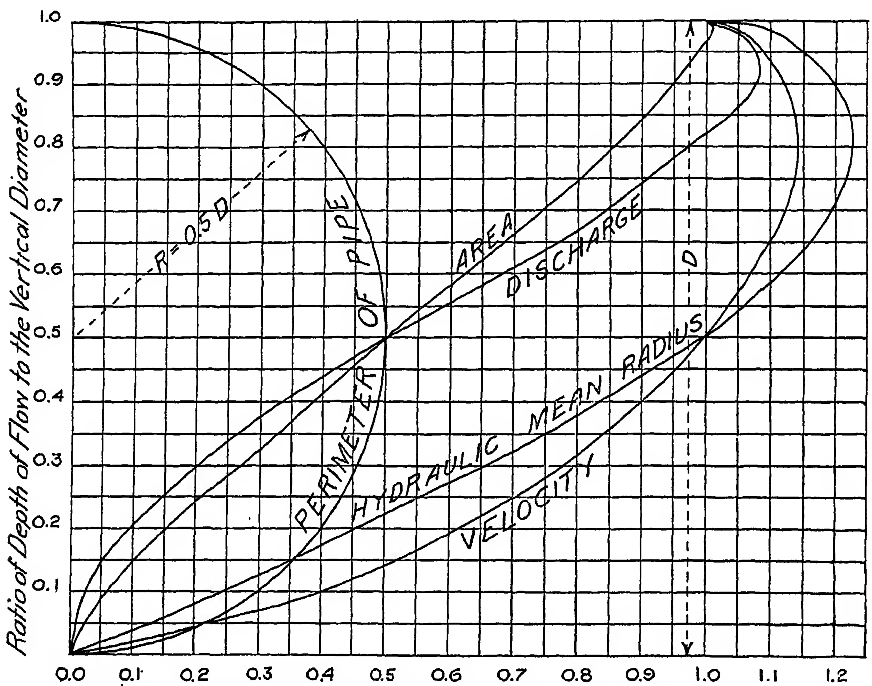


Fig 1917 A

To illustrate the method of using Fig 1918A, let it be required to find the slope and the velocity of the sewage in a 2.5′ sewer discharging 10 cubic feet per second.

Starting at the bottom of the figure at 10 follow the vertical line until it intersects the " diameter in feet " diagonal line marked 2.5. Reading on the left edge of the figure, the slope is 0.6′ per 1000. Interpolating for the velocity between the diagonal velocity lines, we find it to lie between 2.0 and 2.5 and to be about 2.1 feet per second. In a similar manner, if given any two conditions we can determine the two unknown conditions.

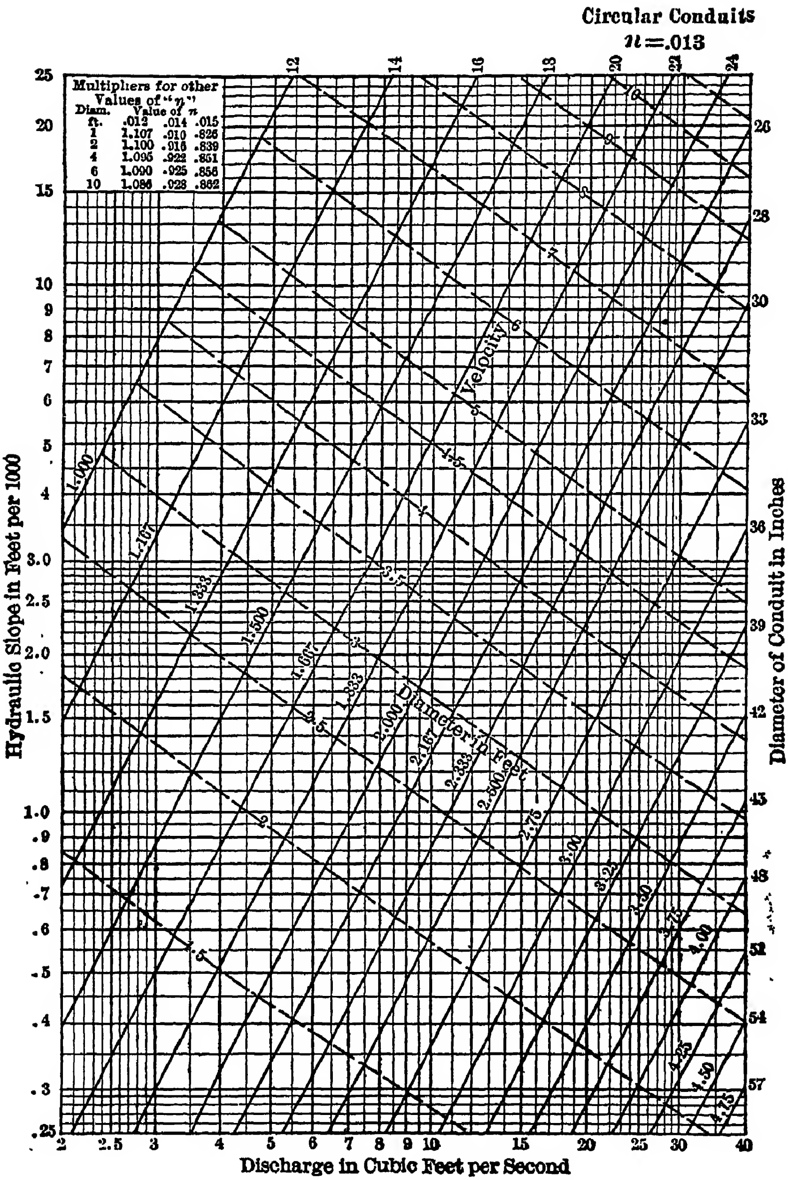


Fig. 1918 A—Discharge of Circular Conduits Flowing Full by Kutter Formula.

1919. **Ventilation of Sewers.**—Since material is apt to settle in sewers and give off offensive and noxious odors, it is necessary to provide for ventilation. The ordinary method is by the use of perforated covers on the manholes which allow a circulation of air through the pipes, thereby diluting the sewer air and keeping it fresh. Disinfectants may be employed in the sewers where the ventilation is not sufficient.

SEWAGE DISPOSAL.*

1920. **Raw Sewage.**—The turbid liquid which flows through the sewers is called raw sewage, and contains animal and vegetable organic matter as well as inorganic matter both in suspension and solution. A chemical analysis has to be made to determine the exact quantity and character of the organic and inorganic substances present.

In the presence of air and moisture the organic matter is reduced to simpler inorganic compounds and elements by a process of putrefaction. It is necessary to regulate properly this process since foul odors are given off, which, if not positively dangerous to human life, are, nevertheless, objectionable to a sufficient degree to be classed as a nuisance.

The reduction process does not alter the amount of chlorine in the sewage. The following characteristics distinguish raw sewage from purified sewage: a large ammonia content, the absence of nitrates, the large amount of oxygen it will absorb, and the amount of matter in suspension.

Bacteria.

1921. The primary aim of every disposal scheme is to kill the bacteria which are dangerous to life, while avoiding the creation of a nuisance.

1922. Raw sewage contains myriads of microscopic organisms called bacteria; several millions are often found in a cubic centimeter. Their function seems to be to break down the complex organic matters in the sewage and with the aid of oxygen to form simple stable compounds and elements. These microscopic organisms are divided into three classes: **aërobes**, which can live and work only in a medium which is well supplied with oxygen, **anaërobes**, which live and work in a medium in which there is no oxygen, and **facultative bacteria**, which live and work in a medium in which oxygen is present but only in a small ratio to the other elements. Each of these three classes consists of many types of bacteria and, as each type has its own mode of action, the successive steps in the reduction of the organic matter are extremely complicated. Each class works under

* Much of the material presented under Sewage Disposal is taken verbatim from Civil Engineering by Col. G. J. Fiebeger.

conditions favorable to itself and extinguishes itself as a result of its own action in purifying the sewage.

1923. **Aërobic Action.**—In general it may be said that if sewage is spread intermittently in a thin sheet over a porous soil, such as coarse sand, the pores of which are filled with air, the conditions are favorable for aërobic bacteria, and the work will be done principally by them. The same is true if the sewage is discharged into highly aërated water. The effect of these bacteria being to oxidize the nitrogen of the organic matter and produce nitrates, the process of reduction by aërobic bacteria is called **nitrification.** It is a process of reduction unaccompanied by foul odors. To prepare sewage for aërobic action it is usually screened to remove sticks, paper, rags, and other material not easily reduced, and is also allowed to stand for a few hours in settling tanks to remove the coarser materials in suspension not removed by the screen, which would be liable to close the pores of the filtering material.

1924. **Anaërobic Action.**—If sewage is allowed to stand in tanks or pools for any length of time, its oxygen is soon exhausted by the aërobes, and the conditions are then favorable for anaërobic action. The anaërobes break down the organic matter and form ammonia, nitrites, and release such gases as nitrogen, hydrogen, carbon dioxide, marsh gas, and sulphuretted hydrogen. As nearly all the vegetable and animal organic matter is liquefied by the anaërobes, the process is called **liquefaction** or **hydrolysis.** During the process of liquefaction foul odors are given off, and the sewage itself is usually in a foul condition and requires the action of the aërobic bacteria to complete its purification.

1925. **Process of Reduction.**—The process of reduction or purification, to prevent a nuisance, is therefore usually effected by bacteria of all three varieties. If the sewage is screened and settled, the reduction, as explained hereafter, may be effected almost wholly by aërobic bacteria, but the other varieties will be present in some stage of the process and will assist. If the sewage is kept in an open or closed tank for 24 hours, the aërobic bacteria will almost disappear, and the process in the tank will be almost wholly anaërobic. In preparing the sewage for anaërobic action it is not so necessary to strain it, nor is it necessary to first pass it through a settling tank. The anaërobic action must, however, always be followed by aërobic action.

1926. **Pathogenic Bacteria.**—Bacteria may also be divided into the disease-producing bacteria and those that are not disease-producing. The principal varieties of the first class, or **pathogenic bacteria,** which may be found in sewage, are those of **cholera, diarrhœa, dysentery,** and **typhoid fever.** They are found in the discharge of patients who have these diseases. If sewage containing such bacteria is discharged into

any stream used as a water- or ice-supply, they at once become a menace to public health.

Methods of Disposal.

1927. Sewage, either in a raw state or after more or less screening and purification, may be disposed of by discharging it into a body of water, upon natural land, upon prepared land, or upon specially prepared filter beds. If the discharge is into fresh water, as is usually the case in inland cities, care must be taken not only to avoid a nuisance caused by too great a concentration of sewage or its deposition on the shores, but also to reduce the number of bacteria in it as much as possible, so that the pathogenic bacteria which may find access to it are destroyed.

1928. **Dilution.**—If raw sewage is discharged into a body of water simply to weaken the sewage and thus render it inoffensive, it is said to be disposed of by dilution.

No nuisance will be created if the raw sewage is discharged into a stream of moderate velocity whose dilution ratio, or the ratio of the volume of discharge of the stream to the sewage, is about forty. If the stream is a rapidly flowing mountain torrent, the factor may be reduced to twenty. These dilution ratios will vary somewhat with the character of the sewage. When sewage is discharged into still water, such as a lake, the discharge should not be concentrated at a single point, as it will then putrefy. Its distribution may be effected by discharging through a long conduit, laid in the bed of the lake, with outlets at intervals along it.

1929. In very still water the mixed water and sewage is so rapidly purified by the process of sedimentation that the pollution of the water does not extend to a great distance from the sewer outlet. In running water the sedimentation is less rapid, and the pollution of the water is observed at a greater distance from the sewer outlet. The reduction of the organic matter is effected by bacteria and other organisms which are found in the sewage, in the water, and in the mud of the bed. In the process of reduction oxygen is extracted from the water to form the new compounds. As the organisms which effect the final change are aërobic, this process is stopped, and offensive decomposition begins whenever the oxygen in the water is too greatly reduced. It is for this reason that the discharge into still water should be distributed over a wide area, and that a stream which can purify a given volume of sewage becomes foul when charged with a greater one. It explains why a torrent which is constantly aërated can have a smaller dilution ratio than a slow stream of equal volume.

1930. The bacteria of the sewage are reduced largely by the process of sedimentation which carries them to the bottom with the suspended material. Other causes of purification are the lack of food and the un-

favorable surroundings in the clarified water. The reduction of the number of these organisms in still water is very great; tests have shown reductions from one million to one thousand per cubic centimeter in 100′. In running streams bacteria must be carried to considerable distances, since it has been shown by experiments that the bacteria of typhoid fever may live twenty-four days even in ice-cold water.

1931. From the above it would appear that raw sewage may be disposed of by dilution without creating a nuisance if the volume of running water or the area of distribution in still water is sufficiently great. It will, however, always be more or less dangerous to discharge it into waters which may be used for water- and ice-supplies on account of the possible presence of pathogenic bacteria. As the volume of discharge of most inland streams is small, and as they are in addition likely to be used as water- and ice-supplies, efforts are being made everywhere to prevent the discharge of raw sewage into small fresh water streams or ponds without previous treatment to reduce the organic matter and the number of bacteria.

1932. If the discharge is into salt water, which cannot be employed for domestic use unless it is distilled, no possible harm can be done except to marine life. The sewage, however, should be screened of all floating materials, and then be discharged below the surface into a current which will carry it away from the shores and prevent deposits which will be exposed at low tides. To prevent the latter it is often necessary to construct reservoirs to hold the sewage which reaches the outlet during flood tide, and discharge it only when the tide is at ebb. Such a reservoir is an element of the Boston sewerage system. The principal danger resulting from the discharge of raw sewage into salt water is the infection of oysters by pathogenic bacteria. Cases of infection thus propagated are of record.

1933. **Broad Irrigation.**—Disposal by irrigation consists in applying screened sewage to the growing vegetation of a sewage farm. The greater part of the liquid is absorbed by the vegetation, and the remainder, after filtration through the soil, may be caught in subsoil drains and conveyed to the natural drainage streams of the country. The solids not removed by previous screening are absorbed by the soil in the same manner as manures and fertilizers applied to land. The main objection to irrigation is the great extent of land required; about an acre is required for every 25 to 100 people who contribute to the sewage. The amount of rainfall and the character and porosity of the soil have to be considered in determining the area required. This method of disposal is, however, extensively employed, especially in Europe.

In level country the sewage may be conveyed in parallel troughs about 40′ apart, raised slightly above the surface of the soil. From these

it overflows the ground, properly sloped, on either side, and the surplus is carried off in surface or subsoil drains midway between the troughs. Another method is to shape the ground in alternate ridges and furrows and let the sewage flow in the furrows. It thus reaches only the roots of the plants which grow on the ridges; subsoil drains may be placed beneath the ridges. As the nitrogen of the organic material must be reduced to nitrates before it can be absorbed by the plants, the bacterial action in irrigation must be wholly aërobic if it is desired to avoid foul odors. This necessitates a porous soil and the even distribution of the sewage; the formation of pools must be avoided. The sewage should be applied intermittently if the pores of the soil show any signs of becoming clogged.

1934. **Intermittent Filtration.**—This is a modified form of irrigation in which the sewage is applied to the land, not for the purpose of utilizing the sewage for plants, but rather to purify as much sewage as possible per acre of land. The sewage is applied intermittently with the view of supporting as large a number of aërobic bacteria as possible and thus avoiding the expense of large irrigation farms. The land used for intermittent filtration requires more thorough underdrainage than that used for irrigation. The underdrains assist in ventilating the soil.

The sewage is usually prepared by screening and then by standing for a few hours in settling-basins. The land is divided into a number of beds, some of which are settling-basins and the others are purifying-beds. The sewage is first conveyed to the settling-basins, where the sludge is precipitated by gravity, and from them to the purifiers. From time to time the settling-basins are emptied and the sludge dried by evaporation. When the sludge dries it is raked up and carted away to be burned, buried, or otherwise disposed of. There should always be sufficient beds to allow each purifying-bed to rest dry some time after it is emptied. This process is extensively employed in this country in regions having a sandy or gravelly soil.

1935. **Sand Filters.**—The natural soil may be replaced by specially constructed filter beds of sand and gravel or other material thoroughly underdrained. The area of such beds will naturally be less than the area of natural soil required for purification. The best results are obtained from filters built of sand, the action being to filter out all suspended matter and to cause the oxidation of the organic matter due to the bacteria present in the sand bed. The effluent from the sand filter, if it is properly built and operated, is very clear and relatively pure. The intermittent action is obtained by using a flush tank as described in paragraph 1912.

1936. **Chemical Precipitation.**—The suspended solids which remain after screening are often removed by precipitating them by some chemical, as lime, copperas, or alum. The substances may be employed alone

or with each other. The kind and the amount of the precipitate best suited to the sewage must be determined by analysis of the effluent.

The chemicals are usually dissolved in water in a separate tank and then allowed to mix well with the sewage before the latter is admitted into the precipitation-tank. The process of precipitation may be intermittent or continuous. In the intermittent process there are three tanks, one being filled, one standing full, and the third being emptied. In the continuous tank the sewage charged with the precipitate flows slowly through the tank either in a horizontal or a vertical direction, depositing its sludge on the bottom. One of the favorite forms is the vertical, or Dortmund, tank. This is composed of two cylindrical concentric tubes of equal length, the diameter of the outer tube being about five times that of the inner. To the bottom of the outer tube is riveted an extension in the form of an inverted cone whose smaller end is equal to the diameter of the inner tube. The sewage, charged with precipitate, is admitted at the top of the inner tube, flows down to its bottom, where it is distributed by radial arms through the cross-section of the larger one. It then flows up through the outer tube and is discharged at its top. The sludge sinks to the bottom and is collected at the bottom of the conical extension of the outer tube; from there it is discharged through a discharge pipe by gravity or by pumping.

If water transportation is available, the sludge is usually received in closed boats and carried out to sea. Otherwise it must be discharged into settling-basins, where it is allowed to evaporate. When nearly dry it is molded into cakes and burned.

The effluent of precipitation-tanks is clear liquid, but unless the process also removes the dissolved organic substances it is subject to further decomposition and cannot be discharged into streams without creating a nuisance. The effluent also usually contains a larger number of bacteria than is considered safe for its discharge into streams which are employed as water- or ice-supplies.

1937. **Land Required.**—The amount of land required for the above processes has been determined in England to be as follows:

	Soil	Number of Acres	Number of Persons
Simple irrigation	Stiff clay	1	25
	Loamy gravel	1	100
Simple intermittent filtration	Sandy gravel	1	100 to 300
Irrigation preceded by precipitation	Clay	1	200
	Loamy gravel	1	400
Intermittent filtration preceded by precipitation	Sandy gravel	1	500 to 600
Intermittent filtration in specially prepared filters, preceded by precipitation and followed by irrigation		1	2000

1938. **Contact Beds.**—Contact beds are specially prepared filter beds arranged in series. They were first installed in England to replace a system of disposal by precipitation and irrigation which had proved unsatisfactory.

The settling tanks, which were about 30′ by 50′, were converted into coarse contact beds by filling them to a depth of 3½′ with burnt clay ballast, or clay burned and reduced to 1″ fragments. It was drained by a 6″ underdrain to which were attached, at intervals of 3′, parallel branches 3″ in diameter. The sewage was fed to the bed through a trough supported above it. The fine contact beds were each about 20′ by 40′, and were filled to a depth of 3′ by various combinations of materials screened to pass through a ⅜″ screen. These materials were burned clay, coke, gravel, and different varieties of sand. The sewage was fed to these beds in the same manner as to the coarse ones.

In operating the beds, screened sewage was first allowed to flow on a coarse bed, whose outlet was closed, until the bed was filled to within 6″ of the surface. The bed was then allowed to remain full for two hours. At the end of that time it was emptied, and the effluent was distributed slowly over one or more of the fine beds; this operation took about an hour. The effluent of the fine bed was sufficiently purified to admit its discharge into a stream which flowed by the sewage farm. The beds were allowed to stand empty for at least two hours before being again used. This allowed time to replace the oxygen necessary to the process. As a rule the interval between the filling of each coarse bed was eight hours.

1939. Schedules, for use with contact beds, vary according to the design and rate of sewage flow, the time of resting depending upon the number of fillings. The following schedule may be assumed to illustrate such a cycle with three fillings per day. Filling, 1.0 hour; contact, 0.75 hour; emptying, 0.25 hour; resting, 6.0 hours; total cycle, 8.0 hours.

While porous material like coke is the best for contact beds, especially for fine ones, beds have been satisfactorily worked with broken stone, slag, coal, etc. In some of the more recent beds a series of three beds instead of two has been employed, coarse, medium, and fine; the effluent of such a series is purer than from a series of two.

1940. **Trickling Filters.**—As the oxidizing power of the contact bed is dependent upon an ample supply of atmospheric oxygen, absorbed mainly during the rest period, the purifying capacity of the bed is limited by this period. Furthermore, less than ½ of the cubic contents of the new bed is available for the storage of sewage during the standing full period and this space is gradually reduced by accumulations in the interstices of the ballast. The storage space is an important element governing the volume capacity of the contact bed. These limitations encouraged efforts to devise beds which would accomplish similar work

without being subject to such restrictions. The result was the trickling filter, similar in construction to the contact bed, except that it is not necessarily built within a water-tight tank. The filter material, usually about $1\frac{1}{2}''$ in size and from $5'$ to $10'$ deep, is placed upon a concrete slab with a system of drains through which the water can flow freely. The sewage is applied to the surface of the filter as uniformly as possible by sprinklers or other devices. It is usual to apply the sewage for a few minutes and then shut off for a short time, as 5 minutes dosing and 10 minutes resting.

This construction offers as large an absorption surface as the contact bed. The sewage does not stand quiescent in the bed but trickles very slowly over the stones and has ample opportunity to give up colloidal and dissolved substances to the bacterial jelly. The method of dosing is such that the bed may absorb oxygen continuously instead of during a brief period as in the contact bed; the danger of anaërobic conditions is thereby reduced. Experience has demonstrated that usually the trickling filters are self-cleansing, while the material of contact beds must be removed and cleaned occasionally.

The theory of action is substantially the same in the two types of beds. The bacterial jelly coats the stones from top to bottom of a bed in mature condition and appears to perform its functions in relays, that at the top acting upon so much of the organic matter as its capacity will permit, that of the next stratum attacking the substances passing by the upper one or perhaps emitted from it only partly oxidized, and so on to the bottom, where oxidized sewage falls through the floor gratings into the underdrainage system.

1941. Knowledge of the particular organisms present, and of the work which each species does, is lacking, but it is certain that the processes are chiefly aërobic, for the failure of the air supply is quickly reflected in the inferior quality of the effluent. It is common to attribute the changes to bacteria, which undoubtedly should have credit for much of it, but the presence of vast numbers of higher forms of life such as worms, varying in size from those almost microscopic to earth worms $2''$ or $3''$ long, leads to the conviction that bacteria are not the only organisms laboring to transform the putrescible organic matter into substances less likely to cause offense and illness.

1942. **Septic Tanks.**—The septic tank is an open or closed tank in which raw sewage is allowed to stand for a period of about 24 hours for the purpose of liquefying its organic matter by anaërobic action. It differs from a settling tank only in the length of time the sewage remains in it. In this tank the sewage putrefies, such gases as hydrogen, nitrogen, marsh gas, carbon dioxide, and hydrogen sulphide are given off, and ammonia is formed. A thick scum forms over the top of the tank

and the sludge is deposited in a fine powder. On account of its odors a septic tank creates a nuisance if placed near dwellings.

Although the amount of sludge is less than in the simple settling tank and the tank therefore requires less frequent cleaning, the sludge produced is of a more offensive character.

The effluent of a septic tank usually requires further purification, to reduce the number of bacteria, by passing through filter beds like those of the contact system. Before it reaches the fine filters it is aërated by passing in a thin sheet over an aërating weir. The weir and beds produce nitrification, which is the last stage of every system of sewage treatment.

1943. Septic tanks have been introduced in this country where the factory wastes can be reduced only by anaërobic action, and where the soil is clayey and not suited to intermittent filtration. The tanks are either open or closed; the latter are best in very cold climates. The sewage is admitted some distance below the level of the liquid in the tank, and usually into a grit chamber separated from the main tank by a low wall over which the sewage flows. It is also withdrawn from the tank by a pipe tapping the tank some distance below the surface of the liquid. The most suitable rate of flow through the tank is determined by experiment.

Difficulty has been experienced with the septic tank due to the fact that sedimentation and septic action are taking place in the same chamber, the greatest difficulties being caused by the evolved gases which have objectionable odors and also as they rise to the surface bring up sludge which contaminates the effluent. This led to the development of tanks in which the two processes take place in separate compartments. The most widely used tank of this type is one developed by Imhof, in which the sewage passes into a sedimentation chamber at the bottom of which is an inverted pyramid with an opening at the bottom which allows the sludge as it settles to pass into a sludge chamber below where its reduction takes place.

1944. **Aëration.**—It has been found by experiment that if sewage containing sufficient oxygen is agitated the impurities form a flocculent sludge. Agitation is accomplished by compressed air which has the added advantage of increasing the oxygen content. The oxygen reduces the number of bacteria. As soon as the flocculent forms, the sewage is allowed to stand until it settles out forming a sludge at the bottom of the tank. It has been found that adding sludge to the fresh sewage helps the process. This method is now being developed.

1945. **Disinfection.**—Where it is necessary to greatly reduce the number of bacteria in an effluent before its discharge, it is disinfected with copper sulphate or some chlorine compound in a manner similar to that

described for a water supply. The disinfectants kill the bacteria in the effluent.

1946. **Disposal in the Absence of a Sewerage System.**—When a building is supplied with running water, but is so situated as to make it impractical to connect it with some sewerage system, the liquid waste may be disposed of by one of the following methods: the tight cesspool, the leaching cesspool, or the septic tank.

A cesspool is a masonry tank constructed underground, into which the sewage drains. If there is danger of polluting a source of water supply the tight cesspool is used. It is constructed of masonry or concrete, care being taken to make it water-tight to prevent leakage. It must be cleaned out periodically, care being especially taken to prevent it from becoming so full as to overflow.

If the ground nearby is porous and there is no danger of polluting a source of water supply, the leaching cesspool will be more satisfactory. It is constructed of masonry or brick placed without cement. The solid matter is reduced to a sludge by septic action, the effluent seeping out into the surrounding ground.

A more elaborate method of obtaining the same result is by the use of some form of septic tank. One form, which is easily constructed, is made of three or four large vitrified pipes set vertically in a single column on a concrete base. It is built below the surface of the ground in a pit and in appearance is similar to a deep catch-basin. The effluent is carried through a tile drain, with open joints, away from the tank. This distributes it over a larger area, thus aiding absorption by the soil.

Care must be exercised with each of these methods to see that the effluent does not contaminate a source of water supply.

For more detailed information see:

Sewerage, Folwell
Sewerage and Sewage Disposal, Metcalf and Eddy
American Sewerage Practice, Metcalf and Eddy
Sewer Construction, Ogden
Elements of Sanitary Engineering, Merriman

PROBLEMS.

P. 1901. A city of 30,000 inhabitants is to construct a sewage disposal plant using intermittent filtration on natural soil. The ground is sandy gravel and worth \$150 an acre. Would it be economical to include a precipitation plant costing \$10,000 in the system?

P. 1902. It is found that in a certain street 250′ long, a combined sewer must be 18″, while if separate sewers are used an 8″ house sewer and a 12″ storm sewer will do. All sewers must be at least 5′ below the surface; the cellars are 15′ deep. If excavation and backfill cost \$0.70 per cu. yd. what would be the difference in cost of earthwork?

P. 1903. If a pipe is running 0.7 full what is the relation between its discharge, area of cross-section of stream, wetted perimeter, and velocity, as compared to these functions of the same pipe running full?

P. 1904. A 24″ sewer is to be laid at a grade of 1′ in 1000′. What will be its discharge and velocity when $\frac{3}{4}$ full?

P. 1905. A sewer is to carry 14,000 cu. ft. per hour running $\frac{3}{4}$ full. If the velocity is to be between 2′ and 2.5′ per second what sized pipe is required?

P. 1906. A 16″ pipe runs only $\frac{1}{3}$ full during dry weather. To what grade must it be laid in order that the velocity shall not fall below 2.5′ per second?

CHAPTER XX.

RAILWAYS.

2001. The first railways were plank tramways or wagon-ways which were used in England as early as the 16th century for the conveyance of coal from mines to shipping points. Subsequently a wooden flange was placed on each plank, to keep the flat wheels on the track. Later, as the wood was replaced by metal, the flanges were placed on the wheel as in our latest railway rolling stock.

2002. The first motive power for railway vehicles was the draft animal. The first high pressure steam locomotive was tried in 1804; but it was 1829 before satisfactory results were obtained, at which time a locomotive built by George and Robert Stephenson drew a train weighing 13 tons 35 miles in 48 minutes along a portion of the Liverpool and Manchester track constructed under the direction of these same engineers.

2003. After 1829, railways rapidly increased. However, no standard was maintained, and each company constructed track and rolling equipment practically without regard to other companies. Even until quite recently a narrow gauge of 36″ was not at all uncommon; butn ow the gauge of 4′ 8½″ is standard throughout the United States and almost the entire world.

The United States leads the world in railways, having over 250,000 miles in operation, which is more than one-third of the total in the world.

LOCATION.

2004. The steps in the location of a railway are the reconnaissance, the preliminary survey, and the location survey.

The **reconnaissance** is a hasty but careful examination of the possible routes between proposed termini, for the purpose of finding the best general route and for obtaining such approximate estimates of cost of construction and operation and amount of probable revenue as may be needed to aid in determining the whole broad question of whether or not to build.

The **preliminary survey** consists in tracing on the ground the route or routes which seem best as a result of the reconnaissance. Along this route, levels and distances are measured so that grades may be known. This work is done hastily, but with a fair degree of accuracy.

The **location survey** is the final staking out on the ground of the actual center line of the proposed railway. In the preliminary survey straight lines only are used, but in the location survey the straight lines are connected by curves wherever a change in direction occurs. This work is done with care and precision.

Thus it may be said that the reconnaissance determines the general route, the preliminary survey furnishes the data for planning the definite line, and the location survey marks this line on the ground.

The whole operation of railway location is the securing of data for the design of the railway, the making of this design, and marking it on the ground. It is the most important and the most difficult branch of railway engineering.

2005. Before the location work is started, the maximum grade and degree of curvature should be decided upon, so that in surveying the line the instrument men will know what their limitations are and will locate the line accordingly. When the level man finds the grade running steeper than the limiting grade, he will take a sufficient amount of side notes to determine the amount of excavation or filling that will be necessary to keep the grade within the prescribed limits; and if he finds the amount of excavation excessive he may have to call back the transit man and have the line relocated. This should not happen frequently, as the transit man should know the maximum allowable angle of slope and should locate his line accordingly.

Reconnaisance.

2006. **Maps.**—The character of the reconnaissance depends very much on the length of the line and the territory within which the line lies. The first essential is the best map which may be obtained.

A study of a contour map will show certain possible routes which join the proposed termini. One or more of these may be selected for further study as being the most feasible of the routes of reasonable directness. From such maps, the practicable grade may be approximately determined in ordinary country. This determines the possible **ruling grade** which is the maximum grade which is permissible in construction. The ruling grade often settles the question of the financial success of the railway, as it limits the weight of the trains which may be handled by one engine.

The length of line and probable amount of curvature may be calculated with some degree of accuracy, but the character of the material to be excavated is not known nor are the minor irregularities shown. Riding over the route on a horse or driving with a car along the road, with side trips at various points, will sufficiently determine the respective characteristics of the routes so that a choice may be made from the map.

2007. If there is no contoured map available, the following suggestions will be helpful:

a. A main ridge may be traced by following along between the heads of streams of adjoining main watersheds.

b. Some sort of a ridge lies between the more or less parallel adjacent streams of the same watershed.

c. When two streams flow in opposite directions, there will generally be a low saddle between their sources. This is especially true in mountain ranges.

d. If the general course of a stream is straight, it probably has a rapid fall and fairly steep slopes on either side. If the course is very crooked with long radius curves, sometimes doubling back and seeming almost to run upstream for a distance, the stream slope is probably very flat, and the stream lies in a flat alluvial valley, whose width is probably considerably greater than the extreme width of the stream windings. If the course is fairly straight with sharp bends at intervals, it is probably running from bend to bend alongside fairly steep bluffs or hillsides.

2008. **Procedure.**—With the best map obtainable, the engineer walks over the one or more routes that seem possible or probable and notes at various points along the streams where the line of the railway will lie with respect to those streams. He notes carefully the highest flood level, as the railway must not be below this level. He selects and notes possible crossings on the streams, estimating what structures must be built and what their size will be. He estimates, approximately, the number of cubic yards of earth and rock per mile which must be moved in the grading, and assumes that a certain minimum grade may be used. From his notes he estimates what the line will cost within five or ten per cent.

2009. Often, when a short line is under consideration, no separate reconnaissance of the route is necessary. But a reconnaissance is made when the preliminary survey is undertaken, keeping a day or two ahead of the preliminary survey. Notes will be made for the locating assistant who is to direct the detailed operations of the preliminary party so as to show him where to run the preliminary line. It is better, however, to make a complete reconnaissance of even a very short line so that the reconnoitering engineer may be certain he has his line along the best route.

2010. For the determination of directions in the field on this reconnaissance, the pocket prismatic compass is the most convenient instrument. The aids that may be useful on such a survey are maps, aneroid barometer, hand level or clinometer, pocket prismatic compass, and book of tables of curves, grades, excavation, etc.

2011. If an airplane is available, photographs made from the airplane will aid very materially in determining many of the requirements of the reconnaissance. By placing barometer elevations on critical points on the photographs, it is possible to determine easily the route that would be preferable from consideration of topography only.

2012. In detail, the reconnoitering engineer should note for each of the routes examined:

a. The character of the territory, probable resources in timber, mineral, agricultural, and manufactured products, making some sort of estimate of the approximate business to be immediately developed and the future possibilities.

b. The probable ruling grade that may be obtained, the length of the line, approximate total rise and fall, total amount of curvature with the probable minimum radius, and relative operating value consequent upon these characteristics.

c. The probable amount of earth work and its classification into earth, loose rock, and solid rock; possible timber and rock supplies for ties and structures; value of the land; number and length of the more important bridges; and consequent probable cost of construction.

2013. **Importance.**—The reconnaissance survey is the most important part of railway location; and, as already stated, railway location is probably the most important item of railway engineering. Mistakes in the details of location can be remedied, mistakes in construction will eventually be corrected, and mistakes in management and operation can be overcome by a new policy; but an error in selecting the route for a railway, if not discovered and corrected before construction, will not be rectified for many years, and it will sooner or later put the railway at a disadvantage with its competitors. The mistake may even be serious enough to handicap permanently the operation of the line.

Preliminary Survey.

2014. The preliminary survey, like a reconnaissance, should be a survey of an area and not merely a line, though the area is narrower and more definitely mapped than that in a reconnaissance. The two methods in general use will be briefly described.

2015. **Methods.**—The first method consists in laying out a line as nearly as the engineer can judge where the final line will be, marking the line with stakes at such intervals, usually 200′, as may seem wise, turning such angles as may be necessary, taking the elevations of the ground along the line at the stakes set, and at such other points as may be deemed wise, making a profile and map of the surveyed line, and from these and a knowledge of the material making a preliminary estimate of

the cost. At each stake a cross slope will be taken, or topography may be sketched in a note book with notes that the line should be so many feet to the right or left.

2016. In the second method the line is mapped on sheets on which are written the elevations of the stations. These sheets are prepared in the field by the topographic party, consisting usually of three men. The topographic party with tape and hand level runs a line fairly close to the probable final line and takes measurements from which contours with five-foot intervals are plotted on these sheets. Measurements are taken on either side of the line for a distance sufficiently large to include the final located line. The final located line is plotted on these same sheets. For this second method, the transit and stadia may be used to advantage. In ordinary country, a stadia contour survey can be made and mapped in much less time and with much fewer men than the ordinary line and level survey. The profile of any line that may be drawn on the map by this method is not so nearly correct as the profile of the line obtained by the first method; but the profile furnished by the second method is likely to be as near the final location profile as in the preliminary line and level profile of the first method.

2017. **Estimate.**—The preliminary survey furnishes more definite information concerning the following items for estimates, viz.: the amount and classification of earthwork, the bridges, culverts, road crossings, right of way, and those items depending on length of line, as track, telegraph, fencing, etc.

The estimate will generally be made by miles. The probable kind and size of each bridge or culvert is noted on the profile; the height of the fill is also noted as it determines the dimensions of abutments and lengths of culverts. Large bridges must be estimated separately.

The estimate of earthwork will be subdivided into earth and rock excavation. It will again be subdivided into various types of cross-sections of different quantities or classes of rock. The computing of quantities is done by tables or diagrams.

2018. **Right of way** is land, usually a strip about 100′ wide, on which the track of the railway is centered. The cost of right of way depends on the value of the land taken and the damage done by the taking. In making preliminary estimates it is wise to figure on at least twice the market value for farm land; and in towns or cities the probable costs should be secured from real estate agents.

The width of right of way depends on the track to be laid, whether single or double; on the depth of excavation or embankment; on the possibility of borrow pits within its width. Fences, buildings, and extra tracks must also be considered in determining the width required.

The general width taken is 100′. The profile must be studied for variations of this width, and an ample width purchased to allow for double-track construction; it will often cost more, later, to get a few extra feet of width than the whole original cost of the first width.

Location Survey.

2019. The important parts of the office work of location are the planning of the line and the fixing of the grade on the profile. With the preliminary profile before him to show where the line should be, the chief of party picks out points on the preliminary map through which the located line should pass. In pencil, he draws straight lines which connect these points. Then with thread for straight line and templates of curves of various degrees he experiments with various lengths of straight lines, connected by curves as few as possible and of as small a degree as possible, until he decides on an approximate location line. This he pencils in on the map. If he is not satisfied with the first location line, he may have its profile called off from the contour map and penciled in on the preliminary profile to note the possibilities of a change in line. A new grade line may be drawn in; and if necessary a further fitting of the line on the map may be made.

Tie-lines from the preliminary line which has been run, in the field, to the finally selected location line are then scaled from the map and prepared for the field party. This is called the paper location. These notes are then used in the field in the actual work of marking the line on the ground; this is called the field location. In staking out the center line, the straight lines are first determined and run to intersections with adjacent straight lines. These intersections determine the vertices of the curves which are then staked out as explained later. For location work a transit and 100′ steel tape are usually employed, stakes are driven at every 100′ point, and all stakes are carefully numbered consecutively and marked. The level party follows the transit party, secures elevations and erects a substantial bench mark every 2000′. At each station the level party places grade stakes on the sides with elevations and directions as to cut or fill to guide the contractor in his work.

2020. **Curves.**—A line of railway is made up of curved and straight lengths: the former are called curves and the latter tangents. Railroad curves are usually arcs of circles. They may be either simple, compound, or reverse. A simple curve is one with a constant radius. A compound curve is one composed of two or more simple curves of different radii curving in the same direction and having a common tangent at their point of meeting. A reverse curve is composed of two simple curves curving in opposite directions and having a common tangent at their point of meeting. The name is also commonly applied to two simple

curves curving in opposite directions and joined by a tangent shorter than the usual length of trains running on the line.

A transition or easement curve is a compound curve, or spiral, used at the ends of a sharp curve to lead gradually from the tangent to the main curve.

2021. A curve with a radius of less than 500′ is commonly referred to by its radius; thus, a curve with 250′ radius is called a 250′ curve. Curves with radii longer than 500′ are usually designated by the number of degrees of arc that a chord 100′ in length subtends at the center of the circle. Thus, a 3° curve means that a 100′ chord subtends an angle of three degrees (3°) at the center of the circle. The number of subtending chords (total curvature divided by degree of curvature) multiplied by the length of each chord may be taken as equal to the total length of the curve L.

That is,
$$L = \frac{\phi l}{D} \tag{2021A}$$

where ϕ = central angle, D = degree of curvature, l = length of chord used.

A 1° curve is considered as the basic curve and its elements are shown in tables for use in computing data for curves of other radii. Its radius is 5729.65′ and a 100′ chord of such a curve subtends 1° at the center of the circle. The corresponding functions or elements of any two curves are proportional to their radii and therefore those for a curve of 3° are found by dividing the corresponding elements of a 1° curve for the same central angle, by 3. This rule holds good as long as the 100′ chord and the subtended arc are not sensibly different in length.

Nearly all railway curves have such long radii that it is impracticable to use the center in locating the curve. Therefore curves are laid out by use of instruments on the circumference in accordance with certain mathematical principles. The methods are discussed in the next few paragraphs.

2022. The various parts of a curve are shown in Fig. 2023A. Moving in the direction FBH, FA is one tangent joined to LH, another tangent, by the curve AGL. The angle between FB extended, and BH is called the external, or central, angle (ϕ). B is the point of intersection, usually designated P. I. A is the point of curve (P. C.). L is the point of tangent (P. T.). O is the center of the arc. AL is the long chord (C), KG is the middle ordinate (M), and BG is the external distance (E). AB and BL are called the tangent distances (T). AO is the radius (r). The degree of curvature is D.

The relations between the various angles and lines can be seen from the figure. Various formulas showing the relation of the different parts to one another are given in tables.

2023. The methods of laying out curves vary greatly, and each depends largely upon the local terrain. A few of the simplest are given below. Either the radius or the degree of curvature must be assumed before the points of tangency (points where the curve begins and ends) can be located. If the ground inclosed in the triangle ABL (Fig. 2023A) is clear and open, practically any curve can be run between the two tangents. Sometimes, however, there is some condition that determines

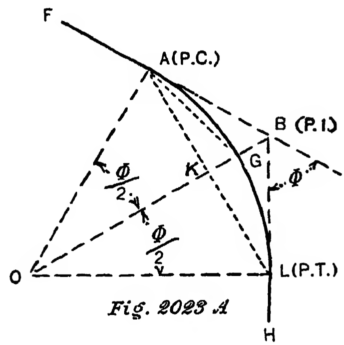


Fig. 2023 A

where the curve shall lie and consequently what the radius of curvature will be. A common condition is that the point G in the curve is fixed by some local condition, thereby fixing definitely the length of the line BG (usually referred to as E).

From Mathematics, we know that

$$r = \frac{E}{\text{ex. sec } \frac{1}{2}\phi} \qquad (2023A)$$

Given r and ϕ, the distance AB and BL can be determined since

$$T = r \tan \tfrac{1}{2}\phi \qquad (2023B)$$

The various related values are given in tables.

2024. **Problem.**—What is the tangent distance of a 3° 10′ curve whose central angle is 16° 26′?

Solution.—Using equation 2023B:

$$T = r \tan \tfrac{1}{2}\phi \qquad (2023B)$$

Substituting values of $\phi = 16° 26'$, and $r = \frac{5729.65}{3\frac{1}{6}} = 1809.4$

we obtain $T = 261.28'$

2025. **Location of Points by Deflection.**—The angle between a tangent to a curve at any point and a chord from that point to any other point of the curve, is measured by one-half the arc between those points; it

is also equal to one-half the angle between the radii to those points. This is the basis for the laying out of a railway curve. Likewise, the angle between two chords intersecting on a point of the curve is measured by one-half of the intercepted arc or by one-half of the angle between the radii drawn to the ends of the intercepted arc. Fig. 2025A shows the above, namely:

$$\angle TLa = \tfrac{1}{2}\angle LOa$$
$$\angle aLb = \tfrac{1}{2}\angle aOb$$
$$\angle bLd = \tfrac{1}{2}\angle bOd$$

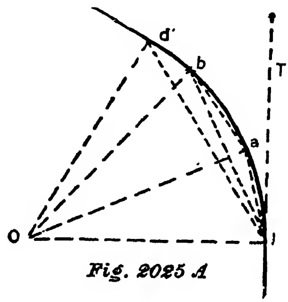


Fig. 2025 A

If $La = 100'$, then by definition the $\angle LOa$ = degree of curvature and the $\angle TLa = \frac{1}{2}$ the degree of curvature. Likewise if the chord $ab = 100'$, then the $\angle aOb = D$ and the $\angle aLb = \frac{1}{2}D$. bd is a subchord subtending the $\angle bOd$, and the $\angle bLd = \frac{1}{2}\angle bOd$. Therefore if a transit is set up at the point L, the point a of the curve may be determined by measuring the proper chord from L in a direction La to a where $\angle TLa = \frac{1}{2}\angle LOa$. The point b may similarly be determined by measuring the proper chord length from a in direction ab to b where $\angle TLb = \frac{1}{2}\angle LOb$. It is generally impracticable to locate all of a curve from any one point, on account of natural obstructions; therefore, the transit must often be moved up to a new station already established on the curve. But the same principles will apply and may be repeated indefinitely. The last point of curve is almost always less than 100′ from the last regular station and must be laid off as a subchord.

2026. **Problem.**—A 4° curve starts at station 53 + 10. The central angle is 18° 40′. Compute the deflections.

Solution.—The first station point is 90′ beyond the point of curve. The subchord angle is therefore $\frac{90}{100} \times 4° = 3°.6 = 3°\,36'$. The deflection from the tangent is one-half of this, or 1° 48′. The deflection for the P.T. is one-half of the total central angle, or 9° 20′. Subtracting 1° 48′, we have 7° 32′ which will allow for three deflections of 2° each and 1° 32′ over, which will require a chord $\frac{1°\,32'}{2°} \times 100 = 76.67'$. The first

subchord is 90′; then three chords of 100′; then a final subchord of 76.67′. The deflections may be tabulated as follows:

Station	*Deflection*
53 + 10	0°
54	0° + 1° 48′ = 1° 48′
55	1° 48′ + 2° = 3° 48′
56	3° 48′ + 2° = 5° 48′
57	5° 48′ + 2° = 7° 48′
57 + 76.67	7° 48′ + 1° 32′ = 9° 20′

which is one-half of 18° 40′ as it should be.

2027. When difficulties of location require that the transit be set up at substations on the curve, the numerical work is calculated on precisely the same principle, but is more involved and chances for errors are greater. The following method of work is as simple as any: Compute the deflections for all stations and substations as illustrated above; set up the transit at the P. C., and locate from it all stations that may be conveniently reached; move up the transit to a forward station, backsight to a previous station with the plates set at the deflection angle for the station sighted at; plunge the telescope and sight at each of the forward stations with the deflection angle computed for that station.

2028. **Laying out a Curve with the Transit, P. I. being Inaccessible.**—It may be necessary to make several changes of direction, as in Fig. 2028A. The point B would be the P. I., but it is inaccessible. The line is run along $AQRL$, RL being the desired direction of the new tangent. The external angle is then equal to the sum of the deflection angles at

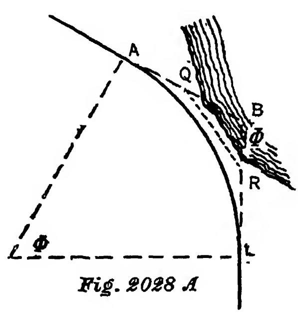


Fig. 2028 A

Q and R. In the triangle QRB, all the angles and the side QR are known. Solve the triangle for QB and RB. Find the tangent distances BL and BA as in paragraph 2023, and lay off from Q, QA equal to BA minus BQ; and from R the distance RL equal to BL minus BR. The points L and A thus located, are the P. T. and the P. C. respectively. The curve is then laid out as heretofore described.

2029. **Laying out a Curve without a Transit, but by Offsets from Tangents.**—(Fig. 2029A). Knowing the P. T., P. C., and r, calculate the tangent distances and offsets. The distance on the tangent LB from the P. T. to the perpendicular offset from the extremity of any arc, La, is equal to $r \sin D$ for the first station and $r \sin (nD)$ for any succeeding

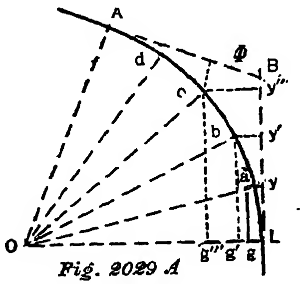


Fig. 2029 A

station (n being the number of the station from the P. T.); and any offset, as ya, from the tangent to the extremity of any arc is equal to r vers (nD). Having the tangent distances corresponding to the consecutive chords for half of the curve and the offset for each, measure off the distances from the P. T. along the tangent and locate each by a peg; then at each peg lay off perpendicular to the tangent the corresponding offset from the column of offsets. This locates half of the curve. Go to the P. C. and locate the other half of the curve from that point.

2030. **Laying out a Curve without a Transit, by Middle Ordinates.**—(Fig. 2030A). P. C., P. T., and r being known, assume some short chord,

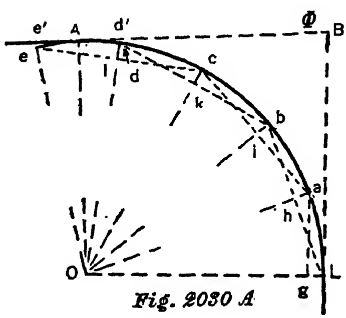


Fig. 2030 A

C', whose ratio to the corresponding arc is practically unity. Using the equation $\sin \frac{1}{2} D' = \frac{C'}{2\,r}$, find D', the angle that this chord subtends for the given radius. Then $\frac{\phi}{D} = N$, the number of such chords in the curve to be laid out.

subchord is 90′; then three chords of 100′; then a final subchord of 76.67′. The deflections may be tabulated as follows:

Station	*Deflection*
53 + 10	0°
54	0° + 1° 48′ = 1° 48′
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which is one-half of 18° 40′ as it should be.

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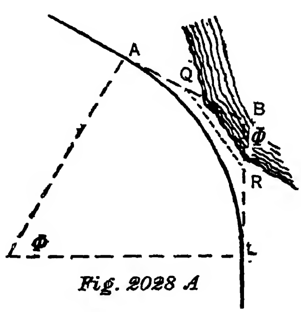

Fig. 2028 A

Q and R. In the triangle QRB, all the angles and the side QR are known. Solve the triangle for QB and RB. Find the tangent distances BL and BA as in paragraph 2023, and lay off from Q, QA equal to BA minus BQ; and from R the distance RL equal to BL minus BR. The points L and A thus located are the P. T. and the P. C. respectively. The curve is then laid out as heretofore described.

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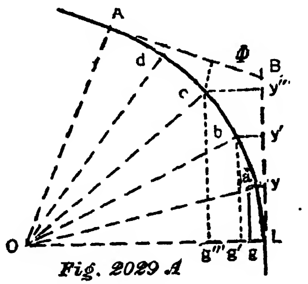

Fig. 2029 A

station (n being the number of the station from the P. T.); and any offset, as ya, from the tangent to the extremity of any arc is equal to r vers (nD). Having the tangent distances corresponding to the consecutive chords for half of the curve and the offset for each, measure off the distances from the P. T. along the tangent and locate each by a peg; then at each peg lay off perpendicular to the tangent the corresponding offset from the column of offsets. This locates half of the curve. Go to the P. C. and locate the other half of the curve from that point.

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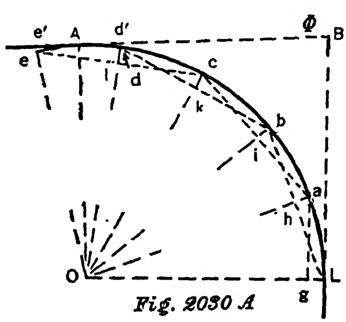


Fig. 2030 A

C', whose ratio to the corresponding arc is practically unity. Using the equation $\sin \frac{1}{2} D' = \frac{C'}{2r}$, find D', the angle that this chord subtends for the given radius. Then $\frac{\phi}{D} = N$, the number of such chords in the curve to be laid out.

Lay off from P. T. toward center the distance Lg, equal to the offset corresponding to the chord C'. Through g extend the line ga parallel to the tangent LB, and with the P. T. as a center strike an arc with a radius C'. The intersection of these two lines is a point in the curve, a.

From a, lay off a distance in the direction of the center, ah, equal to Lg. Extend the line Lh, and with a as a center strike an arc with C' as a radius. The intersection of Lh with this arc is at b, another point in the curve.

If N is a whole number, the P. C. will be one of the points found as above. If N is not a whole number, lay off the whole numbered stations as above until a station, e, is located, just beyond the P. C. Measure Ad and Ae. Find the offset corresponding to each of these chords, dd' and ee', from d and e lay off these offsets, outward from d and e along the directions dO and eO. The line $d'e'$ should coincide with the new tangent and pass through A. This method is only applicable when O can be seen from all points of the curve.

2031. **Grades.**—Grade lines or gradients are designated by the vertical change in 100′. A grade rising of 2′ in a horizontal distance of 100′ is called a plus or ascending 2 per cent grade, commonly written +2.0 grade. Ascending and descending, on an original profile made for construction, refer to the direction in which the line advances.

The grade is one of the most important considerations in railway location. This is evident from consideration of the tractive force of a locomotive, on different grades. The possible draw-bar pull of a locomotive is dependent on the construction of the locomotive and on the adhesion of its driving wheels to the rails. Assuming that its cylinder power is properly designed, the possible draw-bar pull depends on the adhesion of the driving wheels to the rails and may be taken (Frye's Handbook) as one-fourth the total weight on the drivers. This draw-bar pull must overcome the train resistance, which may be taken as 8 pounds per ton of total weight of entire train. (To start, the pull must be about 22 pounds per ton.) On an ascending grade, the draw-bar pull to move the train must also overcome the additional weight, which is the total weight multiplied by the per cent of grade. Thus, on a level the draw-bar must pull 8 pounds per ton, on a 1 per cent grade it must pull 8 pounds plus 1 per cent of 2000 pounds or a total of 28 pounds, on a 2 per cent grade it must pull 48 pounds. (This is deduced mathematically in paragraph 2137.) It is thus seen that the amount of freight that can be hauled on a level is about $3\frac{1}{2}$ times that which can be hauled on a 1 per cent grade and about 6 times that which can be hauled on a 2 per cent grade. A grade of more than 2 per cent is practically prohibitive from an economic standpoint; and much expense in first cost is justifiable in order to decrease the grade to 2 per cent or lower.

Any grade from a 0.0 per cent or level grade to a 0.4 per cent grade is called light; from a 0.4 per cent to 1.0 per cent moderate; from 1.0 to 2.0 per cent heavy; and over 2.0 per cent very heavy. Eastern roads try to get grades of 0.3 per cent or under, while grades of 4 per cent are common in branch lines in the Rocky Mountains. Grades of 6 per cent exist and are operated on some mine and logging roads. The steepest trolley road grades are about 15 per cent.

CONSTRUCTION.

2032. The work of constructing a new railway line or of improving an old one is usually done by a contractor or contractors. When a railway company decides to build a new line, it advertises the estimated amount of work to be done, giving all information known of the character of the work as learned from the preliminary survey. Contractors make bids for this work, naming prices for each item as advertised by the railway company. Bids from all contractors are written, sealed, and mailed to the railway offices. These bids are opened on the day set for that purpose, as indicated in the advertisement, and the contract is awarded. Of those contractors who are known to be reliable, the one bidding lowest for the entire work usually receives the contract. The contract, including specifications for all parts of the work, is then signed by the contractor and the railway company's representatives. A date is set for the beginning of the work.

2033. The **roadbed** is the support prepared for the track. It consists of the foundation and the ballast. The latter should be a material the consistency of which is not affected by water, and especially which does not become slippery when wet. Sand will do if nothing else can be had; gravel is better, and broken stone is best of all. Cinders, shells, burnt clay, and other materials are also used. The surface of the foundation on which the ballast rests is called the subgrade. The total amount of ballast is very great, a single track requiring about 3000 cubic yards of ballast per mile.

2034. The **cross-section** of the roadbed must be decided upon before the level party starts on its work; the level party must know the dimensions of the roadbed for which the stakes are to be set. Figs. 2034A–D show the dimensions of standard-gauge roads. For any other gauge slopes will be the same, but the shoulders outside of the ties will be different. The difference will therefore be the difference in the gauge of the tracks, plus twice the amount that the shoulder can be reduced in width. In excavations, plenty of depth should be allowed for ditches in order to insure a dry roadbed.

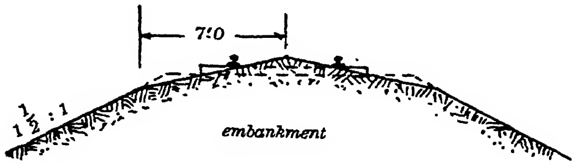


Fig. 2034 A Roadbed Earth ballast

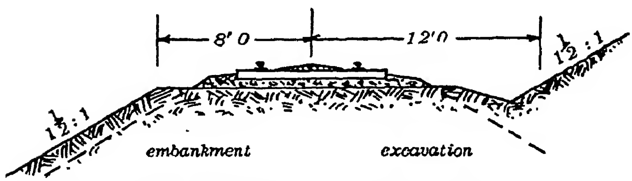


Fig. 2034 B Roadbed Stone or Gravel ballast
Straight line

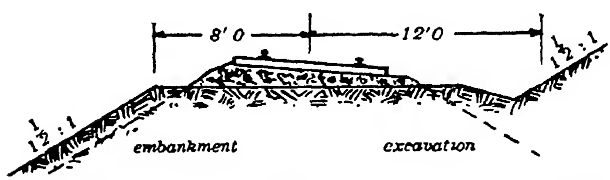


Fig. 2034 C Roadbed Stone or Gravel ballast
Curved line

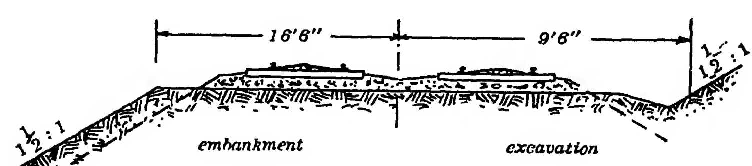


Fig. 2034 D Roadbed Stone or Gravel ballast
Straight line-Double track

Track Materials.

2035. **Ties** are made of various kinds of wood, steel and infrequently of concrete and reinforced concrete. The kind of wood used varies with the locality. Often the best local wood is used by the railways in preference to transporting more suitable wood from great distances. Oak and creosoted pine are quite commonly used. Redwood and fir are largely used in the west and on the Pacific Coast while many soft woods as cedar and tamarack are also used.

Wood Ties.—Wood, for ties, should be hard. It should hold a spike well, be fairly straight-grained and stand the weather well. The ordinary dimensions of wood ties are from 7′ to 9′ long, 6″ thick, and 8″ wide.

Wood ties fail in one of three ways, through decay, railcuts, and spike wear. They fail most frequently by decay. The life of a tie depends on many things, but may be averaged at 8 years.

The life of a tie may be lengthened by the use of metal plates, to protect it against being cut by the rail; by the use of screw spikes as they do not split the tie or cause it to " spike wear " so quickly; and by treatment with some preservative, creosote being the most effective but also the most expensive.

Concrete and Reinforced Concrete Ties.—Concrete and reinforced concrete ties have been experimented with but have not as yet justified their expense. They are not resilient and are particularly subject to fracture when struck by a derailed car.

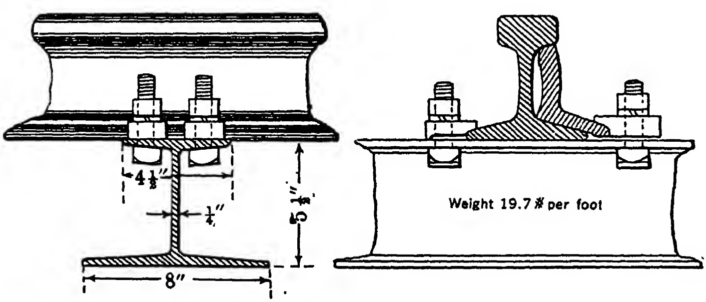


Fig. 2035A Carnegie Steel Tie

Metal Ties.—(Fig. 2035A). Metal ties have been used for about 30 years chiefly in Europe and very largely in Germany. Several railways in the United States have experimented with various forms of metal ties but their use has never passed the experimental stage.

The Carnegie metal tie is probably the best known in America. So far it has not been possible to determine or estimate the life of these

metal ties and until this is done no real comparison as to their economy can be made with wood ties.

The "longitudinal" tie shown in Fig. 2035B is a form of metal tie or rail support largely used in Germany. The track consists of the ties, the rail and the attachments of the latter to the former and to each other.

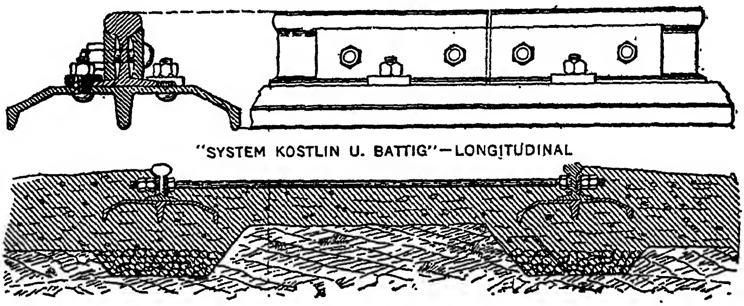


Fig. 2035 B

2036. **Rails.**—Standard rails are 33′ long and are designated by their weight per linear yard. Thus an 80 pound rail weighs 80 pounds per yard.

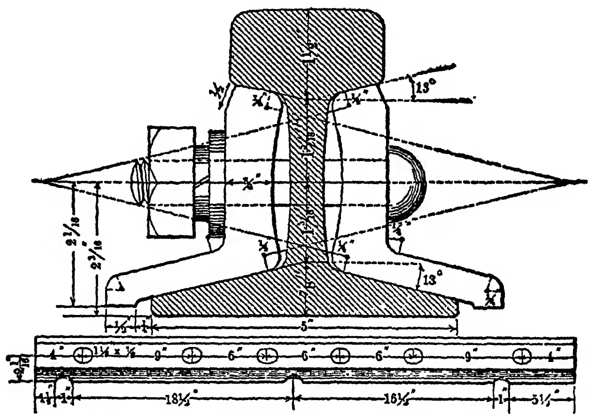
For the very heaviest traffic, as in large city terminals, 110 pound rails are used; for main traffic lines 90 to 100 pound rails are laid; for light traffic 70 to 80 pound rails are satisfactory.

The life of rails cannot be foreseen, as it depends upon the amount and distribution of the loads. Engines are supposed to cause about half the wear of the rails. Rails which would last for many years on an average track may be worn out very quickly in traffic when laid at the entrance to a busy yard; this is because there is more traffic at this place and because a large part of this traffic is engine traffic. The life of a rail may be roughly stated as from 300,000 to 500,000 trains. All things considered, rails should last about ten years on trunk or main lines. In many cases, rails must be removed and replaced because of bent or battered ends before the expected wear has occurred.

2037. **Joints.**—The ideal joint should have the same strength, stiffness and elasticity both vertically and horizontally as the rails joined by it. The joint must also permit longitudinal movement of the rail to allow for expansion and contraction due to changes of temperature. The allowance for expansion should be 0.0000065 of the length of the rail per degree Fahrenheit. The ends of the rails must be held in true surface under the passing wheels so that one or both ends may not be battered.

There are two general types of joints used on American railways: (1) The angle bar; (2) the bridge joint. Of the first type there are several varieties as the ordinary angle bar (Fig. 2037A), the Bonzano (Fig.

2037B), and the 100 per cent joint (Fig. 2037C). The continuous (Fig. 2037D) and the Weber (Fig. 2037E) are examples of the bridge type.



—STANDARD ANGLE-BAR—80-LB. RAIL.
Fig. 2037.

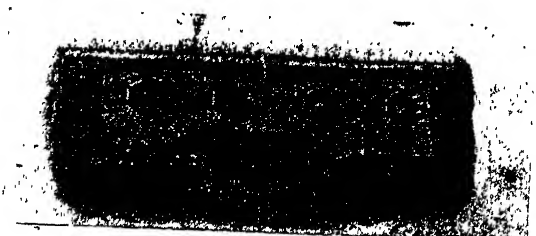
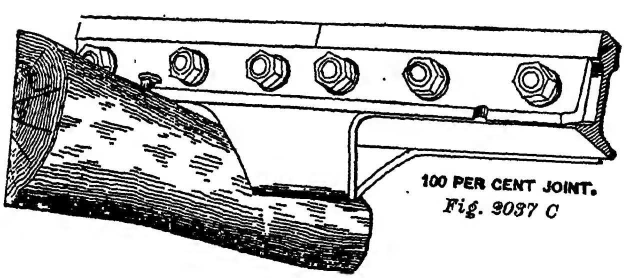


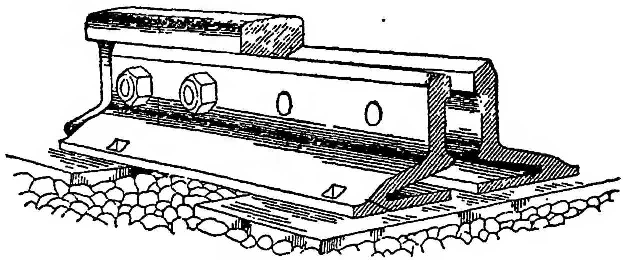
Fig. 2037 B Bonzano Rail Joint



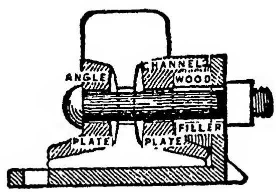
100 PER CENT JOINT.
Fig. 2037 C

The bolts used for fastening joints are oval under the head. Bolt holes are punched oval in shape to fit the oval part of the bolt, thus preventing the bolt from turning when the nut is screwed home. Their diameter is $\frac{3}{4}''$ or $\frac{7}{8}''$ and length from $3\frac{1}{2}''$ to $5''$. The nuts are square or

hexagonal, generally square. A nut-lock similar to a lock washer holds the nut in place. Joints are placed opposite or staggered (alternate); the latter arrangement is most commonly used.



CONTINUOUS RAIL JOINT.
Fig 2037 D



WEBER RAIL JOINT

Fig. 2037 E

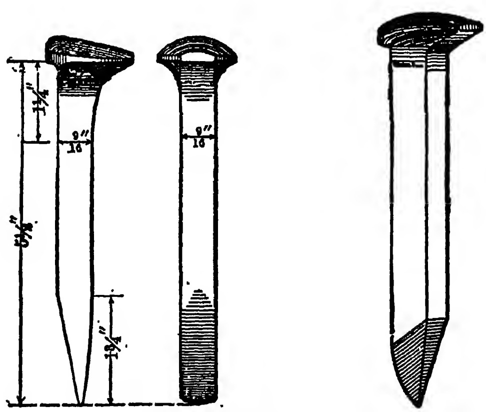


Fig. 2037 F

Spikes hold the rails to the cross-ties. There are two forms: hook-headed spikes and screw spikes. Screw spikes are largely used in Europe but infrequently in America. The common spike is $\frac{9}{16}''$ square by $5\frac{1}{2}''$

long with a head which is hooked or overhung to catch the flange of the rail (Fig. 2037F.)

Tie Plates are metal and are placed between the rail and the top of the tie. They lengthen the life of the tie by preventing the rails from cutting into the wood. Tie plates are made $\frac{1}{2}''$ thick and $6''$ wide. (Fig. 2037G.)

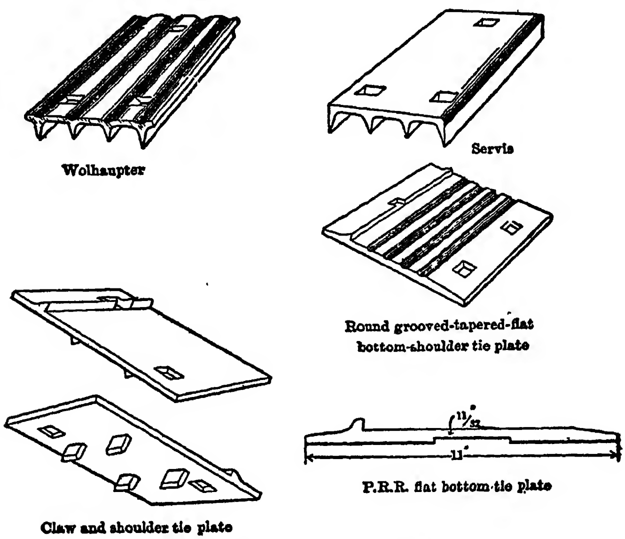
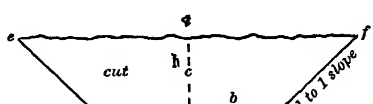


Fig. 2037 G · VARIOUS FORMS OF TIE-PLATES.

Earthwork.

2038. **Cross-sections.**—The ordinary forms of cross-sections are as shown in Figs. 2038A to F. In determining cross-sections, enough dimensions must be taken to define the line of the ground surface. There



Level Section
Fig. 2038 A

are many different varieties of sections such as level, three-level, five-level, irregular, cut and fill, based upon the number of level points used in the section, the surface being assumed straight between these points. Compound sections are sections where materials of different classification

occur in the same section. The ordinary classifications of materials in a section are earth, loose rock, and solid rock.

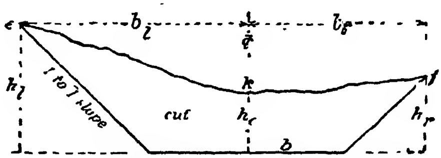


Fig. 2038 B Three Level Section

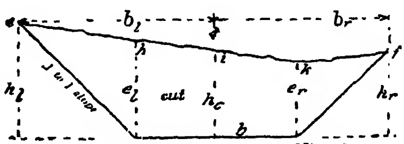


Fig. 2038 C Five Level Section

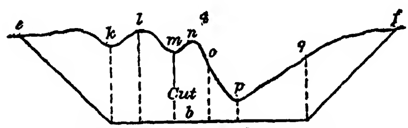
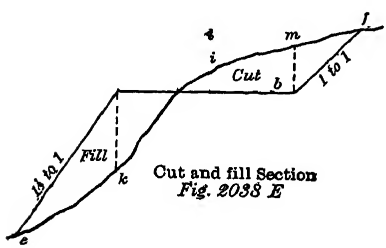


Fig. 2038 D. Irregular Section



Cut and fill Section
Fig. 2038 E

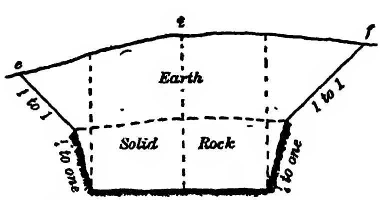


Fig. 2038 F Compound Section

2039. **Methods of Computation of Earthwork.**—To compute the volume of earthwork between two parallel cross-sections, let A_1 and A_2

be the end areas, A_m the area of a section midway between the ends, l the length, and v the volume.

By Average End Areas $\quad v = \frac{l}{2}(A_1 + A_2) \qquad$ (2039A)

By Prismoidal Formulas $\quad v = \frac{l}{6}(A_1 + 4\,A_m + A_2) \qquad$ (2039B)

2040. **The areas of sections** must be found in order to use the above formulas. Let b = base, h_c = center height, s = side slope (horizontal by vertical). Let h_r and h_l be side heights, b_r and b_l be side distances. Then,

(Fig. 2038A), For a level section $A = h_c\,(b + sh_c)$

(Fig. 2038B), For a three-level section $A = \frac{1}{2}\left[\frac{b}{2}(h_r + h_l) + h_c(b_r + b_l)\right]$

(Fig. 2038C), For a five-level section $\quad A = \frac{1}{2}(h_c b + e_r b_r + e_l b_l)$

where e_r and e_l are the intermediate heights. Irregular sections are usually plotted on cross-section paper and their areas are determined by the planimeter.

2041. **Haul** is usually computed as so many cubic yards hauled 100′. The average length of haul is the distance between the center of gravity of a mass of earth in cut and the center of gravity of the same mass after it has been deposited in fill. The distance, in feet, times the quantity hauled, gives the haul in units of cubic yards, hauled one foot.

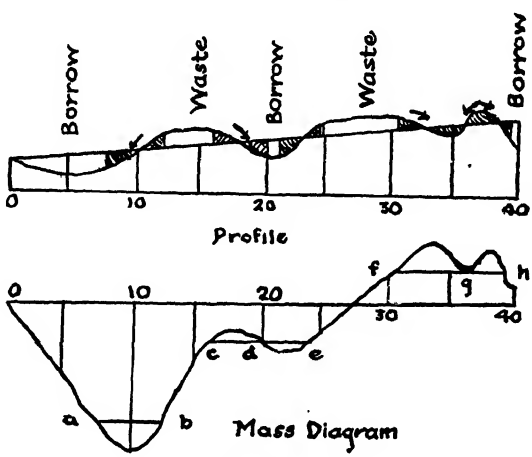


Fig. 2041 A

The Mass Diagram (see Fig. 2041A) gives the best means of measuring haul and of making studies of different schemes of haul and of excavating and filling. The figure represents a portion of a railway profile and its mass diagram which is plotted as follows: At station 0,

the ordinate to the curve of mass diagram is zero; at station 10, the ordinate represents the algebraic sum of cut and fill between station 0 and station 10; at station 20, it represents the quantity from station 0 to station 20; similarly, at any station the ordinate represents the algebraic sum of all the quantities from station 0 up to that station, excavation being considered positive and embankment negative. In this mass diagram it is seen that from station 0 to station 28 the cut and fill balance each other, and that from station 0 to station 40, there is a little excess of cut.

2042. Knowing the volume of each cut and fill, the number of cubic yards may be written on the profile below each, and from the resulting diagram the most economical direction of haul of the material can be determined. If a fill is so far from the adjacent cuts that the material cannot be economically made from these cuts, it is made from a borrow-pit which is secured in its vicinity; if the earth from a cut cannot be economically placed in one of the adjacent fills, it is placed in a spoil-bank on ground secured in its vicinity.

Earth excavated from a cut will shrink in volume about 10 per cent when placed in an embankment which is thoroughly compressed. Rock excavated from a solid ledge will expand in volume about 50 per cent when placed in an embankment. The volume of the earth fills must therefore be increased 10 per cent and that of rock must be decreased 50 per cent, in the diagram above described. In contract work the specifications should state the amount of compression or expansion which is to be considered in computing volumes.

2043. A study of the mass diagram brings out the following properties:

a. High and low points of the curve occur at the points of no cut and fill on the profile.

b. Descending lines denote embankment; ascending lines denote excavation.

c. The difference in length algebraically between any two ordinates is the measure of the excess of cut or fill between the stations at which the ordinates are drawn.

d. Excavation equals embankment between any two points where a horizontal line intersects the mass diagram (as *ab* and *cde*).

e. The area between a horizontal line and the curve of the mass diagram is a measure of the haul between the two stations where the horizontal line cuts the diagram.

2044. **Execution of the Earthwork.**—Having cleared the right of way of trees and brush, the material of the cuts is loosened by plows, excavated by hand or by machinery, and moved to the fills in wheelbarrows,

drag-scrapers, carts, wagons, wheeled scrapers, or dump-cars. Steam excavators are employed when the cut is deep. Barrows and drag-scrapers are employed when the material is to be moved less than 200′; carts, wagons, and wheeled scrapers when the material is to be moved less than 600′; cars are employed for removing the material more than 600′, especially if the amount is large.

The side slopes of excavations are preferably 1 on 1; but in making deep excavations it is frequently advisable to vary this normal inclination of the side slopes to meet local conditions. If very hard material is encountered, as gravelly soil, the inclination may be increased; if on the contrary loose sand or soft clayey earth is encountered, the inclination must be reduced. If it is necessary either on account of the cost or on account of encroachment on private property to maintain the material at a slope steeper than its natural slope, the side slope of a deep cut may be terraced by leaving horizontal benches a few feet wide at vertical distances of 10′ or 12′; this will usually localize the sliding of the material of the slopes. The slopes may also be revetted, or supported by retaining-walls on either side of the roadway.

Track Laying.

2045. **Final Center Line.**—As soon as the roadbed is completed, the transit party runs over the line and sets center stakes 100′ apart on tangent and 50′ apart on curves, and marks again the center of the track by tacks in the heads of the stakes. The road is then ready for the track laying gang. This center line must be accurate, as bridge and track work will begin simultaneously.

2046. **Track Laying.**—The track laying party now follows, and the plans for all preceding work should be such that the track laying can go on continuously. The rapidity of track laying is governed by the rate of supply of material at the working point. This can be accelerated on sidetracks and on double-track work by unloading ties, rails, and other material directly from cars on the track alongside.

On new work, however, the rails, ties, and all other materials must be distributed along the line by the use of wagons or must be carried on a train and the track built ahead of the train, the ties and rails, etc., being unloaded and placed in position by laborers.

2047. On ground that is fairly level and free from streams the method of distributing materials from wagons is the most rapid. The following example gives a working force and its distribution of labor which in actual work laid an average of 4.27 miles of track per day of 10 hours during the month of August. The force was distributed as follows:

Hauling Ties	75
Loading Ties	28
Distributing Ties	10
Spacing Ties	4
Lining Ties	2
Marking and Placing Joint Ties	4
Unloading Flats and Loading Iron Cars	24
Unloading Iron Cars and Placing Rails	13
Hauling Rails	3
Spikers	38
Nippers	19
Strappers	12
Distributing Spikes	2
Lining Track	8
Total	242 men

The ties were unloaded from the construction train, placed in wagons, hauled to the front; and then thrown into place, lined and spaced by the first 123 men in the above list. The rails were unloaded from the construction train on to small rail cars, hauled to the front and unloaded from these cars and placed on the ties by the next 40 men of the table. Rails were spiked to the ties, the bolts put in, and the track lined by the remainder of the crew.

The method just described is the most rapid that can be followed, but is more expensive and therefore is not generally followed in commercial lines. In the second method, all the track material is carried on a train and distributed ahead of that train. The ties, rails, etc., can be carried by hand from the cars to their place on the roadbed, or use can be made of a track laying machine.

Bridges. (See also the chapter on bridges.)

2048. The loads which must be carried by railway bridges are very heavy, and a special design is usually required for each bridge. For wide and deep crossings, truss bridges are generally necessary. Suspension bridges are unsatisfactory, principally because of lack of stiffness. In crossing navigable streams, the requirements of navigation make necessary a long central span; this naturally makes it necessary to place the piers near the bank, which has one advantage in that the depth to a good foundation is usually less.

2049. Standardization of railway bridges is usually impracticable because of the different lengths of spans and consequent different sizes of members. It is not often practicable to make spans of standard

length for different bridges, the principal difficulty being the impracticability of placing the piers at equal distances apart. However, some standardization is being obtained, even though it makes greater difficulty in building piers, as the advantages of standard sizes for repair and first construction render this system advisable in some cases.

2050. Trestle bridges are quite common in railway construction over shallow and dry ravines. They are easy to construct, and the expense is small.

Quite often, a portion of a low trestle is replaced by an embankment, a procedure which is generally unsuitable with a bridge. While the relative cost is very variable, depending on the local price of timber for the trestle, the proximity of a sufficient supply of available filling, and the methods to be employed, yet as an approximate figure it may be said that depths as great as 25′ may be filled with earth as cheaply as a trestle can be reconstructed. In practice, however, it is noted in addition that the average amount of timber required annually for repairs of trestles is about one-eighth the volume of timber in the trestles, also that the labor involved in maintenance of trestles is very great while it is almost insignificant for an embankment, so that the height at which it becomes economical to fill with earth instead of reconstructing the trestle increases until it may reach 50′.

TRACK MAINTENANCE.

2051. Constant attention is necessary to keep track in good condition. Special care must be taken to keep all ditches and drains clear, and to deepen them rather than to allow them to grow shallow; to keep spikes and bolts tight, rails in line and grade, and the ends of ties solidly tamped; and to remove worn out or broken rails and ties.

Tamping in maintenance is a slightly different operation from tamping new track, and other tools are used. The space to be tamped is that between the ties and a trough in the well packed ballast. The tie is nipped up, as in new work, and the tamping is tight under the rails and only snug at the middle. Surfacing will be required in the spring if the track is on dirt or gravel ballast, and in any kind of ballast if the drainage is not good and the frost is deep. If there are especially bad spots on the section, they should be attended to first; otherwise, it is best to begin at one end and to work continuously to the other. Men are sent ahead to set up bolts, nip up ties, and set spikes, and, if necessary, to gauge the track so that when the surfacing gang follows it has nothing to do but line and tamp. With tamping bars, two men should work on a tie opposite each other, striking simultaneously.

2052. To renew a tie, the spikes are drawn with a claw bar, material is dug out under the tie until it drops clear of the rail, a pick is struck in one end of the tie and it is drawn. The new tie is slipped in its place, spiked and tamped.

To renew a rail, the new one is placed alongside the old one. The inside spikes are drawn, and when all is ready the old rail is slipped out and the new one is lifted into place, and spiked.

2053. On military roads it is very important to have the most complete facilities quickly available for removing wrecks and repairing extensive damage to bridges and track, such as would be the result of successful raiding expeditions. Civil roads have wrecking trains prepared to start at short notice. They are manned by regular employees taken from shops and other places, the regular work being interrupted.

Wrecking and repair trains are, in part, identical, but they differ enough to make it advisable to have one of each made up, loaded and manned ready to start at a moment's notice. These trains should stand on a double Y, from which they can pull out on the main line, headed either way without delay. There should be one at each division terminus, and it should work half the length of a division in each direction. If the trouble is serious, a train may be sent from each side, but in any case a locomotive should be sent to the wreck from the side opposite the wrecking train.

RAILWAY AUXILIARIES.

2054. A complete railway requires many engineering works in addition to the main line. These auxiliaries are necessary for the proper operation of a railway, and they are always included in the estimate of total cost of the railway construction. The most important of these auxiliaries are:

Switches
Yards and Terminals
Structures

Switches.

2055. A device for connecting auxiliary tracks to main tracks or to each other is called a switch. It consists of several parts, which are shown and named in Figs. 2055A and B.

There are three general classes of switches:

a. Stub switches, in which both main line rails are cut.
b. Split or point switches, in which but one main line rail is cut.
c. Special switches, in which neither main line rail is cut.

The **stub switch,** Fig. 2055A, is the oldest and simplest form, but has disadvantages, especially for high speed traffic, which have caused its virtual exclusion from main line tracks. A train trailing the switch, — approaching from the direction of the frog, — must be derailed if the switch is misplaced, while trains from the other direction are likely to be derailed if the switch is not completely closed for either the main line or

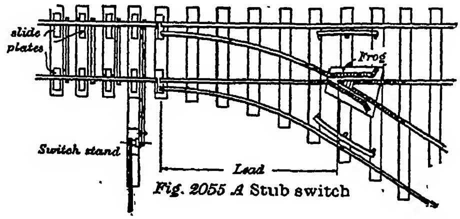


Fig. 2055 A Stub switch

siding. The switch rails have one end spiked to the ties for a short distance and are straight when in the main line. When the switch is thrown, these rails take a curve which assists in the change of direction. The deflected part is not spiked and has no support except what it receives from the spiked ends and from the tie-rods (Fig. 2055A), of which there are usually four.

The **split switch** (Fig. 2055B) has one rail of each track continuous and spiked. The moving rails when in position rest against and are supported by the continuous rails. If the rail section is heavy and the lead rails short, they are usually jointed at the point where the spiking ends.

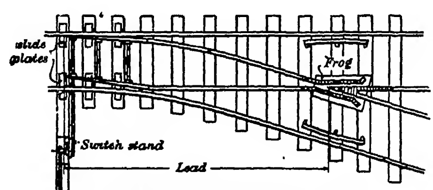


Fig. 2055 B Split or point switch

Special switches which leave both the main rails unbroken have the lead rails elevated above the main rails at the point of crossing, enough to permit the wheels to pass over the latter without striking their flanges. Parts of the lead rails move laterally so as to join over the main rail or clear it entirely.

Yards and Terminals.

2056. A yard is a number of sidings and spurs, usually parallel to each other, although often not parallel to the main track. These auxiliary

tracks must be sufficient in number to permit the convenient and rapid breaking up of trains, classification of cars by contents, destination, or otherwise, and making them up into new trains in accordance with the new requirements. Yard tracks are divided into groups according to their purpose. A certain number near the main line at one end of the yard are called receiving tracks, and trains arriving pull in on them. In convenient proximity is a caboose track, where cabooses are stored when not in trains. A group of repair tracks are convenient to the shops, and the engine track leads to the engine house, near which should be the coal and watering stations and the ash-pit.

All yard tracks except repair and team tracks should be open at both ends, so that all traffic over them may be in the same direction. This permits all traffic through the yard to be in one direction, which saves much confusion and delay. The standard method of arranging yard tracks for greatest convenience and compactness is by the use of ladder tracks (Fig. 2056A), which are oblique tracks at the proper distance apart to accommodate the other tracks between them. Each receiving, distribution, and departure track connects with a ladder track at each end by an ordinary switch. The dead end of each spur track should be provided with a bumper to prevent cars running off.

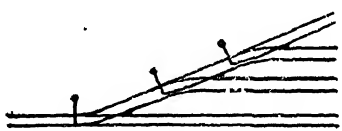


Fig. 2056 A Ladder track

The train on the receiving track is broken up and its cars are switched onto the distribution tracks, selected so that the cars on each of these tracks will belong in the same outgoing train. There should be enough distribution tracks to permit the convenient classification of freight cars according to their contents and destination, and of passenger cars according to character, as baggage, express, coaches, tourist, and standard sleepers. In most cases, outgoing trains are completed on the distribution tracks and pulled from them onto the main track, when authorized by order to do so. It is better, when possible, to have a third group of tracks at the other end of the yard, which may be called the departure tracks. When a train is completed on a distribution track it may be pulled out onto a departure track, where a caboose and engine are added, the designated crew takes charge, and the train is ready, on receipt of orders, to pull out on the main line without delay and without crossing or interfering with any of the yard traffic. Departure tracks permit the

distribution tracks to be fewer and somewhat shorter. If there are departure tracks, they and the receiving tracks should be divided into two groups and designated for traffic in opposite directions. Trains which go through or return without change go direct from the receiving tracks to a distribution or departure track, the returning trains first going through a loop or Y to turn them around.

2057. **Gravity or " Hump " Yards** are simply yards on a down grade in which the cars are started by a locomotive and then run by force of gravity to their desired position. Theoretically the movement of all cars through a yard should be forward, but this condition would require a layout somewhat like Fig. 2057A. The lead *A* runs into a receiving

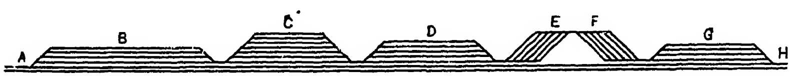


Fig. 2057 A Conventionalized terminal layout

yard *B*. The train is broken up and the cars are classified as to destination, etc., on the tracks of the yard *C*. The cars are grouped into trains in yard *D*. The train is sorted and arranged in order, as required by regulations, in yards *E* and *F* and is then run out into yard *G* ready to proceed. Such a yard layout is not altogether practical, but an understanding of the uses of each part of this layout may assist in planning other yards.

2058. **Freight** will usually go into storehouses or into compact piles at designated points for temporary cover, pending issue. **Platforms** will be used to the degree to which time and materials at hand permit; but the main reliance for discharging cars will be **ramps** (sloping board walks) of suitable form to lead from the car floor to the ground, which should be provided in profusion and so distributed that it will be next to impossible to set out a car at a time or place where a ramp cannot be procured within easy carrying distance.

Storehouses should be narrow and long enough to permit all the cars of a train to discharge simultaneously. If there are no houses, the ground on which the supplies are discharged should be of similar shape for the same purposes.

2059. **Storage Tracks.**—The number and length of these tracks depend upon the importance of the station, the number of depot storehouses that may be located at that point, etc. A study of these items shows that each question must be dealt with as it arises; and, as it is impossible to tell how important a station may become at any future time, available track room will be left for additional loading platforms and storage tracks. In locating the storage tracks, room should be left between the various tracks so that at least one and preferably two lines

of wagons can drive in and receive freight from cars on the storage tracks.

Structures.

2060. **Stations.**—The number of stations, or sidings, that will be necessary depends upon the number of trains that will be run and the speed at which such trains can move. The distance between sidings will vary according to the grades, but having the profile in mind and knowing the speed of the locomotive in miles per hour for certain grades and loads, the distance apart of sidings can be determined approximately as follows:

Let N = the total number of trains necessary to pass over the line in a day;

T = the time in hours necessary for a train to pass from any station to the next one, and return;

Then if $24/T$ is less than N for any two stations, some measure must be taken to decrease this maximum value of T. This can be done by putting in an intermediate siding or by installing extra locomotives in places where they can assist in increasing the speed. The next alternative is double tracking. Sidings should be spaced so that the time required for a train to cover the interval both ways between all adjacent sidings will be approximately equal. Four miles is about the minimum allowable distance between sidings except under very special conditions.

The amount of construction work that must be done at any station depends upon the location of the station, the number of buildings, and the length of auxiliary track that will be needed at that station to suit the local conditions. The questions of water supply and storage room at a station must be especially considered.

The location of stations may be determined by any number of causes beyond the control of the locating engineer, but where such conditions do not fix the station, and the geography of the ground does not compel the location at any certain point, a study of the grades will be made and the stations located where there are no grades. The amount of siding, or passing tracks, that will be necessary at a station will have to be figured in each special case, and the minimum amount should be about two and one-half times the maximum train length contemplated on that division. The passing tracks are usually parallel to the main line.

The station sites should be as level as practicable, and if possible so located that the yards can be seen from trains approaching from either direction. An open space should be left near the tracks to facilitate the loading and unloading, by allowing a systematic and orderly arrangement of loading vehicles in the immediate neighborhood of the station.

2061. **Platforms.**—On every train length of siding there should be a platform at least 12′ wide and long enough to load three or four standard cars at one time. The tops of these platforms should be on a level with the bottoms of the car floors, and the platforms should have ramps at both ends leading to the ground and one in the middle, if the other tracks permit. These platforms are particularly useful in loading and unloading animals and vehicles and facilitate the handling of freight.

2062. **Coal Stations.**—The rapid and easy coaling of locomotives requires a gravity supply. This is obtained by raising the coal either by hoisting it into pockets or by running the coal cars up an incline and dumping the coal into bins. One coal station should be located in about 60 to 70 miles of track and at division terminals.

2063. **Roundhouses.**—At division terminals temporary roundhouses will be constructed, or storage tracks will be laid, for the proper cleaning and repairing of engines. A turntable is desirable at such points, but if the number of engines is not great and no roundhouse is available, a few parallel tracks and a Y (Fig. 2063A) for turning engines will answer the purpose of a roundhouse with a turntable.

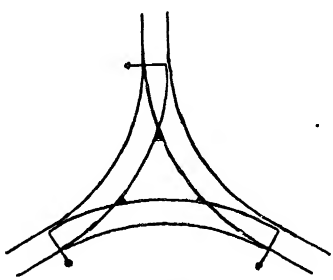


Fig 2063 A "Y"

2064. **Buildings.**—The buildings at the station may be constructed after the line has been finished. The railway official in charge at each station must see that there is sufficient storage room for all railway purposes, so that under the most adverse circumstances there will be no excuse for leaving material stored in cars. Buildings should be rented temporarily in emergency for this purpose when not enough are available.

2065. **Shops.**—At division terminals there would also be located machine shops large enough to keep in repair all the locomotives that would be stabled in the roundhouse, and car repair shops large enough to make all necessary repairs to cars damaged on that division.

For more detailed information see
Railroad Construction, Webb
Military Railways, Connor

PROBLEMS.

P. 2001. What is the difference in length between a chord 100′ and its corresponding arc on a 1° curve?

P. 2002. Prove the principle of paragraph 2025.

P. 2003. Prove the principles of paragraph 2028.

P. 2004. Prove the principles of paragraph 2029.

P. 2005. Prove the principle of laying out a curve by middle ordinates.

P. 2006. Show method of laying out curves by offsets from the long chords of the curve.

P. 2007. Deduce an equation for the relation between the total possible weight of a train and the total weight on the driving wheels.

CHAPTER XXI.

HIGHWAYS.

2101. A **highway** is an open way or passage for public travel, forming a communication between two places some distance apart.

A path or track over which a person can travel on foot is the simplest form of highway. A line having been marked out or "blazed" between two places is soon beaten into a well defined path by constant use. A person traveling over a highway like this will find nothing but a beaten path on the surface of the ground, with few or no modifications of its surface, and generally with no conveniences for crossing the streams or rivers which intersect it.

As the travel over a highway of this kind increases and beasts of burden begin to be used for packing the merchandise, baggage, etc., which are to be carried over the route, modifications and improvements of the path become necessary. For convenient passage of the animals, the path must be widened, the brush and undergrowth removed, temporary bridges constructed or means of ferriage provided for crossing streams of any considerable depth, and steep ascents and descents must be modified and rendered practicable for the pack animals. The term "trail" is used to designate the original path and also the path when improved so that it can be used by pack animals.

Since transportation by wheels is cheaper and more rapid than by pack animals, the next step will be to improve the road still further so that vehicles on wheels can pass over the route. This necessitates a still further widening of the trail, a further reduction of the slopes so as to render them practicable for carts and wagons, the providing of means to cross the streams where they cannot be forded, and the raising of the ground in those localities where it is likely to be overflowed. In this condition the trail is called a "road."

As the travel over this kind of road increases, the wants and conveniences of the community demand a further improvement of the road so that the time taken in going over it and the cost of transportation shall be reduced. This is effected by shortening the road where possible, by reducing still further the ascents and descents or by avoiding them, and by improving the surface of the road. The ultimate result of continued improvement is a paved road or city street.

2102. **Definitions.**—A **street** is a highway of a village or city designed for general use; an **alley** is a narrow highway designed for the use of the owners of abutting property.

A **road** is a country highway which connects villages and towns. The **roadway** is that part of a highway designed for the use of vehicles. The **sidewalks** are the parts of a highway reserved for the use of pedestrians.

A **pavement** is a layer of hard material placed on the surface of a roadway or sidewalk. The term is also applied to the block or sheet coverings forming the hard surface of the complete pavement.

Gutters are shallow depressions along the sides of a street. They are usually paved to catch the surface drainage of the street and conduct it to the sewers.

A **ditch** is a deep drain along the side of a road. It catches the drainage of the road and of the adjoining lands which slope towards the road and conducts it to the nearest water course intersecting the road.

A **curb** is a border of stone or concrete which supports one side of the pavement or gutter of a street and separates the roadway and sidewalk.

The **grade** is the rate of rise or fall of the highway. It is usually indicated by its rise per hundred feet assuming the grade to be uniform for this distance. A grade of 1/100 is called a 1 per cent grade; a grade of 2/100, a 2 per cent grade, etc.

A **cross-section** of a highway is the intersection of its surface by a plane normal to its center line.

The **crown** is the rise in cross-section from side to center of the roadway.

The **profile** is a longitudinal section of the highway, generally taken along the center line. It is the natural profile of the country over which the road passes, modified by **cuts** and **fills** to reduce the irregularities in the profile and the inclination of its slopes or grades.

DESIGN.

2103. The object to be obtained in the design of a highway is to provide a hard surface, so constructed and supported that it will be durable under the action of the traffic which it bears. In addition, it is also desirable that the highway be smooth without being slippery, noiseless, dustless, elastic without losing its shape, impervious to water, free from decaying substances, and well drained so as not to be seriously affected by freezing temperatures.

2104. The design of a highway depends upon the requirements of the highway and the money available. For example, a country highway bearing a small number of animal-drawn vehicles would not have a paved surface or a specially prepared foundation, whereas a main high-

way having a large automobile traffic must have a paved surface and a strong foundation.

2105. **Cross-section.**—As a result of the different requirements of the highway and different amounts of money available, there are many different designs of highways, as illustrated by the figures in this chapter. These designs shade into each other, each approaching the perfect highway as closely as the money available will permit. Fig. 2105A shows an asphalt road and illustrates the important elements in a complete highway.

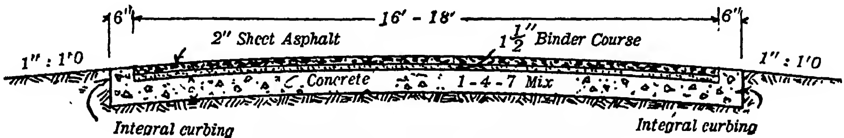


Fig. 2105 A Sheet Asphalt Road

Subgrade.

2106. The subgrade is the base on which the road material rests. In case of an earth or a sand and clay road, the subgrade is practically the same as the bearing surface of the road.

2107. The subgrade must have sufficient bearing power. For an earth road the subgrade itself furnishes directly much of the bearing power, and it is easily seen that the bearing power must be quite strong so as to support the wheel load on the surface pressed by the wheel. For a road with large stones laid as foundation, the subgrade need not be so strong, as the bearing surface is large under each stone and also the several stones and surface materials are so placed as to transmit the load to several of these bearing surfaces at the same time. For a road with concrete as a foundation, the subgrade can be still weaker, as the load is transmitted over a still broader bearing surface.

Drainage.

2108. The paramount question to be dealt with in road design is the drainage, as this primarily affects the supporting power of the subgrade and hence of the entire road. The supporting power will be a maximum when the soil is sufficiently damp to compact well and yet not wet enough to yield considerably under the pressure. Theoretically, it is not desirable to remove all the moisture from the soil, because if this is done it loses its compacting power, and any particles dislodged from cohesion to adjacent ones remain on the surface in a friable condition, refusing to reunite under pressure until moisture is supplied. However in practice an effort is usually made to remove all water and keep it removed.

The water to be disposed of in connection with any road is: (1) that

which flows toward the road from adjacent slopes; (2) that which falls on the surface of the roadbed; (3) that which finds its way beneath the surface, commonly called **ground water.**

The supporting power of the subgrade is increased in two ways: first, by removing the surplus water and keeping it out; and second, by introducing rigid material, or a combination of materials, which will afford a proper bearing surface and so distribute the pressures as to reduce them below the supporting power of the wet soil. The application of methods involving one or both of these principles constitutes the bulk of road work, whether of construction or repairs.

2109. **Side Ditches.**—Surface water flowing toward the road is intercepted and carried off by ditches along one or both sides of the road, according to the direction from which the surface water comes. If on a side hill, the water is carried under the road — across it, in some cases — and discharged down the slope, preferably in a gully or a natural drainage line. The best form of side ditch is shown in Fig. 2109A. Its advantages are that it is favorable for a variable flow of water at relatively

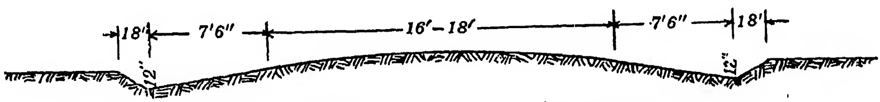


Fig. 2109 A Wide and Flat Ditch

uniform velocity; that it does not fill up by caving or from the wash of earth from the road; that if a wagon is run into it accidentally or in an emergency no especial trouble follows; and that it furnishes earth to crown the road. This form is suited to a road which has ample width and is on good ground. If these conditions are reversed, the road narrow, and the ground wet, a ditch of the form shown in Fig. 2109B will be better. It takes less space and is deeper. It will fill up more rapidly and will require more work to keep it open. The ditch in Fig. 2109A can be dug with scrapers and graders; that in Fig. 2109B must be dug with shovels.

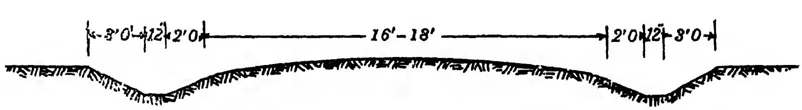


Fig. 2109 B Ditch with Trapezoidal Section

2110. **Drainage of the road surface** takes care of the water which falls on the roadbed itself and is effected by making the surface of the roadbed smooth and compact and giving it regular slopes longitudinally and laterally. Compacting or consolidation of the road surface reduces

the rate of absorption of water, and smooth regular slopes cause the rainfall to run off promptly. Compacted earth absorbs water very slowly. By digging in a beaten road or footpath it will be found, even after a hard rain, that the ground is wet for a slight depth only: the surface stratum when wet seems to form an impervious coating which keeps the rest dry. If the surface is disturbed during the rain, as by traffic on a road, the protection of the surface stratum is lost, and the water penetrates deeper. An earth road in constant use during wet weather will become muddy no matter how much attention is paid to drainage, but with proper drainage a road will not become muddy so soon, not stay muddy so long, nor will the mud get so deep.

2111. The crown of an earth road should be 9″ for a road of an ordinary width of 18′. Theoretically the crown should increase with the grade, but this is a refinement unnecessary in practice. The convenience in construction of a fixed crown outweighs any advantages of a variable crown. If the grade is so steep that water flows too far along the road, causing scour in the wheel ruts, it is better to build low ridges across the road at intervals to turn the water to the side than to attempt to produce the same effect by a greater crown. The ridges may be made wide and flat and high enough to be effective, and yet not so abrupt as to materially disturb traffic.

The best distribution of the crown is to give $\frac{5}{8}$ of it to the outside quarters and $\frac{3}{8}$ to the inside quarters of the road. The resulting crown is nearly an arc of a circle. With inexperienced men it may be necessary to use a form for a crown, the form being simply a plank of the proper length and curvature provided with a convenient handle for manipulation.

2112. **Subdrainage** is resorted to when it is desirable to lower the surface of the ground water. By the ground water level, at any point, is meant the depth at which the soil becomes fully saturated. If it is 4′ or more below the surface, it will not affect the condition of a road in good soil. Ground water rises in wet and falls in dry weather. It probably rises when the ground is frozen, regardless of the rainfall. Many roads have their surface broken by the freezing and expansion of the ground water.

2113. Subdrainage is best accomplished by a very deep side ditch or by tile drains laid on one or both sides of the road under the side ditches (Fig. 2113A). The tile should be of the bell and spigot pattern, laid with open joints, the bell upgrade. As water flows along the outside of the pipe as well as on the inside, it should be surrounded by porous material such as broken stone or gravel.

Substitutes for the tile must often be used. The essential requirement is a continuous conduit into which water may percolate through

the sides and along which it may flow with a relatively high velocity. Broken stone, plank or layers of fascines or brush will do much good. Any substitute form except a pipe or box tends to quickly choke up with fine silt washed into the interstices. This may be partly prevented by interposing a layer of filtering material such as straw, turf, grain sacks, etc., between the material of the drain and the surrounding earth, especially on the top. If turf is used, the grass side should be placed towards the drain.

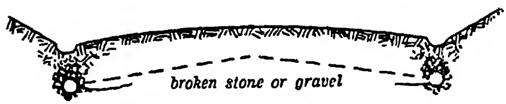


Fig. 2113 A Subdrainage

If the road is wide, a drain under the center line will obviate the necessity of very deep side ditches or side drains (see Fig. 2113B).

The expense of proper subdrainage is so great that it is rarely practicable. Except in very unfavorable soil, reliance is placed upon the side ditches and upon the strength of the surface to resist weight of vehicles above and action of frost below.

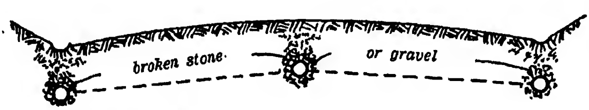


Fig. 2113 B Subdrainage for wide road

2114. **Water-breaks.**—Water-breaks are small ditches or embankments which are constructed diagonally across an earth highway on steep grades, to prevent the water from running in the wheel ruts and thus eroding the surface. They are unnecessary on hard surfaced highways whose surfaces are properly crowned. Small cross-drains, covered by a grating and connected with the sewers or ditches, are sometimes constructed across such highways on long grades to prevent the accumulation of too much water at the foot of the slope.

2115. **Culverts.**—The term culvert is limited to spans of about 12′: when the span exceeds 12′, the structure is a bridge. The area of cross-section of a culvert should be sufficient to carry off the water which reaches it in the heaviest rain and when its drainage area is in condition to discharge this water most rapidly.

The area of the culvert will depend, therefore, on the maximum rate of rainfall, the area of the drainage basin above the culvert, the slope of the basin, the condition of the soil as affecting the amount of water

which it will absorb in a unit of time, and the condition of its surface as interfering with the free flow of the water.

2116. The solution of the problem cannot be accurate; reliance must be placed upon empirical formulas modified by good judgment. The one commonly employed is:

$$\text{Area of cross-section in square feet} = k\sqrt{\text{drainage area in acres}}$$

in which k is a coefficient whose value depends on the character of the surface: it is 1 for a flat country, 1.6 for a rolling country, and as high as 4 for a mountainous country. The flood height and the area of cross-section of a stream in time of flood can often be best determined by making inquiries of people living near it, or by a careful inspection of its banks and the culverts or bridges along its course.

2117. For a very important culvert, whose failure to carry off the water promptly might cause serious damage, the following formula is used:

$$Q = Cv\sqrt[4]{\frac{S}{A}} \qquad (2117A)$$

in which, Q = cubic feet per second per acre reaching the culvert;
C = coefficient dependent upon the amount of obstruction to flow, and varying from 0.75 for paved city streets to 0.30 for the drainage of gardens.
v = maximum volume of rainfall in cubic feet per second per acre, which is approximately equivalent to the maximum rainfall in inches per hour.
S = slope of ground in feet per thousand feet.
A = number of acres in the drainage area.

To apply this formula, a contoured map of the drainage area is necessary.

2118. Culverts should always be made in solid ground, and never in the made ground of the fill. If constructed in made ground, they will be broken when the embankment settles.

Small culverts up to about 2′ span are usually made of glazed sewer-pipe or cast-iron water-pipe. To increase the capacity of the culvert, two or more of these pipes are laid side by side.

Medium sized culverts are usually rectangular or box culverts or arch culverts. The box culvert is preferably made of dry or cemented stone walls, and is covered with slabs of stone. The bottom of the culvert is paved with stone, preferably laid in mortar. When stone is not available, box culverts are also made of beams and planks or of square timbers. When the span exceeds 4′, there is placed a central vertical partition which divides the culvert into two channels, thereby making it possible to use smaller timbers.

Foundations.

2119. The **foundation** is the base underlying the wearing surface. The object of the foundation is to support the wearing surface, and transmit and spread its load to the subgrade.

2120. **Earth, sand-clay** and **gravel** roads have no separate foundations (Figs. 2120A and 2120B). There is some arch effect secured by the rolling or packing of the materials, but such roads will soon break down under heavy traffic.

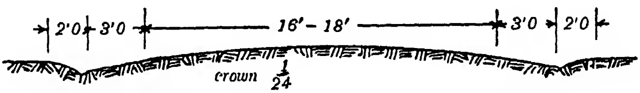


Fig. 2120 A Cross-section of Earth Road

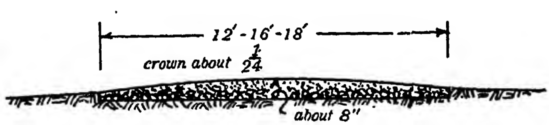


Fig. 2120 B Cross-section of Gravel Road

2121. **Macadam roads** have a separate foundation of large stones. Ordinary macadam has the stones placed with the smallest dimension vertical (Fig. 2121A). **Telford macadam** has the stones placed on edge, and fitted carefully by hand with the smallest dimension horizontal (Fig. 2121B). Telford macadam is more expensive than ordinary macadam foundation.

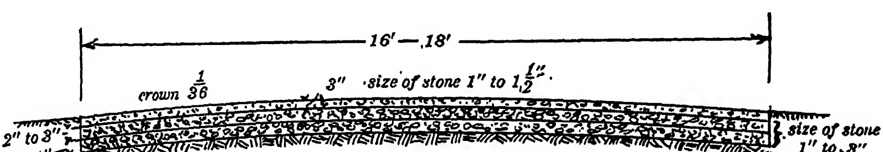


Fig. 2121 A Cross-section of Macadam Road

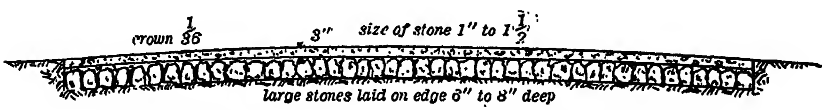


Fig. 2121 B Cross-section of Telford-Macadam Road

2122. **Concrete foundations** (Fig. 2122A) are the best for all purposes. A layer of concrete is placed on the rolled subgrade. The vehicle load is transmitted by the concrete foundation to the subgrade and the weight

of the load is thus spread to a much broader subgrade support. Also, in case the subgrade is weak in places, the concrete in such places acts as slabs or beams and transmits the load to the firmer subgrade.

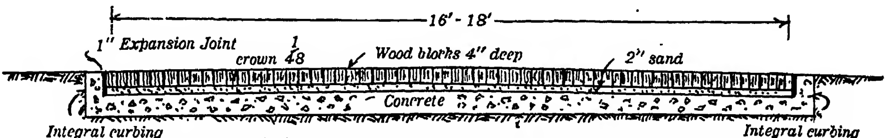


Fig. 2122 A Cross-section of Wood-block Pavement
Note: In place of 2″ sand, mortar or tar joint can be used

Surfaces.

2123. The **wearing surface** of highways has the double duty of lessening resistance to vehicles and of preventing the penetration of water to the foundation and subgrade.

2124. Earth, sand-clay, and gravel roads have not satisfactory surfaces. The passage of vehicles forms ruts, due to lack of hardness of surface, thus increasing the resistance to vehicles. The surface is not impermeable: and with ruts it is even less so, as water collects in the ruts and makes the surface still softer and the subgrade weaker.

2125. Macadam roads have a surface consisting usually of smaller stone placed on the larger stone and rolled until packed and smooth. A suitable sand or sandy gravel is placed on the surface and rolled into the interstices between the small broken stone. This material is called a **binder:** it binds the surface together and renders it impermeable. Stone screenings from the crusher make the best binder.

2126. Asphalt, brick, wood, and stone are the most commonly used materials for paved surfaces. Each is described fully in paragraphs 2164 to 2172. The foundations for these surfaces may be of macadam, but are more generally of concrete.

2127. A concrete surface is simply an extra thickness of the concrete foundation. The added thickness is not separate: but the total thickness is made sufficient for both foundation and wearing surface. This highway is in very general use. It is simple in construction, less expensive than many of the other hard surfaced highways and generally satisfactory as a wearing surface.

2128. Tar and gravel or tar and broken stone are often placed as surface material on the earth subgrade without any special foundation. This surface is impermeable until it is broken through by vehicles; and it has been found suitable for cheap roads bearing light traffic. Effort has been made to obtain a suitable surface by use of tar alone on an earth subgrade, but its supporting power is so small that it is immediately broken and becomes permeable, thus losing its only valuable quality.

LOCATION.

Surveys.

2129. **General Considerations.**—To determine the best position of a road which is to connect any two points, it is essential as with railways to study the topography of the country between the points with the aid of topographical maps.

If no maps are available, the routes which, from observation or inquiry, seem to be the most favorable must be carefully reconnoitered. This may be done with a sketching case, watch, and aneroid barometer. The selection of the routes to be examined is governed by the same general considerations as explained for location of a railway.

The reconnaissance, preliminary survey, and location survey, are conducted essentially as already explained for railways. Airplane photographs are even more useful in road work: with complete photographs of the country and a few elevations, it is possible in some cases to lay out the road in almost its exact position for the locating party to place their stations on the ground.

2130. **Marking Adopted Line.**—Having selected the best of the trial lines, it is staked out on the ground. With a transit and chain or stadia, the general line of the axis of the road is laid out upon the ground as a broken line, and a stake is driven at every 100′ and at every angle. The elevations of the ground at these stakes and at the salient and reëntrant points in the profile, above an assumed datum plane, are determined with a level. At every stake, and at intermediate points if the surface is very irregular, transverse profiles are taken of the entire right of way.

2131. **Curves.**—The center line of the road consists of the straight lines or tangents, connected by curves usually circular. Sharp curves

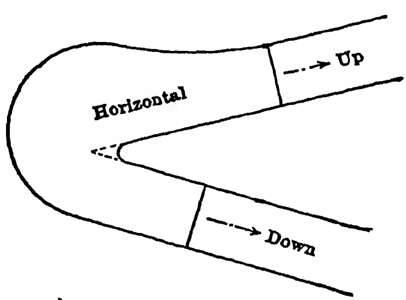


Fig. 2131 A Curve on Steep Grade

on steep grades or at the foot of steep grades are not safe; good practice calls for a minimum radius of 300′ or 400′ for these cases. Right angle

turns even on level stretches are inconvenient and often dangerous. Where sharp curves occur on steep grades the roadway should be widened and the grade decreased so as to give space for vehicles to pass or stop (Fig. 2131A). Curves of opposite curvature should be separated by a straight stretch or tangent at least 60′ in length.

Steep grades, sharp curves, and knolls that obstruct the view ahead should be avoided in the interest of safety. There should always be a clear view ahead of at least 250′ and if a curve exists on a hill, the grade should be flattened around the curve so as to permit a quick stop in case of emergency. In the case of a short and sharp curve in a deep cut, the high bank on the inner side of the curve is cut away, if possible, to allow a view of approaching traffic. On automobile roads, the curves have great radii and consequently little cutting of the inner bank is necessary.

2132. The rules governing curves on the French highways are:

a. Make the radius of curvature at least 100′ if possible.

b. Avoid sharp curves on steep roads.

c. Separate curves of opposite curvature by a straight track at least 60′ long.

2133. The profile is plotted on cross-section paper to convenient scales. In order to magnify the irregularities, the vertical scale is about ten times the horizontal scale. The transverse profiles are plotted either above or below their actual positions on the profile as shown in Fig. 2133A. From a study of the plan and profiles it is possible to determine what changes should be made in the position of the axis of the road in plan.

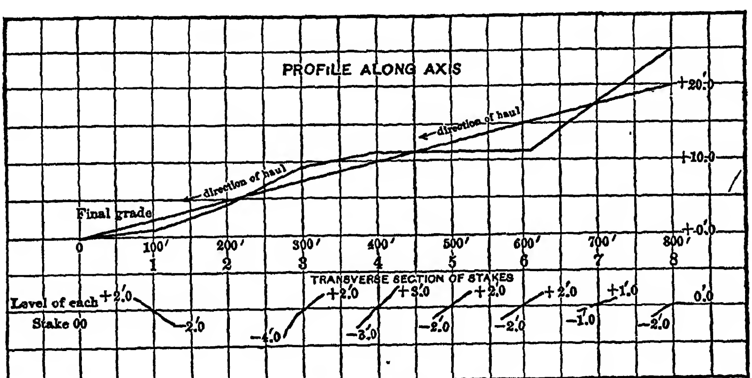


Fig. 2133 A

Grades.

2134. The maximum permissible grade is that which can be allowed without making it necessary to reduce the ordinary size of load hauled over the road. Since the automobiles and trucks in general use have

sufficient power to take them up any firm surfaced grade that has heretofore been considered suitable for horse traffic, it is evident that teams for heavy hauling still govern the selection of maximum grade on roads where there is horse-drawn traffic. This grade will depend on the tractive power of the horse and on the load and resistance of the vehicle which he draws.

2135. **Tractive Power of a Horse.**—The tractive power exerted by a horse at any moment is approximately measured by the tensile stress in the traces. The working tractive power is that which he can exert throughout a working day when he moves at a walk on a level road. This is now generally assumed as 120 pounds for a horse weighing 1000 pounds, or as about one-eighth his weight.* His maximum tractive power, which he can exert for a few moments, is at least twice his working tractive power and is equal to about one-fourth his weight.

The amount of daily useful work performed by a horse is the product of the tractive power by the distance over which he exerts this power. It is generally assumed that a horse can, without undue fatigue, walk 20 miles a day on a good level road, and at the same time pull a wagon whose constant resistance is 120 pounds. This is equivalent to 12,672,000 foot pounds of work. The normal horsepower employed in Mechanics is based on a horse whose tractive power is 150 pounds moving at the rate of $2\frac{1}{2}$ miles per hour, or 15,840,000 foot pounds per day of eight hours.

The maximum tractive power of a horse on a slope is less than on a level; the action of his weight is inclined to the surface; this reduces the friction of his shoes and also reduces his pulling effect. Also, since part of his muscular effort is expended in raising his own weight and that of the loaded vehicle, he has less strength in daily useful work than on a level road.

2136. **Tractive Resistance of Vehicles.**—The tractive resistance of a vehicle on a level surface equals the force which must be applied to the vehicle, parallel to the surface upon which it rests, to move it at a uniform rate of speed. This resistance is due to the frictional resistance between the axles and hubs, and the resistance of the circumference of the wheels caused by irregularities in the road. The total resistance may be expressed by the formula:

$$R = \mu W + \frac{Wk}{\sqrt{r}} \qquad \text{(2136A)}$$

in which, R = total resistance in pounds.
μ = coefficient of friction of surfaces of axle and hub.
W = weight on wheels (weight of vehicle and load).
k = a coefficient dependent on character of road surface.
r = radius of wheel.

* Recent experiments (1924) at Iowa State College give a higher value.

The term μW represents the resistance of the axle, and $Wk/\sqrt{r}$ represents the resistance due to irregularities in the surface of the road. Both are determined by experiments and hence the formula which is the result of these experiments may be given in the general form

$$R = \mu W \tag{2136B}$$

in which, R = total resistance in pounds.
W = total weight of vehicle and load in pounds.
μ = coefficient dependent on the character of the road surface and speed.

Extensive experiments over a period of years give the following results for the average animal-drawn vehicle moving at $2\frac{1}{2}$ miles per hour:

Surface	Tractive rolling resistance in pounds per ton of 2000 pounds	Calculated value of μ
Steel railway	8	.004
Steel tramway	16	.008
Sheet asphalt	20	.010
Concrete	22	.011
Brick	25	.012
Stone blocks	44	.022
Macadam	50	.025
Gravel road	90	.045
Earth road	100	.050
Loose gravel	148	.074
Loose sand	320	.160

Recent experiments (1924) at Iowa State College, on gravel, concrete, wood block, and asphalt roads, give somewhat smaller rolling resistances (10 per cent to 40 per cent) for motor vehicles moving at the same speed.

Assuming the working tractive power of the horse at 120 pounds, he can, on a level, pull with equal ease, if he can secure a good foothold:

15 tons on a railway
7.5 " " a tramway
6 " " sheet asphalt
5.4 " " concrete
4.9 " " brick pavement
2.7 " " stone block pavement
2.4 " " a macadam road
1.3 " " a gravel road
1.2 " " an earth road
.8 " " loose gravel
.4 " " loose sand

The resistance is not altogether independent of the speed, but it is usually so taken. Also, it is to be noted that the above values are for rolling pull. The resistance to starting of animal-drawn vehicles, as shown by the Iowa State College experiments, varies from 6 to 10 times that of rolling. There are no suitable data available as to comparative resistance of starting and rolling of motor vehicles.

2137. In pulling a vehicle up a grade, the tractive force applied must overcome the same resistances as on a level, and in addition must also overcome the component of the weight of the vehicle parallel to the slope. The total tractive force must therefore be (Fig. 2137A),

$$T = \mu W \cos \phi + W \sin \phi \qquad (2137A)$$

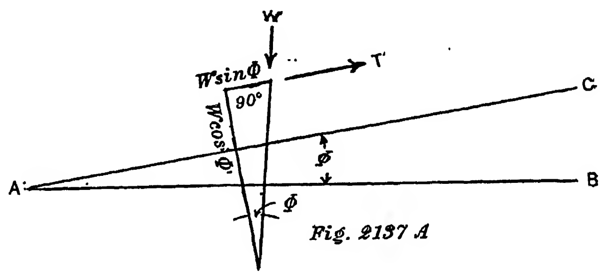


Fig. 2137 A

in which ϕ is the angle of inclination of the slope. Since this angle is never very large, we assume $\cos \phi$ equal to unity, and $\sin \phi$ equal to $\tan \phi$ or equal to h/l, in which h is the rise and l is the horizontal length of the slope. Making these substitutions, we have

$$T = \mu W + \frac{Wh}{l} \qquad (2137B)$$

The value of Wh/l, the tractive resistance due to the slope alone, is independent of the surface of the road and varies directly with the weight of the vehicle and the inclination, h/l. We have therefore the following table:

Resistance per Ton (2000 pounds) of Load due to Grade:

1 per cent grade.....	20 pounds	6 per cent grade.....	120 pounds
2 " " "	40 "	7 " " "	140 "
3 " " "	60 "	8 " " "	160 "
4 " " "	80 "	9 " " "	180 "
5 " " "	100 "	10 " " "	200 "

These resistances are added to those given in the previous table, to determine the total tractive resistance of a vehicle which is being pulled

up a slope. It is seen that the relative effect of a slope is greater on a good road than on a poor one. Thus on a hard asphalt pavement the total resistance on a 2 per cent grade is 200 per cent greater than on the level, while on an earth road with the same grade the total resistance is only two-fifths greater than on the level. Therefore, it is more important to reduce the grades on good road surfaces than on poor ones.

2138. In moving down the slope, the force Wh/l acts in the same direction as T, and the expression for T becomes

$$T = \mu W - \frac{Wh}{l} \tag{2138A}$$

If $\mu W = Wh/l$, then T reduces to zero. Therefore, in moving down a slope whose inclination, h/l, is equal to the coefficient μ, no tractive force will be required.

If the road has a macadam surface, for which μ is equal to $1/40$, no tractive force is required to pull a load down a slope of $1/40$; and a tractive force twice that required to pull a load on the level will be required to pull the load up this slope. For these reasons, a slope of $1/40$ is considered the ideal maximum grade of a macadam road for animal-drawn vehicles. For motor vehicles the ideal maximum grade is slightly less. Similar slopes can be obtained for other surfaces.

2139. **Maximum Grades.**—In road construction it is necessary to fix the maximum and minimum grades in the profile. On account of the expense of construction it is impossible to adopt the ideal grade as the maximum grade.

A grade of 5 per cent corresponds to the ideal grade of a smooth earth surface. Grades of 8 per cent and 10 per cent are not infrequently encountered on earth roads and do not necessitate reducing the size of loads hauled. As a matter of practice many highway engineers take 6 per cent as the maximum grade to be obtained, permitting steeper grades in rare instances only when topographical conditions necessitate.

2140. **Ruling and Average Grades.**—In determining grades on long sections of roads in rolling country, there will be many small hills where the maximum may easily be obtained by little work. However, it is evidently foolish to spend money to make a slight reduction in grade on one small hill when near it is another large hill which cannot economically be reduced below the original grade of the first. It is good practice to select some maximum grade for all hills, and to reduce these only when the expense is justified.

2141. **Minimum Grades.**—The minimum grade in the profile is that which will permit satisfactory drainage. However, a road need have no slope at all, if it is constructed with a proper crown and its surface is

properly maintained, because the slope from the crown to the side ditches insures drainage for the surface of the road.

2142. **Undulating Roads.**—When roads are in rolling country, many slight grades are encountered, and as a general rule no real economy results from removing them or reducing them. Experiments have shown that with an automobile the travel down hill compensates somewhat for the travel up hill. Even with a horse, the experiments show that there is a compensation by reason of the rest to the muscles while traveling down hill.

Widths.

2143. Roads constructed for horse-drawn vehicle traffic can be made narrower than those which are subjected to both horse-drawn and motor traffic. In the former case a width of 14′ to 16′ gives sufficient clearance between two passing teams; but motor vehicles require greater widths. This is due not only to greater width of vehicles, the width outside to outside of the average touring car being about 6′, and of large motor trucks as much as 8′; but is due also to the need of a wider road in order to allow the machines to pass each other at a fair rate of speed, with the proper clearance, and still keep on the improved surface. Many instances have been observed where a 14′ width of macadam has not been sufficient to prevent the automobiles in passing each other from moving off the edge of the macadam, on to the earth shoulder of the road, so that ultimately the edge of the road has been broken down. For main interstate and intrastate highways a width of improved surface of 20′ would probably be none too great, while a width of 14′ to 16′ would be ample for those roads which act as feeders to the classes just mentioned, the smaller width to be used only in case of a very light traffic.

2144. In France the width of the carriageway is fixed by an order of the Royal Council dated in 1776. This order divided the public highways into four classes and fixed the width of roadway of each.

The first class comprised the great roads which connect Paris with the principal cities, ports, and industrial centers. These are called the **national highways (routes nationales)**. The second comprised the roads which connect the principal provinces, cities, ports, etc., with each other. These are called **departmental roads (routes dépertmentales)**. The third and fourth classes comprised the roads which connect the towns of one province with those of an adjacent one, and also the roads which connect the towns and villages of each province. These are called the **local roads (chemins vicinaux)**.

Exclusive of the width of ditches and side slopes, the width of first-class roads was fixed at 45′, the second at 38′, the third at 32′, and the fourth at 25′. The roads consist of a central paved strip between

unpaved strips. The normal width of the different parts, as actually constructed, is given in the following table:

	Macadam	Each Unpaved Strip	Total
National highways........	16.4′ to 22.9′	8.2′ to 11.5′	32.8′ to 45.9′
Departmental highways...	13.1′ to 16.4′	4.9′ to 8.2′	26.2′ to 32.8′
Local roads..............	9.8′ to 16.4′	4.9′ to 6.5′	19.6′ to 26.2′

2145. There are no standard widths of roads or of cross-sections in the United States. The Bureau of Roads, Department of Agriculture, is in charge of the disbursement of the Federal appropriations by which each state is aided by funds appropriated by the National Government. The usual widths and cross-sections required by the Bureau of Roads are:

20′ and over for earth, sand-clay, and feather-edge gravel.

16′ to 18′ for gravel, macadam, brick, asphalt, and bituminous concrete.

16′–18′–20′ for plain concrete and reinforced concrete.

On special roads these widths are exceeded. For example, the New York State reinforced concrete road from Albany to Schenectady has a paved width of 24′.

2146. The Bureau of Roads has planned a complete system of national highways crossing the United States, and certain other national highways connecting principal cities and industrial centers. The most important are:

a. Lincoln Highway, from New York to San Francisco, via Philadelphia, Pittsburgh, South Bend, Chicago Heights, Omaha, Cheyenne, Salt Lake City, and Sacramento.

b. Roosevelt National Highway, from Washington to Los Angeles via Charlottesville, Lexington, Louisville, St. Louis, Kansas City, Denver, Salt Lake City, and Tonopah.

c. Dixie Highway, east division, from Detroit to Miami via Dayton, Cincinnati, Knoxville, Augusta, Savannah, Jacksonville, and Palm Beach.

d. Dixie Highway, west division, from Chicago to Tampa via Indianapolis, Louisville, Nashville, Chattanooga, Atlanta, Macon, and Tallahassee.

Many parts of each of these highways are paved and all parts are kept in good condition for travel throughout all seasons.

CONSTRUCTION AND MAINTENANCE.

2147. Laying out the line, placing pegs to mark the limits of cuts and fills, and execution of the earthwork are in all essential particulars the same as in railway work. Less accuracy is required, curves are often marked by eye without the aid of instruments; slopes are not so accurately determined; fills on side slopes need not be so carefully constructed.

2148. On the other hand, the subgrade must be more carefully prepared. In railway construction, it is sufficient to provide for drainage, and then to place the ballast after some or no rolling. With roads, the subgrade is carefully cleared of all unsuitable materials and then rolled thoroughly so that the foundation will have a hard and compact earth support.

2149. **Macadam Roads.**—(Fig. 2149A). The construction of a macadam road is described in detail, to show the general system of construction of paved roads.

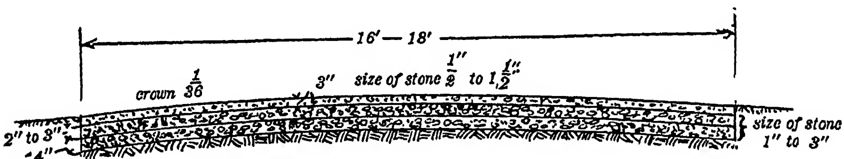


Fig. 2149 A Cross-section of Macadam Road

A trench is first excavated to hold the broken stone. The bottom of the trench is rolled with the heaviest roller obtainable and all its soft places are filled with sandy earth or ashes. The subgrade after being thoroughly rolled should be parallel to the finished surface of the road, and at the proper distance below it.

The surface of the trench is then graded to a lateral slope of about $\frac{1}{30}$. As sand, sandy earth, or any other soft and dry material makes the best subgrade for a road, the broken stone is placed directly on the ground in the bottom of the trench. On the best roads, there are three layers of stone. The first consists of any sound stone with longest dimension 1″ to 3″; this layer is 4″ thick and is compressed by a roller. The second layer is of stone similar to the first, is 2″ to 3″ thick and is similarly rolled. The third layer is of $\frac{1}{2}''$ to $1\frac{1}{2}''$ granite or other hard stone, and 3″ thick.

The three layers given above are correct and best. Generally, however, there are only two separately rolled layers, the first and second layers as above being combined and rolled as one layer, thus saving time and expense. Quite often the stone of all three layers is of the same size, 1″ to 2″; but this gives poor support and the road gradually sinks

into the subgrade and more broken stone is constantly needed. Cases are known of such foundation courses where top layer after top layer has been added until there existed a mass of small broken stones 8′ in depth. It is thus seen that suitable large stones in the bottom course are necessary.

After this top layer is thoroughly rolled, screenings are spread on the surface and wetted down, and the road is thoroughly rolled until the interstices have been filled and there is a solid surface apparently all of one piece. In fact, certain stone screenings act somewhat as a cement to form a kind of concreted surface. However, in the case of the ordinary screenings, the rolled hard surface is only a mechanically packed mixture and not a chemical compound.

2150. **Telford Macadam.**—In the Telford macadam road (Fig. 2150A), the lower or foundation course consists of stones placed on edge. A size of stone that can readily be handled by one man is suitable. The vertical dimension should be within an inch of the required thickness of the foundation, which may be 6″ or 8″. The width of these stones is from

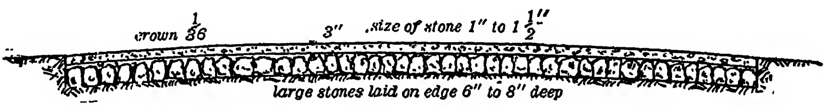


Fig. 2150 A Cross-section of Telford-Macadam Road

5″ to 12″ and the length from 8″ to 15″. The Telford foundation course must be carefully placed by hand. The greatest length of each stone is laid across the road and it rests on its widest edge so as to break joints as much as possible. All chinks between the stone are filled with gravel, or crushed stone. The lower course is then thoroughly rolled with a heavy roller before the upper layer is placed on it. The Telford macadam road is like ordinary macadam, with the exception of the special foundation course as described above.

2151. In the Telford foundation, the interstices are open and the water can flow away if the subgrade has any slope: hence the ground-water does not reach the binder of the top surface to soften it. In the macadam foundation, the interstices are usually filled with binder or dust, which prevents the escape of the water and allows it to be drawn up and distributed through the binder by capillary attraction, thus softening the pavement. Macadam pavement being the cheaper is therefore satisfactory when the soil is sandy and allows the escape of water through it. The Telford pavement, being less likely to be affected by ground water, is therefore most necessary when a pavement must be laid on clayey soil, especially in cuts where the water is likely to accumulate.

2152. Macadam surfaces may be treated with light refined tar or asphaltic oil by sweeping the road clean of dust and applying the liquid with sprinklers. The surface is then covered with pea gravel, stone or slag screening, or a sharp coarse sand. In parts of New York the cost has ranged from two to three cents per square yard including sweeping, materials, and labor.

Bituminous flush coats are applied by sweeping the macadam carefully to remove all surface dirt as well as the stone or sand filler to a depth of about one half inch below the top of the larger stone fragments. On this rough, clean, dry surface is spread about 0.2 to 0.8 gallon per square yard, of hot heavy tar or a bituminous residuum. It is spread either by hand or by sprinklers and then covered with a layer of about $\frac{3}{4}''$ dustless screenings and thoroughly rolled. A well constructed surface resembles asphalt. It protects the macadam, is waterproof, forms a surface which takes the traffic from the large stone fragments and gives a smooth road. But it cannot be laid in wet or cold weather and like asphalt is slippery and unless laid evenly will develop sharp waves or humps.

2153. **Earth Roads.**—The wearing surface on earth roads is the earth excavated from the sides of the roadway or from the ditches. It is not a good wearing surface, unless well drained and kept in thorough repair. Even then, the tractive resistance is large. The quality of the wearing surface depends upon the material; it is best when composed of sand or gravel with just sufficient clay or loam to bind it thoroughly; it is worst when sand or clay alone is used.

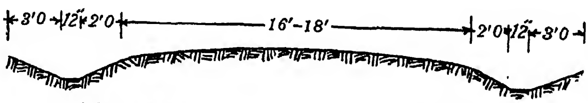


Fig. 2153 A Cross-section of Earth Road

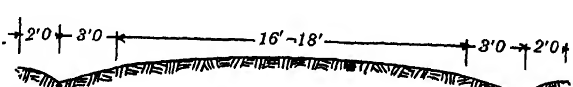


Fig. 2153 B Cross-section of Earth Road

If the cross-section is of the general form shown in Fig. 2153A, the roadway proper should have a crown of $\frac{1}{24}$, and the side ditches should be deep enough to assist in draining the roadbed. As this cross-section necessitates the excavation of the ditches by hand, a more common form is shown in Fig. 2153B which can be easily constructed by machinery. As shown in both of these cross-sections, the earth road has a high crown; this is necessary in order to prevent the surface water from sinking into the roadbed when the road is worn by travel.

The surface drainage can be kept intact only by filling the ruts with good material as soon as made. Large stones should never be used in making these repairs.

As heat and free circulation dry the roadbed, a clayey road should never be shaded by overhanging trees; a sandy road, being better in a moist than in a dry condition, should have as much shade as possible.

2154. Besides the grading machinery employed in the construction of the roadbed, the principal machines employed in its improvement are the **simple** and **elevating scraping graders.** The simple scraping grader is a four-wheeled vehicle which supports a heavy steel blade, either plane or curved, which may be so set that its cutting edge is oblique both to the axis of the road and to the horizontal plane. As it is pulled along, it cuts the earth to an inclined plane and moves it from the sides towards the center. The elevating scraping grader (Fig. 2154A) has, in addi-

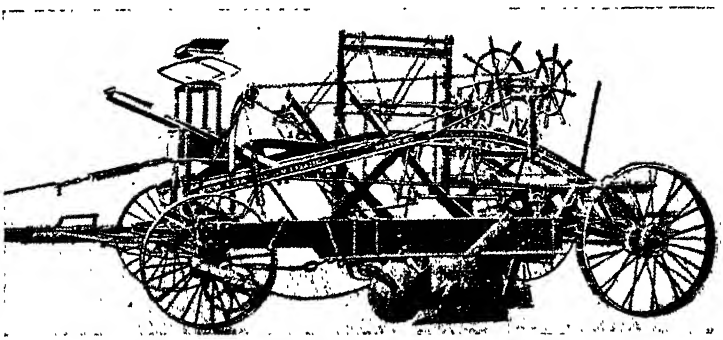


Fig. 2154 A. Elevating Scraping Grader

tion, a carrier by means of which the earth excavated from the side ditches is conveyed directly to the middle of the roadway. **Road drags** are probably more commonly used than any other tools for the maintenance of earth roads. They are very simple in their essential features. A chain is attached so as to cause the drag to follow the team or machine drawing it so that its long axis makes a slight angle with the perpendicular to the axis of the road. Drags are from 6′ to 9′ long. They are made in several types such as the split log drag, the plank drag, and many forms of steel drags, in which the blades can be tilted to different angles. The drag most commonly used and the simplest in construction and use is the split log drag (Fig. 2154B). A log is split in half and the two halves placed parallel and braced together about 3′ apart. An iron strip is nailed to the lower corner to form a cutting edge. A platform of 1″ boards is built on the braces for the driver of the team.

Drags are used to clean out the side ditches which become filled with

grass and débris; but their most important use is to keep the surface of the road in shape. The drag does its best work when the soil is moist but not sticky. It crowns the road and fills up the ruts with the earth which is worn or washed to the edges of the road.

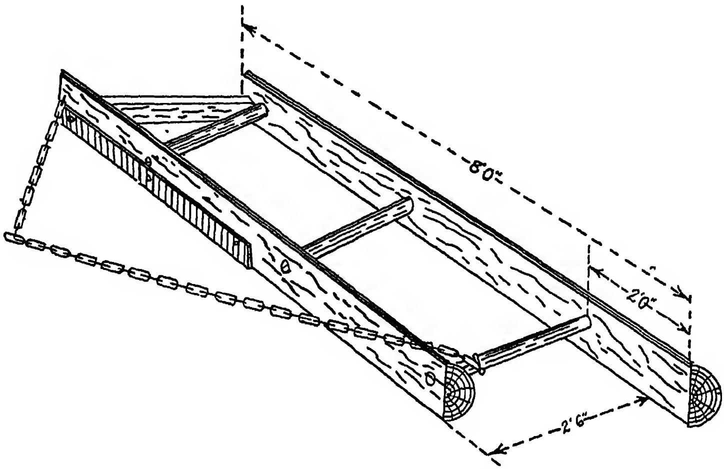


Fig. 2154 B. Split-log Drag

2155. **Sand-Clay Roads.**—The name sand-clay is given to a type of road surface (Fig. 2155A) which consists of sand and clay, loam, or gypsum, or all of these materials. Such surfaces are constructed on clay, gumbo, or loam soils, and on deep sand.

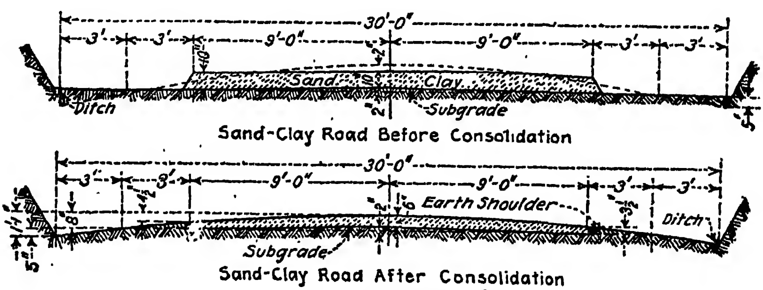


Fig. 2155 A

The essential requirements for successful work in building such a road are that the materials be reasonably suitable in character and quality and that they be thoroughly mixed. The aim is to secure, on the surface of the road, a layer or crust which is made up of sand into which has been mixed a binder of clay, of loam, or of gypsum and clay. The

amount of this binder needed is about the amount which will fill the voids in the sand.

The sand-clay road is resilient, dustless, and will be smooth if properly built. It becomes only slightly muddy, is serviceable for moderate traffic, and requires little care after it becomes finally solid. It will be damaged by traffic during long spells of dry weather.

2156. **Gravel Roads.**—(Fig. 2156A). Care should be taken to make the distribution of fine and coarse gravel as even as possible, the coarser particles being placed in the bottom. The gravel is thicker at the center than it is at the sides, and after it is spread to the desired surface is

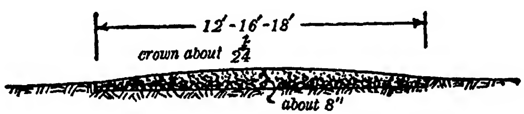


Fig. 2156 A Cross-section of Gravel Road

thoroughly rolled and compacted. The best form of construction is to build a gravel surface in a shallow trench in a manner similar to that used in building macadam roads. The gravel is spread on this trench in two or three layers, which are separately rolled. The thickness of the separate courses varies, but the total thickness is about 8″ at the center decreasing to about 6″ at the sides. Water should be used to help the consolidation of the surface, but care should be taken not to use an excess of water, since it may wash out the natural binding material or soften the subgrade.

2157. **Concrete Roads.**—(Fig. 2157A). Concrete as wearing surface for roads is an excellent development in the use of this material. It has long

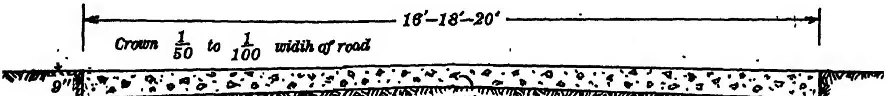


Fig. 2157 A Concrete Road

been used as a base for various types of pavements where its function is to distribute the load stresses to sufficient area of earth in the subgrade to insure stability. As lately developed, the concrete is made a little thicker and is itself the surface.

When concrete is used as wearing surface it must perform all the functions required in the concrete foundation and in addition must resist the abrasive action of traffic, and the tensile stresses due to changes in temperature.

The concrete road is commonly constructed with a small crown. Experience and theory show that the outer edge of the concrete surface is more likely to crack than the inner sections; and consequently, the subgrade is curved and the thickness of the concrete is less at the middle than at the edge. For heavy traffic, the usual thicknesses are 7″ at center and 9″ at the edges; for light traffic, 6″ and 8″ are suitable.

Where concrete is employed for city pavements, the thickness is uniform, being at least 6″ throughout, and the crown is gained by making the subgrade with the necessary elevation. Curbs and gutters are usually made as integral parts of the pavements: by this method considerable expense is saved over separate curbs and separate gutters. The mixing is done by machinery, chiefly operated by gasoline engines. After these operations, the concrete is "cured," that is it is kept moist for a few hours by canvas covers which are constantly wetted and thus the concrete is prevented from drying out too quickly. Upon the removal of the canvas, the surface is covered with a layer of sand or earth, which is kept thoroughly moist for a period of two weeks.

Expansion joints should always be provided to take care of the expansion and contraction of the concrete due to changes of temperature, variation in the moisture content of the concrete, and variation in the condition and character of the subgrade. If expansion joints are omitted, the concrete will crack upon sufficient decrease in temperature to overcome its tensile strength opposing the contraction; it will likewise crush or bulge upon increase of temperature. The width of the expansion joints depends upon the distance between them. It is better to construct narrow joints at short intervals rather than wide ones far apart. Transverse joints are placed about 50′ apart. Also, a longitudinal expansion joint is placed down the middle of a road more than 16′ wide.

2158. **Placing the Concrete.**—Before laying the concrete on the earth subgrade, the latter must be thoroughly plowed, shaped, rolled, and preferably subjected to the effect of traffic and weather so as to diminish the unequal settling. This will diminish or eliminate the cracks in the pavement.

The mixing may be done right on the job, at the point needed, by a batch mixer; or it may be done at a central plant and hauled by truck to several points or jobs where concrete is being laid, simultaneously. As the concrete is being laid, it is shaped to the specified cross-section, tamped, and smoothed. The movable concrete paver is coming into general use. It mixes the concrete, distributes it over the width of the road, and moves along parallel to the axis of the road.

2159. **Reinforced Concrete Roads.**—The advisability of reinforcement in concrete pavements is a matter of opinion rather than the result of definite experiments. It is often recommended that when roads

exceed 20′ in width, reinforcement of some kind be used to prevent large cracks and to distribute the stresses due to temperature and moisture.

Two general types of reinforcing are used: mesh, and rods. The reinforcing is not primarily intended to strengthen the concrete slab as a beam, but to prevent parts of the slab from being displaced vertically.

It prevents the widening of cracks which occur under stresses as described above.

2160. **Characteristics of Concrete Roads.**—A well constructed concrete road has a granular, uniform, easy riding surface which affords excellent traction for motor vehicles and fair foothold for horses. It is free from dust but is trying to the eyes in bright sunshine because of its white color. A good concrete road resists the abrasion of traffic very well, but when a crack occurs deterioration is rapid. As to its ultimate life, data are not available, but recent experience indicates that, if properly built and properly maintained, it will be a very durable pavement.

Pavements.

2161. Sheet and block pavements are suited to city streets because of the ease with which they may be kept clean, the infrequency of necessary repairs, and the comparative ease with which needed repairs may be made. Also, they are fairly free from dust and mud. The above properties make them likewise suitable for country roads, but the item of expense makes it impracticable to use them often for this purpose.

2162. **Wearing Surfaces.**—The ordinary wearing surfaces are made of blocks of wood or stone, and sheets of asphalt concrete or asphalt mortar.

All block pavements are laid in a similar manner, in rows of blocks perpendicular to the line of travel. The blocks break joints in the direction of travel. They are placed on a sand cushion about 2″ thick and the surface is made perfectly smooth by pressing the blocks, if irregular, to the proper depth into this sand.

2163. **Foundations.**—The best foundation for all forms of pavement is a bed of concrete, laid on the subgrade, which has been graded and then thoroughly rolled with a heavy roller. The normal thickness of the concrete foundation is 6″, which may be reduced to 4″ or 5″ with a rich mixture on an especially good subgrade. The normal mixture is 1 : 3 : 6.

When concrete is too expensive, the foundation may be made of a bed of gravel or cinders about 6″ thick and rolled with a very heavy roller. This form of foundation will always develop depressions over weak places in the subgrade. These depressions will ultimately cause weak places in the surface; and such a foundation will probably be more expensive in the end than a concrete base.

2164. **Stone Block Pavement.**—(Fig. 2164A). A pavement of granite blocks on a concrete base is the most durable of all pavements. It requires no attention for many years after it has been laid. It is the most suitable type of pavement for streets subjected to very heavy loads, such as dock and warehouse districts. The earlier stone block pavements were rough and noisy; but the blocks are now made smoother and fitted like bricks with the result that the stone block pavements constructed in recent years are high class surfaces of great durability.

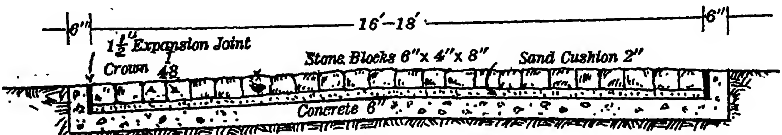


Fig. 2164 A Stone Block Pavement

The blocks are about 6″ deep, 4″ wide, and 8″ long, and made of granite, trap, and sandstone. The granite blocks are best, since they are more durable than sandstone and less slippery than trap. The blocks are laid on a 2″ cushion of sand resting on a 6″ concrete base.

The joints in the best pavements are filled with hot gravel and asphalt cement, tar, or pitch; those of inferior pavements are filled with gravel only.

The points to be noted about a stone block pavement are its great durability and long life. It will withstand the heaviest traffic and the hardest wear. It becomes slippery with wear, is always noisy, does not present as pleasing an appearance as many other types, and is difficult to keep clean. It is very expensive in first cost, being in this respect the most expensive of all pavements.

2165. Pavements with a stone wearing surface were introduced into Europe by the Romans in the construction of their military roads and in the paving of their city streets. Their pavements consisted of irregular shaped slabs usually of large area, which were supported by a firm foundation of flat stones covered by a layer of a kind of concrete.

2166. The **cobblestone pavement** is a stone pavement which is made of egg-shaped stones from streams or fields. It is very rough, very slippery, hard to keep clean, and destructive to vehicles. The noise is much greater than over stone blocks. The pavement is made of stones 4″ to 6″ long, 2″ to 4″ in diameter, laid with close joints, covered with sand, and then tamped with a heavy hammer until they are firmly bedded. Practically no new cobblestone pavements are being constructed.

2167. **Brick Pavements.**—(Fig. 2167A). A brick pavement is like a stone block pavement with bricks replacing the stone blocks. Bricks are less durable than stone blocks, but are uniform in size whereas stone

blocks are not exactly so. The resulting pavement therefore has narrower joints and a smoother surface and is less noisy. It is also cheaper than stone block, since the cost of the brick is about one-half the cost of the stone block for the same area of street.

A great number of the paved streets in the United States are of brick.

The bricks for pavements are made of selected clay, and are burned at a high heat just short of vitrification, producing what is called a vitrified or pavement brick.

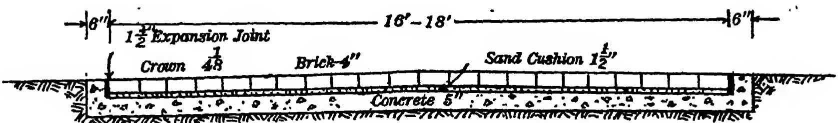


Fig. 2167 A Brick Pavement

Pavement bricks are usually subjected to two tests, the absorption and the rattler tests. The absorption test consists in immersing the brick in water for forty-eight hours in order to determine the amount of water by weight that it will absorb. As the manufacturer can make his brick impervious by a suitable coating, it is necessary to break the brick into two parts. The rattler test consists of rolling the brick and some metal blocks around in a barrel, called a rattler, for the purpose of determining the toughness and durability of the brick. The edges of the bricks are worn away by concussion and friction in the rattler and are broken if too soft. The amount of such wear determines the lack of excellence of the brick.

In laying a brick pavement, the bricks are placed by hand on edge on a cushion of sand about 1″ to 2″ thick. The surface is then tamped by dropping a heavy tamp on a short thick plank placed over the brick surface. The joints are filled with cement grout. Longitudinal expansion joints are placed at intervals on each side at the curb; transverse expansion joints are placed at intervals of about 50′. The expansion joints are filled with bituminous cement and are impervious.

2168. **Wood Block Pavements.**—Wood block pavements are extensively used in certain parts of the country. Their principal advantage is their noiselessness. The blocks commonly used are about 8″ long, 3½″ wide, and 4″ deep. In the United States they are most commonly made of southern yellow pine, Norway pine, black gum, and tamarack, and are treated with creosote to prevent decay.

The wood blocks are generally laid on edge in a cushion of sand with grain vertical. However, the sand cushion has not always proved satisfactory as it has been prone to wash from under the wood blocks or to shift when dry, thus rendering the pavement uneven. This is particularly noticeable on streets carrying surface lines, the jar of the cars

being sufficient to loosen the blocks along the track and make an opening for water. As a result, it is considered best practice to use a mortar cushion instead of sand. In some cases, a thick coating of tar is spread on the concrete foundation and covered lightly with sand, the wood blocks being placed on the mastic thus obtained. When wood blocks are thus laid, it is important that the concrete foundation be finished with the surface true to shape of the final pavement as the mastic does not permit leveling of the surface by increased pressure of the blocks by hand, as in laying in the sand.

The foundation is about 6″ thick as in brick pavements. There are the same expansion joints on each side of the curb, and the same transverse expansion joints, except that they may be placed farther apart than with brick, say at a distance of 100′. The joints between blocks and the expansion joints are filled with the same material as in brick pavements.

The wood block pavement is adapted to any sort of traffic and is exceedingly durable even under the heaviest traffic. It is smooth, resilient, and quiet, but its one undesirable characteristic is its slipperiness. It is not suitable for grades in excess of about 4 per cent and if subjected to heavy horse-drawn traffic should not be used where the grade exceeds 2 per cent. The slipperiness can be overcome to a large extent if the pavement is sanded at the proper time. Some difficulty is encountered from expansion due to the wood swelling when wet, but this is eliminated by the use of suitable expansion joints.

2169. Plank and corduroy roads are crude forms of wood block pavements; they are employed to carry a road across a wet place into which the wheels of a vehicle would sink to a considerable depth were they not supported by some form of platform. The foundation consists of two or more rows of longitudinal sleepers. In the plank road these are covered with a floor of 2″ or 3″ plank: in the corduroy road by saplings with the upper surfaces of the saplings roughly hewn, or covered with brush or grass and a layer of earth.

2170. **Sheet Asphalt Pavements.**—(Fig. 2170A). The foundation of a sheet asphalt pavement is preferably 6″ of concrete as with block pavements. Upon this foundation is laid the binder course of about $1\frac{1}{2}''$ of sand and asphalt cement or of broken stone and asphalt cement. The asphalt is melted and mixed with the sand or broken stone at the mixing plant, located fairly near where the pavement is being laid. The mixture is then brought on in carts at a temperature of 250° to 350° F., deposited on the foundation, and then compacted about 40 per cent by a roller. The **wearing surface** consists of an asphalt mortar made of an aggregate of finely graded sand with a filler of stone dust mixed with hot asphalt cement. It is brought on at about 300° F., dumped and spread on the binder, tamped around manholes and gutters, and then rolled

throughout. A continual rolling is very necessary, as constant kneading action is necessary for a well compacted surface.

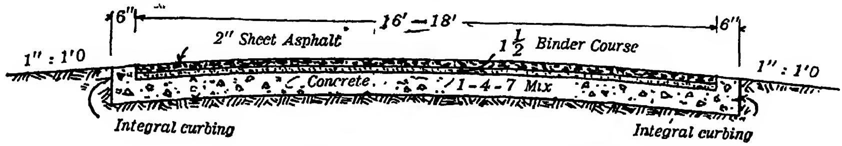


Fig. 2170 A Sheet Asphalt Road

2171. **Asphalt Block Pavements.**—These are made of asphalt blocks, laid in the same general manner on the same foundation as brick or wood block pavements. The asphalt blocks are made by compressing in molds a hot mixture of asphalt cement, mineral dust and trap rock screenings. The blocks are about the same size as brick, though other sizes are in use.

The asphalt block pavement has much the same appearance as the sheet asphalt pavement but is less slippery and somewhat more durable.

2172. **Asphaltic Concrete Pavements.**—There are other varieties of asphalt pavements known as asphaltic concretes. Of these there are two distinct classes, the Bitulithic and Topeka types of asphaltic concrete. Both are mixtures consisting of graded stone, sand, and rock dust or other fine material which is cemented together by a bituminous material. The chief difference between the two types is the proportion of voids allowed due to the grading of the stone. The Bitulithic is a patented pavement.

The asphaltic concrete is resilient, dustless, quiet and easily cleaned. It is less slippery than the sheet asphalt. It has about the same durability as sheet asphalt.

COMPARISON OF DIFFERENT KINDS OF ROADS AND PAVEMENTS.

Characteristics.

2173. **Earth roads** are inexpensive and easy to build and maintain. They are usually dusty. The surface is not hard, and consequently they are not satisfactory for heavy traffic, especially when wet. Generally, earth roads are the most economical of construction, though this may not be the case over a period of many years of repair.

Sand-clay roads are more durable and have harder surfaces than earth roads. If properly built, they are better than ordinary earth roads; but they are more expensive in first cost.

Gravel roads are excellent for purposes where the pavement is not subjected to much traffic. They are cheaper than macadam roads, but are not as smooth or durable. Water penetrates such roads.

Macadam roads are satisfactory for horse-drawn vehicles and are suitable for motor vehicles, but they wear down quickly under constant motor traffic. They are practically impermeable and consequently are more durable than gravel roads.

Brick pavements are durable, but their greatest objection is due to the noise of travel on them.

Wood block pavements are less noisy than brick. They are subject to decay, and even when properly treated are not as durable as brick. They are not so easily cleaned as the other pavements.

Stone block pavements take the heaviest kind of traffic. They are noisy, and slippery in wet weather, and the edges of the blocks chip off and make the pavement rough after several years.

Sheet asphalt pavements are the most satisfactory for all general purposes. They are not as suitable for heavy traffic as stone block. They become soft in hot weather, thus increasing the traffic resistance but increasing the foothold of animals. In cold weather, the reverse is the case: the traffic resistance is much decreased, but the pavement is very slippery. When decay once starts, it spreads very rapidly; therefore, repairs should be made very promptly upon appearance of weak spots.

Concrete pavements are smooth but noisy. They are easy to clean, are not slippery, and give good foothold for horses and excellent traction for motor vehicles. They are average in cost, require some maintenance, but are very easy to repair.

2174. The following table sums up, numerically, the comparison of various roads and pavements:

TABLE OF RELATIVE CHARACTERISTICS OF HIGHWAYS (AGG.)

	Initial Cost	Durability	Sanitary Qualities	Noiselessness	Slipperiness	Dustlessness	Appearance	Ease of Cleaning	Tractive Resistance	Ease of Maintenance
Gravel	1	7	7	4	1	8	7	7	7	6
Water-bound macadam	2	8	7	4	1	8	8	7	7	3
Cement concrete	3	6	1	6	3	4	5	4	1	3
Vitrified brick; grouted	6	3	5	6	3	4	4	5	1	8
Wood block	7	2	3	1	6	1	3	1	3	1
Sheet asphalt	4	4	1	1	8	1	1	1	4	2
Asphalt block	5	4	4	1	7	1	1	1	4	7
Granite block	8	1	6	8	3	4	6	6	6	3

Costs.

2175. The expense of construction of different types of highways varies greatly with the section of the country, and in the same section of the country the relative costs of the different types vary greatly with the materials present for the types in question. For example, the expense of constructing any kind of road will depend in any section upon the labor costs in that section and the facilities for transportation to the site of the necessary equipment and material.

For the same site of highway, the expense of construction will depend greatly upon the presence of suitable materials in the vicinity. In one section, as in the lumber states, a wood block pavement may be rela-

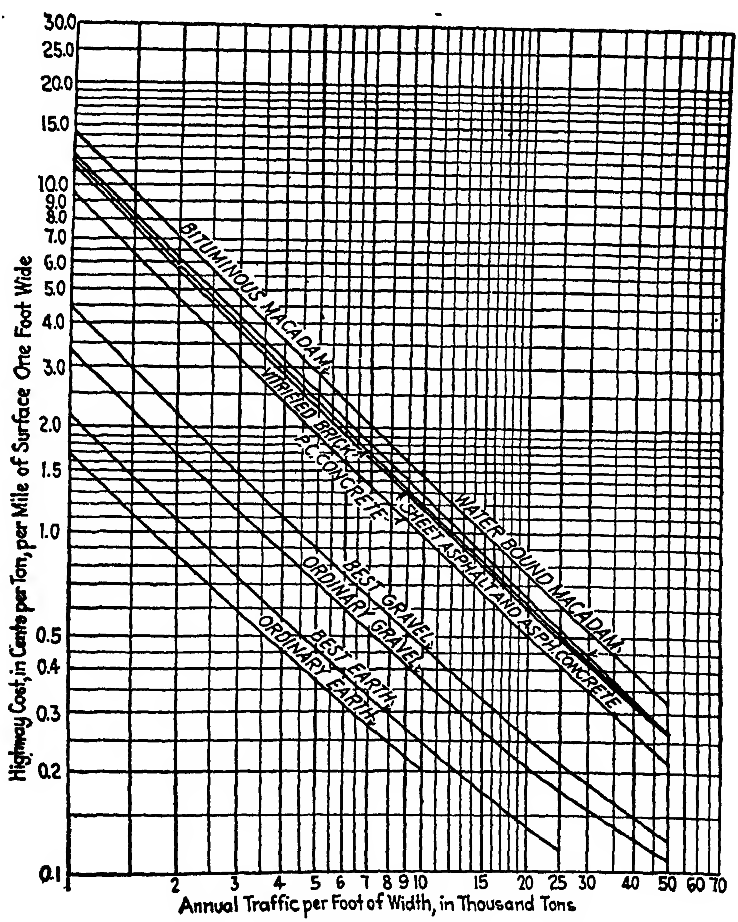


Fig. 2176 A —Diagram showing annual highway cost.

tively less expensive than it would be in sections where the wood for blocks must be brought from a great distance.

2176. The following table shows the average life without extensive repairs and the average cost per square yard of the different kinds of highways, excluding cost of grading.

	First Cost	Life—Years
Earth	$0.10	1
Sand + Clay	0.20	2
Gravel	0.40	3
Macadam	1.00	4
Telford Macadam	1.20	6
Concrete	1.50	10
Sheet Asphalt	1.80	16
Brick	2.00	20
Wood Block	2.50	18
Stone Block	3.00	30

At first it would appear a simple matter to determine the relative costs by obtaining the annual expense for the useful life of any highway; but the calculation is complicated by many things. For example, the annual cost of maintenance varies greatly, some of the highways have surfaces which can be sold after useful life, the interest on additional first cost must be calculated, and the amount of ton-miles of traffic which have been sustained by the different kinds of highways enter into the calculation of actual service rendered. The chart in Fig. 2176A (taken from Agg's Construction of Roads and Pavements) represents a comparison of average cost of actual service rendered.

For more detailed information, see

City Pavements, Besson
Principles of Highway Engineering, Wiley
Construction of Roads and Pavements, Agg
Roads and Pavements, Baker
American Highway Engineers' Handbook, Blanchard.

CHAPTER XXII.

CITY ENGINEERING.

2201. The engineer of a city government is confronted with a great variety of engineering problems. A railway engineer must know principally about railways; an electrical engineer must know about lighting, heating and power; a sanitary engineer must know about water purification and about sewerage; but a city engineer should know about these and many other branches of engineering. In a large city, the senior engineer has assistants who are specialists in the various branches of engineering; but in a small city, he should himself possess this broad knowledge because he may have no engineer assistants.

2202. In recent years, there have been many changes from the old system of city government by elected aldermen. First, there was the change to Government by a small number of Commissioners elected by the people. The first such commission form of city government was adopted by the voters of the City of Galveston in 1901, after their experience in the disaster of the Galveston flood had forced them to see the necessity for change from the old system. Fig. 2202A shows the commission form of government adopted in 1907 for Des Moines, Iowa. This is very similar to the commission form of government which has been in use in Washington since 1874, in which city there are three commissioners, two of whom are civilians and the third an officer of the Corps of Engineers, all appointed by the President. Although this Washington Commission form of government was in successful operation and was satisfactory for 26 years before the Galveston flood, yet its success did not attract much attention because its commissioners were appointed and not elected by the voters.

2203. A study of Fig. 2202A shows that in Des Moines one of the Commissioners is in charge of the Department of Streets and Public Improvements, in which practically all questions require a knowledge of engineering. The work of the Department of Parks and Public Property is also almost entirely engineering; and a small portion of the work of the Department of Public Safety requires a knowledge of engineering. The difficulty of inducing competent engineers to seek election as commissioners and the fact that the total amount of engineering was greater than that of all others combined led to the City Manager system, a system wherein the Commissioners are elected as before and they employ an engineer as City Manager and hold him responsible for all engineering and most of the other problems of the City government.

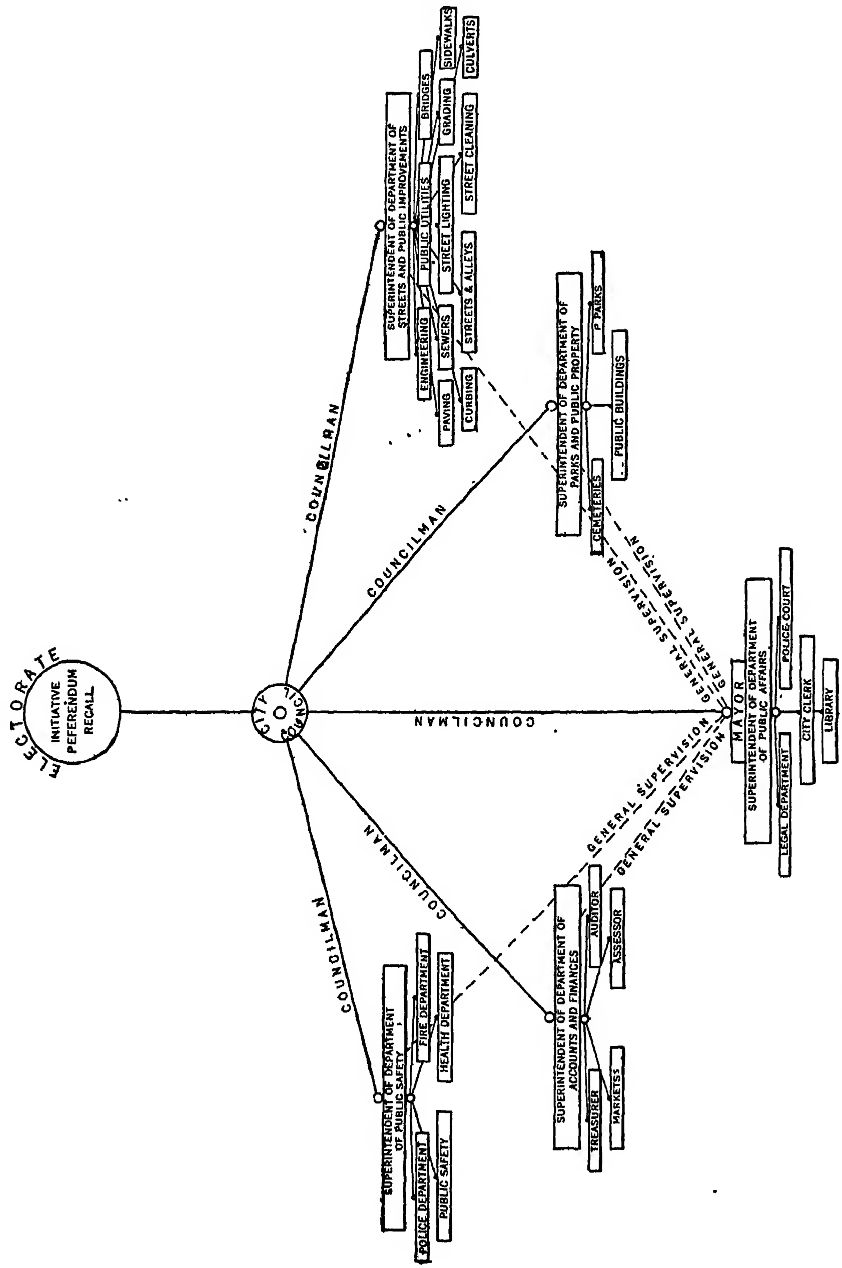


Fig. 2203 A Diagram of Commission Government of Des Moines, Iowa

CITY PLANNING.

2204. It is not often in modern times that an entirely new city is to be built on an unoccupied site, as was the case when Washington City was built on the banks of the Potomac, and recently when Canberra was designed as the new capital of Australia. However, certain principles of City Planning are now so generally accepted that all cities should have a comprehensive plan and should force all future activities to the consummation of this plan. With an entirely new city, no restrictions would be necessary; with an old city, the activities are guided to this plan by certain restrictions such as refusing to issue permits for factory construction in certain areas, setting aside certain streets for street-cars, designating selected areas for railway yards and tracks, and limiting heights of buildings along certain streets.

Surveys.

2205. Nearly all cities in Europe and many in the United States have grown without any comprehensive controlling plan. Therefore a survey must show not only the topography of the city, but also its present social and economic condition. Further, so as to be satisfactory, this survey should be revised each year in order to keep up with the growth of the city.

2206. The topographic survey requires first and foremost a contoured map of the ground. This is needed for determining the lay of the ground as affecting lines, curves, and grades. But there are other considerations affected by topography: watercourses suitable for canals or for water supply; natural advantages for harbors and piers in cases of navigable streams or harbors; good centers for the junction and crossing of highways and railways.

The survey must show the character of the sub-soil, with reference to its suitability for foundations, subways, and sewer lines.

City surveys must be more accurate than other surveys. Errors of 1/5000 must not be exceeded in the suburban districts, and errors of more than 1/20,000 cannot be allowed in the congested districts.

2207. The social and economic information must comprise every type of existing public property, such as streets, parks and public buildings; all public services, such as transportation, lighting, and water supply; distribution of population and industries; the housing of the people, size and type of houses, size of yards, distances to parks.

Zoning.

2208. The survey completed, the next step is to anticipate as far as possible the future lines of normal growth of the city, to provide a comprehensive plan for taking care of this normal growth, and to link up this

plan with the necessities of improving the existing conditions of the city not only for its present requirements but for its future normal growth.

2209. **Zoning** is the first requisite for such a comprehensive plan: it consists in definitely setting apart certain sections of the city as factory or mercantile or residential districts, and restricting these sections to such uses. In addition, zoning prescribes height limits and the amount of ground to be covered. This plan has been in use for many years in European cities, but its introduction into America has encountered many difficulties. However, the necessity of zoning is being gradually accepted in the United States. Practically every city has its fire limits within which new buildings must be of fire resisting material. Boston has heights of buildings prescribed for certain districts, other cities have certain zoning restrictions vitally necessary to themselves, Washington and Los Angeles have practically complete zoning restrictions. In 1923, 22,000,000 people comprising 40 per cent of the urban population of the United States lived in 183 cities which had accepted zoning systems.

Residents in many cities have combined to form their own private zoning restrictions, where their cities would not do it for them. As a result, practically every city has some zoning restrictions, public or private.

Street System.

2210. It is easily seen that the street system is a complement of the zoning system. The duty of streets is to provide communication between different parts of the city: the streets must be wider or there must be more of them where the traffic is greatest, viz., at the railway and central street-car stations, on the main business streets, and on main lines from central business districts.

2211. The basis of all well-regulated street systems is the grid-iron, making each block a rectangle. Where the topography renders this impracticable, the streets follow curves or diagonals until the difficulty is overcome, and the gridiron system is then resumed.

In the original plan of Philadelphia, the city was laid out in squares with sides of 400′. In New York the blocks are 200′ × 800′.

2212. A strict adherence to the gridiron system is not necessary nor advisable. The distance of travel from one point to another not on the same street would always be along the two sides of a rectangle instead of along the shorter hypotenuse. If the two sides of the rectangle are the same, the distances are as 2.0 to 1.4. Therefore, diagonals are run from the places of greatest congestion. These diagonals are usually made wider than other streets, because they contain the street-car tracks and are used by bus lines and wheeled traffic proceeding to greater distances.

For special localities, as railway stations, or post offices, several diagonals are suitable.

2213. Fig. 2213A shows the plan of the City of Washington as prepared for President Washington by Pierre Charles L'Enfant, a distinguished French engineer, selected to lay out the new Capital of the United States.* (The actual surveying and laying out was done by Andrew Ellicot, later Professor of Mathematics at West Point.) A glance at this plan shows the gridiron system, the diagonals, and the circles at the most important junction points. This L'Enfant plan has been satisfactory and has been generally adhered to for over one hundred years: even the advent of street-cars and automobiles has not rendered necessary any vital change in it.

2214. The simplicity of the gridiron street system has tremendously increased its use. By numbering its streets successively in order, and by numbering the houses in hundreds corresponding to the street numbers, it is a simple matter to find any building. This numbering plan is so satisfactory that in Washington the buildings on the diagonals (avenues) are numbered in accordance with their locations between certain streets.

Likewise, property lines can be more clearly described with the gridiron system. Areas must be defined by straight or curved or broken lines. The gridiron system makes possible the description of property by a statement of four lines. With curved lines the description is very difficult: and with broken lines there must be a description of each of the broken lines.

STREETS.

2215. **Classification.**—As previously explained, all streets are not the same width. Those which carry the most traffic should be the widest and have the best pavement. In less important districts, wide streets are not desirable, as the area can be more useful for other purposes.

W. B. Munro, in his book on Principles and Methods of Municipal Administration, classifies streets as follows:

> (1) The arterial streets, or traffic thoroughfares. In this class come the streets which are the direct route of through-the-city and across-the-city traffic, particularly the automobile traffic. They may be retail, wholesale, or residential streets from the viewpoint of use by abuttors; but their prime characteristic is their use as trunk thoroughfares.

The L'Enfant plan covered only a portion of the District of Columbia. The present Highway Plan covers the whole of it.

Fig. 2213 A Part plan of Washington City as prepared by L'Enfant

Every city has a few such streets; the smallest community has at least one of them.

(2) The retail business streets. These are usually the most congested by pedestrian, vehicular, and street-car traffic. They bear also the largest number of impediments in the way of poles, signs, street clocks, and so forth.

(3) The streets of the wholesale, financial, shipping, market, and office districts. Here the foot traffic is not so large, nor is the street-car always a factor in congestion; but slowly moving vehicles are more numerous and the streets should be adapted to their use. The practice of backing trucks up to the curb for loading and unloading should also be taken into account. Vehicles so placed take up about fourteen feet of roadway.

(4) The main streets of residential districts. In such street areas the traffic of all kinds is relatively light, but much of it is quickly moving, and a large portion of it is cross-city or through traffic.

(5) The minor streets of residential districts. These streets bear very little through traffic, and what they do carry is highly diversified. They form by all means the most numerous class of streets in any city.

(6) Boulevards, esplanades, and parkways. Roadways of this type are used almost wholly for pleasure driving; heavy traffic is usually excluded, and hence no unusual strain need be provided for.

(7) Alleys, lanes, courts, and passageways. This class includes the narrow public ways that are used almost altogether by delivery wagons, garbage collectors, and so on.

2216. It is evident that the classification of the street determines the engineering connected with it. Some streets will require good pavements and constant care; others may not require more than drainage and sanitary precautions. Some streets are selected for the main sewers though they may not at all be the most important. Each street requires special care in accordance with its own special problems.

2217. **Width.**—Many points must be considered in determining the proper width of a street. Its classification will determine its principal use and will be the main factor in determining its width, but other factors are of great importance. For example, a double-track street-car line with due clearance requires twenty feet. The usual allowances are as follows:

Street cars, per track	10 feet
Vehicle traffic, one line	8 "
Standing vehicles, one line	7 "
Sidewalks	9–16 "
Gutters, each	2 "
Strips for trees	5 "

The above figures must be varied somewhat, in each particular case, but they can be used as a basis for calculation. For example, a minor residential street with one street-car line would require a width as follows:

One street-car line	10 feet
Two zones for vehicles	16 "
One zone for standing vehicles	7 "
Two sidewalks	18 "
Two gutters	4 "
Two strips for trees	10 "
Total	65 "

2218. The above figures show that the width of the street is not directly dependent upon the vehicular traffic upon it. The recent remarkable increase in automobile traffic has brought additional complications. Parking space must be provided for these automobiles, and the streets seem more satisfactory than areas set aside, especially as there are generally no such areas available. A principal thoroughfare should not be encumbered with these vehicles; therefore, it appears that a side street near such a thoroughfare may actually require as great a width as the much more important thoroughfare.

Similarly, the streets of residential districts require special consideration. These streets usually have special strips for trees, whereas trees are not desirable in the most congested business districts. Some of these streets have street-car lines, and require extra width on that account. It is thus seen that even in the suburbs each street requires special consideration.

2219. The existing widths of streets (including sidewalks) in different cities give some idea of the requirements. Most streets in New York have widths of from 60′ to 66′; Chicago streets are about the same. These widths are satisfactory in some cases; but are generally unsatisfactory due to the automobile traffic, and one-way traffic regulations have been found necessary. Washington's numbered and lettered streets are about 90′ wide and most of the avenues (diagonals) are 160′ wide. Streets of European cities are usually very narrow, but some special streets are quite wide: Unter den Linden in Berlin is 193′ wide and the Avenue des Champs-Elysees varies from 230′ to 260′ wide.

2220. **Area.**—As a rule the streets constitute from 25 per cent to 35 per cent of a city's entire area. The streets of Boston, Philadelphia, and New York are about 30 per cent of the area: those of Washington are about 45 per cent.

2221. **Sidewalks.**—In some cities, standards have been adopted whereby the sidewalk is a fixed percentage of the total width of the street. As a rule, the width of the sidewalk is about 40 per cent. However, there are always exceptions to such a standard: for example, a street in a wholesale district will require a comparatively small sidewalk and a street in a warehouse district will require practically no sidewalk at all.

2222. **Pavements.**—The pavements in city streets do not differ in any essential from the pavements of the most important highways. Their foundations and wearing surfaces have already been described in the chapter on Highways.

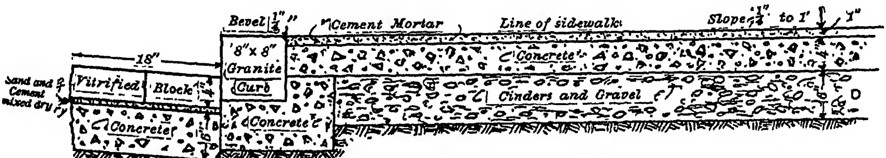


Fig. 2222 A Pavement for a sidewalk

However, the pavings of sidewalks are somewhat different (see Fig. 2222A). A study of this figure shows that the foundation is not rigid, as no great weight is to be borne, and the wearing surface is not thick for the same reason. To prevent a sidewalk being one slab and consequently breaking when slightly moved by expansion of subgrade, joints are placed about 5′ apart along the sidewalk. These should be completely through the sidewalk so that each small slab can be raised by changes in temperature of subgrade without raising the adjacent slab or causing it to crack.

2223. The care of city pavements is more expensive than the care of highways. They must be kept cleaner and in better repair. Also an additional expense arises from the constant tearing up of pavements for laying of pipes or sewers. This expense is considerable: in Boston, a count shows that about 15,000 such openings are made each year. Most of them do not occupy the entire width of the street, but their total dimensions in direction of traffic were $33\frac{1}{3}$ per cent of the length of the city's streets. Of course the torn place is renewed without expense to the city, but the new pavement rarely fits well with the old and the city is put to additional expense besides losing the torn area for a short time.

2224. Pavements next to street-car lines are quickly worn out by the shock of the moving cars. Special paving is therefore laid next to the car tracks. Payment for this is usually required of the street-car companies.

PARKS AND PLAYGROUNDS.

2225. Each city usually has at least one large park for general use of all of the inhabitants, and small parks with playgrounds for use of the people in their immediate vicinity. Of the two, the small parks with playgrounds are undoubtedly the more valuable, and cities without them have in many cases been forced recently by public opinion to buy land and make them.

2226. It appears that about 5 per cent of the area of a city should be set aside for local parks and playgrounds. William Penn, in his plan for Philadelphia, set aside about 4 per cent for this purpose. His successors did not maintain these areas, and now only about 1 per cent is left: the result is that Philadelphia's local parks are very inadequate and great expense will be necessary to make them adequate.

2227. The size and location of the large park depends on the topography. In New York, it includes the rocks and difficult places: in other cities, it includes the streams, where building sites are not so satisfactory. However, the large park must include areas suitable for athletic fields and public assemblages. These are often in separate parks, making two or more large parks in the city.

BUILDINGS.

2228. Each city of any importance has a building code of its own. All of these codes are similar, prescribing allowable strengths of materials, materials which may be used in different zones, wind and snow loads and heights of buildings in the different zones. City building inspectors check the plans of proposed buildings and inspect them during construction to see that they are safely constructed in accordance with established principles of engineering.

2229. Safety against the commencing and spreading of fires is one of the most important requirements of the building code. In certain zones most of the materials of construction must be non-inflammable, so-called "fire-proof". The principal requirement is that the framework shall not be of wood.

The heights of buildings are prescribed in all zones. Roughly the height of the building must not be greater than the width of the street. This results also in improvement of appearance. Under its zoning

regulations, Washington City is divided into four districts with respect to height of buildings. These districts are:

- *a.* 40′ districts, for buildings not over 40′ or over 3 stories high: these districts contain the small one-family houses.
- *b.* 60′ districts, for buildings not over 60′ or over 5 stories high: these districts contain the small apartment houses.
- *c.* 90′ districts, for buildings not over 90′ or over 8 stories high: these districts contain the large apartment houses and lower industrial structures.
- *d.* 110′ districts, for buildings not over 110′ high: these districts contain the high office buildings and high industrial structures.

There are certain rare exceptions to these height limits, each exception being granted only after special consideration.

2230. In planning public buildings, special allowances are made. Preferably such buildings are erected in parks so that extra space will be available around them. Also, as these buildings are usually of greater architectural merit, the street system is often so arranged that the public building is the center of view from several approaches.

PUBLIC UTILITIES.

2231. In some cities, the public utilities are owned by the city: in others, they are owned by private capital. In either case, they should be controlled for the benefit of the public by regulations and laws governing their operation.

2232. **Light, Heat, and Power.**—Standardization of electric current is preferable for general purposes: yet there are so many needs for special lighting or heating or power that both direct and alternating currents are found in all large cities. For lighting and for ordinary household uses, 110 volt, 60 cycle alternating current is generally used. For outlying districts, alternating current is always used, as it can be transmitted at high voltages for great distances much more cheaply and then stepped down to 110 volts AC for household use. This plan also applies to motors in these districts, even though direct-current motors especially in small sizes are somewhat more efficient. In central and closely settled districts, direct current is a little cheaper; but in small cities this slight advantage does not overcome the disadvantage of having two systems, two sets of generators, two sets of wires, and two systems of supplies for users. Therefore, the tendency at present is to place more and more reliance on the alternating current system, and one system is preferable for small cities.

2233. Regulations cover the insulation in buildings and transmission through the streets. In buildings, the wires must be visible or at least easy of access: each wire must be covered with prescribed insulating material and supported by porcelain or other insulated knobs. In the streets, the wires may be overhead or underground. The overhead system is cheaper but is exposed to interruption by storms, snow, or accidents. The underground system requires a trench in which the wires may be placed directly if covered with sufficient insulation, or conduits of wood or fiber may be laid and the wires drawn through them. The latter system is preferable.

2234. Gas for lighting is being replaced by electricity; in fact, it has been replaced altogether for this purpose except in localities where natural gas is abundant and very cheap. However, it is much cheaper for heating and it is extensively used for cooking and for furnaces. The first cost of installation of pipe is greater than for electricity, and for sparsely settled suburbs, the cost of the necessary pipe system is prohibitive.

2235. **Water Supply.**—This must be sufficient in quantity: and the principal difficulty in connection with city water supply is the prevention of waste so that the supply may be always sufficient. Even if the supply at first is more than sufficient, it will soon cease to be so unless regulations are enforced to stop leaks, breaks, and carelessness in use.

The source of water supply is usually a reservoir from which the water is led into the distribution pipes. This is often supplemented with cisterns and wells. As the reservoir water can be more properly controlled and purified, it is customary to forbid the use of these local supplies when the water pipe system is laid to these sections.

2236. The pressure of water at the street level varies between 40 and 110 pounds per square inch. A higher pressure will burst the pipe system; a lower pressure will not force water to the top floors of the houses. Also, a lower pressure renders the system practically useless for fire purposes without fire-engines: yet, as fire-engines should be available for throwing water to such heights as necessary, the pressure for fire purposes is not a primary consideration.

2237. **Waste Disposal and Sewerage.**—The amount of waste of every sort, including liquids, in a city amounts to about a ton per day per person. The greater part of this goes through the sewers; but much of it consists of ashes, papers, refuse, and garbage which must be taken away by special arrangements. In some cities, this removal is done by city employees; in others by contractors paid by the city; in still others, by licensed scavengers paid by the individual householders. Most of such waste can be burned: but there is always enough waste even from incinerators to make the disposal of it a difficult problem. In coast

cities, it has been carried to sea and dumped overboard, but this was not satisfactory. Generally, valueless or cheap land is set aside as a dumping ground.

2238. Sewers are discussed in a separate chapter. In large cities, the sewers vary from 4″ at a house outlet to many feet in diameter for the main sewers. Efforts have been made to place gas pipe and electric wires in these large sewers, so as to save the trouble of tearing up streets for changes or repairs of these lines, but results have generally been declared unsatisfactory, principally because of initial expense in constructing sufficiently large sewers.

The largest city sewers are enormous. In Paris, the largest sewer is 30′ in breadth by 16′ in height, in a system of 725 miles; the largest sewers in Paris carry the gas pipes and electric wires.

2239. **Street-cars.**—Special track construction is necessary: rigid car tracks are best for the pavement but are hardest on the passengers. Concrete and metal ties have been tried, placed on concrete or ballast foundations. European practice now uses these ties, placed usually on concrete foundations; but (Figs. 2239A and B) American practice calls

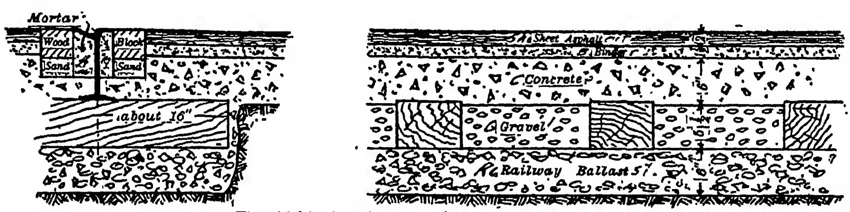


Fig. 2239 A Street-car track construction

for wooden ties laid on a very well packed railway ballast. The rails are preferably grooved girders as in the figures, thus fitting better with the adjacent pavements, though traffic wears them out more quickly.

Fig. 2239 B Street-car track construction

The pavement is laid between the tracks with concrete foundations as elsewhere: and the rails are grouted to this concrete foundation, thus providing more rigidity. With sheet pavements, blocks are laid next to the rails where the shock is greatest.

2240. For many years, except in a few of the largest cities, the transportation of the public has been accomplished by surface cars. Later, because of the introduction of elevated lines and more especially of subways in the congested areas, and because of the extensive use of the automobile, the surface car has been losing ground. Even with the skip-stop which has been almost universally introduced to speed up traffic, the surface line is slow and tends to delay vehicular traffic. The minimum time between cars is 20 seconds in New York and 30 seconds in Washington. The subway, the elevated, and the fast suburban trains with an uninterrupted right of way, can run more closely and are not delayed by cross traffic. With their multiple car trains, permanent stations and loading platforms, and speedy service, they are at present the only solution for the greatest traffic.

Motor busses are growing rapidly in favor. At the present time they usually have a slightly higher fare than the surface cars, but they are faster, more comfortable, and do not slow up automobile traffic. The busses usually pay less to the city in taxes, but this is gradually being adjusted. In 1923, the street-car companies in Washington, D. C., paid much more tax per revenue passenger than the bus company. However, a gasoline tax of two cents per gallon has been initiated, so that this discrepancy will largely disappear.

WHARVES.

2241. In the interior of harbors and on navigable rivers, bulkhead and pierhead lines are established by the United States Government. The bulkhead line is the line to which solid or solid-filled structures may be built. The pierhead line is the line to which open or pier structures may be built, and beyond which is the fairway for movement of vessels. The bulkhead line is usually the line of mean low tide, as it is desired to allow no obstructions to the natural flow of water in the channel.

2242. Piers should be long enough to berth the longest ship to use them; and should be so spaced that two ships, one alongside each pier, may be accommodated in the slip. The width of piers depends upon the use to which they are to be put, and varies from 40′ to 100′ to 250′ in extreme cases. The live loads in design are quite large, 600 pounds per square foot being the average, special designs being necessary for railway tracks and heavy cranes.

For more detailed information see

Principles and Methods of Municipal Administration, Munro
City Planning, Robinson
Commission Government, Bradford
Reports of the Commissioners of the District of Columbia

PROBLEMS.

P. 2201. Calculate the exact percentage of street area in a square mile of the rectangular system of New York City streets, assuming each street to be 66′ wide and that there are no diagonal streets.

P. 2202. Same for Philadelphia.

P. 2203. Same for Washington lettered and numbered streets, assuming one avenue (diagonal) across the center.

P. 2204. Compare the distance via street from one corner to the opposite corner in the three cases above.

P. 2205. What is the height to which water will ascend from a 60 pound per square inch pressure in the street. Disregard friction.

P. 2206. Same for 40 pound pressure.

CHAPTER XXIII.

CONSTRUCTION EQUIPMENT.

2301. In every engineering enterprise, construction equipment involving the use of power driven machinery is essential to a systematic, economical and speedy completion of the project in hand. No fixed rules can be given, however, as to the most economical form of power to be used. This will be a function of many elements largely dependent on the conditions imposed by the location and accessibility of the site of the construction work, the feasibility of transporting fuel, and, in general, the kind of project. The best choice may be steam, gasoline, gas, electric, or water power, or any combination of these with mechanical, pneumatic or animal power. This wide choice as to the source of power enlarges the field from which the construction engineer or contractor may select equipment to meet his need in each individual enterprise. The selection of construction equipment therefore becomes a matter largely dependent on the experience and personal preference of the contractor, and, with the advance in methods of construction and consequent continued improvement in construction equipment, it necessitates his keeping thoroughly informed as to the types of modern construction equipment available.

However, the scope of the work will govern the extent of the plant required. Often, young engineers on small construction projects, inspired by too much enthusiasm, build up plants to aid in construction which could have been accomplished with little or no machinery and in less time than was needed for the construction of the plant. Hence, a careful study must be made to decide the character of plant which will effectively perform the required work at least net cost of the structure plus the original cost of plant less its selling value at the close of construction operations.

CLASSIFICATION.

2302. In its most general classification, construction equipment may be grouped according to the operation in which it is used. We shall therefore consider it as being divided into

(1) Power Producing Equipment.
(2) Equipment employed in Excavation (tunneling, shafting, boring, drilling, dredging).
(3) Crushing Equipment.
(4) Lifting and Handling Equipment.
(5) Hauling, Conveying and Distributing Equipment.
(6) Concrete Mixing Equipment.

POWER PRODUCING EQUIPMENT.

2303. Of the power producing engines available, the steam engine, gasoline engine, and electric motor are the types usually employed. The principle governing their operation is well known; the economics of their operation is beyond the scope of this text. On construction work these engines are of a simple type, preferably light enough to be readily transported from place to place on the job. Heavy equipment having to do with the utilization of power is generally provided with a complete power unit forming an integral part of the equipment itself. Thus we find steam shovels, cranes, road rollers, concrete pavers, all operated under their own power.

2304. **Steam Hammer.**—Chapter XIII on Foundations gives a detailed discussion of the steam hammer used as a pile driver. This is an example of the application of steam to develop a driving force rapidly and repeatedly applied. The same end is also accomplished in smaller hammers by compressed air.

2305. **Air Compressors.**—Air compressors may also be considered in the nature of auxiliary machines developing their own form of energy from power of another source. They are usually run by gasoline engines

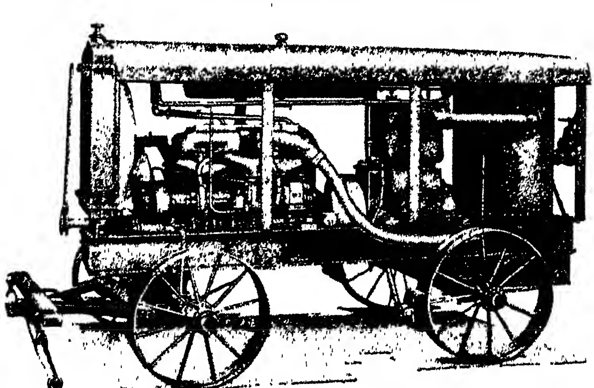


Fig. 2305 A Portable Gasoline-Driven Air Compressor

mounted with the compressor on a carriage which may be freely moved from place to place. This obviates the necessity for an extensive piping system to connect with the implements (drills, hammers, riveting machines, and other pneumatic tools) utilizing the air. Fig. 2305A shows a portable gasoline driven air compressor consisting of engine, compressor

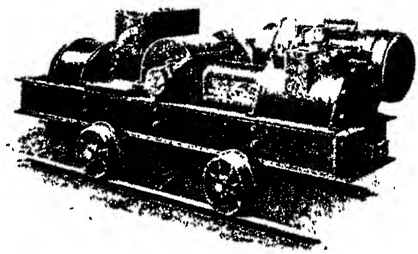


Fig. 2305 B Mine-Type Electric Compressor

and air tank. Smaller sizes may be had of many different types, varying with the make of the machine. Compressors driven by electric motors mounted on small wheeltrucks are considerably less bulky and can be carried up very near to the work in hand, obviating the necessity for a long piping system to deliver the air (Fig. 2305B).

EQUIPMENT USED IN EXCAVATION.

2306. **Steam Shovel.**—The steam shovel is employed in the excavation of earth and loose rock. As shown in Fig. 2306A, this consists of a steel plated, steel toothed scoop called a "dipper" attached to the end of a long arm by means of which the dipper may be forced into the earth at the same time that it is lifted up by a hoisting cable, thus causing a scooping effect which fills the dipper with the material undergoing excavation. The dipper arm is supported by a boom, which, at its lower end, is connected to a platform. This platform carries a cab, housing a steam boiler and engines which provide power for hoisting, swinging the dipper, and "crowding" (forcing dipper into the earth). The crowding engine is shown on the boom in the figure, its gears in mesh with a rack on the dipper arm. When the dipper is full, the platform is revolved on its base, swinging the dipper to a position above waiting trucks or wagons. It is lowered to a convenient height above these, a door in the bottom of the dipper is released and its

contents emptied into the waiting trucks. This operation is then repeated; and the amount excavated may average fifty or sixty cubic yards of material per working hour. After a section is excavated, the steam shovel is moved by its own power to another portion of the embankment. The shovel shown in Fig. 2306A is provided with a caterpillar mounting, which type of traction enables the shovel to move up quickly without any planking, and also enables it to climb grades even up to 30 per cent.

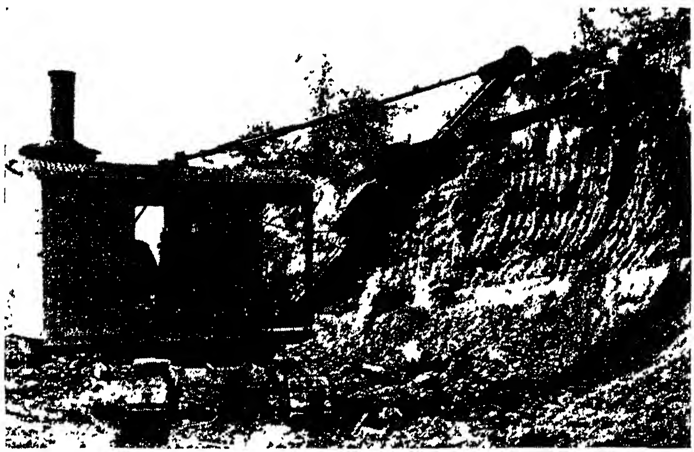


Fig. 2306 A. Steam Shovel

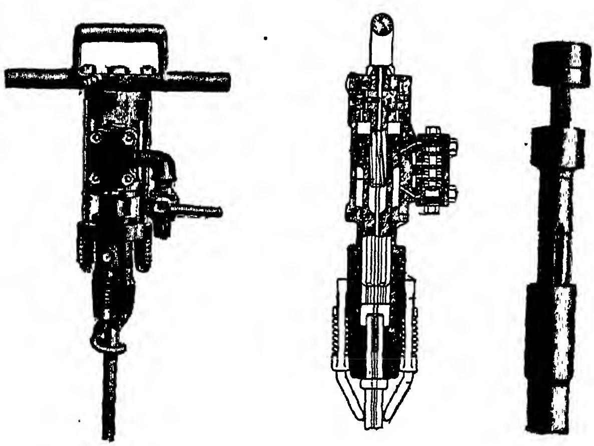
2307. **Dragline Excavator.**—Dragline excavators are similar to the steam shovel. In dragline excavators, however, the dipper and dipper arm of the steam shovel are replaced by a cable carrying a heavy toothed scoop which is dragged along the material to be excavated and is filled up by reason of its own weight and sharp lower edge. They are obviously limited in their use to excavation work which is comparatively shallow and level, whereas the steam shovel may be used in excavation of side slopes practically vertical. The latter may also be controlled with far greater accuracy as regards the limit and grade of the excavated slope.

2308. **Excavation of Earth in Road Construction.**—In the excavation of earth in road construction, many forms of scrapers, graders, etc., are available. These are employed in the preparation of the earth subgrade, after which it is compacted by the use of road rollers, which are merely heavy rollers moving under their own power and wide enough to properly distribute their load.

2309. **Excavation of Solid Rock.**—In the excavation of solid rock as in tunneling, drills and blasting equipment are necessary, the type of drill

bit used being suited to the quality of the material excavated. The principal kinds of drills employed include hand hammer drills, mounted hammer drills, electric air-drills, churn drills, and core drills, together with many special kinds of deep well drilling machinery and their accessories.

2310. *Hand Hammer Drills.*—Hand hammer drills are employed in shaft sinking, tunneling, trenching, mining, quarrying, and general utility work of contracting. They are economical in their operation, easy of manipulation, portable, durable and rapid in their action. They are available in several sizes whose depth capacities range up to twenty feet, each size being especially favored for its own particular type of work. Compressed air being the usual actuating power for these drills and conveniently available, the drill steels are made hollow and a blower tube is provided which sends an air jet through the drill to the cutting point. This acts as a blower to remove the rock dust and chips from the bottom of the drill hole during the operation of drilling. Similarly



A B C

Fig. 2310 Rotating Hammer Drill

a water jet device is used in combination with air. The wet method is employed in underground work* while the dry method is generally used in outdoor work. In Fig. 2310A is shown a Rotary Hammer Drill and in Fig. 2310B, a diagram of its parts in cross-section. This drill is of the reciprocating piston type, its rotating mechanism being shown in Fig. 2310C. The blower tube may be seen passing through the center

* Dust is thereby avoided.

of the drill. For use in the wet method this would be connected to a tube from a water tank and the water introduced under a pressure regulated by a needle valve in the top head of the drill.

2311. *Mounted Drills.*—Mounted hammer drills are similar in principle to the hand drills, being especially adapted to drilling horizontal holes and deeper vertical holes than the hand drills. They may be mounted on tripod mounts or on columns or bars clamped into position between projecting ledges of rock, thus forming a substantial support for the drill. Tripod mountings are more or less obsolete in present day construction work, the column mountings being adaptable to drilling under conditions in which the tripod could be used and also to drilling for which the tripod is not suitable. Fig. 2311A shows a column mounting.

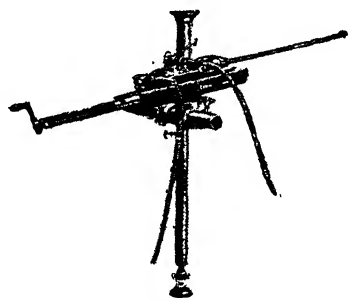


Fig. 2311 A Column Mounting

2312. *Electric Air Drill.*—Another type of drill called an electric air drill is supplied with compressed air by an electric pulsator. The system consists of a closed air circuit connecting the pulsator with the drill by two tubes, each alternately acting as supply and exhaust, a constant compression being maintained by compensating valves.

2313. *Churn Drills.*—Churn drills are operated by a drilling machine fitted up with a derrick, cable, pulleys, etc., and a power unit which raises and drops the drill, thus producing a churning effect from which the name is derived. These drills are to be had with depth capacities running as high as several thousand feet.

2314. *Core Drills.*—Diamond drills are a type of core drill utilizing diamonds as the cutting tools. In Rotary Shot drills, steel shot replaces the diamonds, acting on the same principle. Core drills are discussed with Wash Borings and Augers in Chapter XIII on Foundations.

2315. *Drill Sharpening and Repair Machinery.*—Drill sharpening and repair machines are, of course, necessary to the contractor who has

a large amount of drilling to do. Numerous types are procurable on the market, as well as many types of special drills not dealt with herein.

2316. *Underwater Excavation.*—Underwater excavation involves the selection of equipment to conform to the special method adopted in dealing with the problem. Several methods are given in the Chapter on Foundations.

2317. *Dredging.*—Dredging is a method in favor where the excavation is carried on over a considerable area as in the deepening of channels. A Dipper Dredge consists of a barge equipped with a steam shovel built on large proportions in order to meet the requirements of deep dredging. Bucket Dredges use a chain of buckets for the hoisting of the dredged material. Clamshell Dredge, Orangepeel Dredge, etc., are names indicating the type of container used with derrick equipment to accomplish the dredging.

2318. *Hydraulic Dredging.*—Hydraulic dredging is used in the underwater excavation of material such as sand and gravel or material which has been broken up into comparatively small fragments. In this method, pumping machinery is employed as a part of the equipment on the dredge boat. The end of a pipe line is brought into contact with the material and an "agitator" stirs up the material. The pumps draw water through the pipe line and this force draws the agitated material from the bottom along with the current of water, and through the pipe line. The material may be thus piped to waiting barges where the solid material sinks and remains in the barge while the water overflows, or it may be piped ashore to any desired dumping site. The latter method has been found economical in filling in marsh lands and extending property out to seawalls by a "hydraulic fill" composed of the material dredged from the bottom of the channel. It has been extensively employed by the Corps of Engineers in reclaiming Government shore lands and extending harbor fronts to previously built sea-walls.

CRUSHING EQUIPMENT.

2319. In modern engineering, crushed rock enters into nearly every type of construction work with which the civil engineer has to deal. The crushing equipment, when completely installed, comprises crusher, conveyer, separating screen, and storage bin, with an engine to furnish the motive power.

2320. **Rock Crusher.**—A rock crusher consists of a substantial metal housing containing a crushing mechanism usually operated by a belt drive connected to the flywheel of a steam engine. This mechanism is, in principle, merely the application of sufficient compressive force to overcome the internal resistance of the material, thereby crushing it

into smaller fragments. The smaller stones, emerging from the crusher, are caught and carried up by a chain of buckets into an inclined, revolving cylindrical or conical screen perforated with holes of graded size. The small stones pass through the nearest holes in this screen, medium sized stones are carried on down to the next sized holes, and so on with the several sizes. A bin forms the support to the screen and catches in each of its partitioned compartments a single graded size of stone. Fig. 2320A shows a crushing plant and accessories. The bin is seen to

Fig. 2320 A Gyratory Crusher Plant

be provided with small chutes, which when raised allow no crushed stone to emerge from its side openings. The bin is elevated in order that wagons may pass underneath the chutes, the loading being then effected by the force of gravity when a chute is lowered. Rock crushers are of many types, with crushing mechanism ranging from revolving discs, hinged hammers and oscillating jaws to corrugated gyratory cones. Of these, the two principal types are Jaw Crushers and Gyratory Crushers.

2321. *Jaw Crusher.*—A typical crusher of the jaw type is shown in section in Fig. 2321A. Stone is fed into the opening between E, which is a corrugated stationary jaw, and C, the movable jaw. The latter is given an oscillating motion by the cam I which is rotated on the belt-wheel shaft H. This grinds the stone between the jaws until it is broken into pieces small enough to drop through the opening at the bottom of the jaws, which may be adjusted to produce the desired size of stone. The principal disadvantages of this type of crusher are its limited capacity and the lack of uniformity in the size of its output. Below a capacity of twelve tons per hour, however, jaw crushers weigh less and are cheaper. This type is in general use, therefore, where the output is not necessarily of great volume and where a high percentage of uniformity in the size of the stone crushed is not essential. These disadvantages are to a great extent eliminated in the gyratory crusher.

2322. *Gyratory Crusher.*—A gyratory crusher is shown in Figs. 2320A and 2322A. In the sectional view (Fig. 2322A), the main shaft is shown supported by a bearing centrally located above the mouth of the crusher

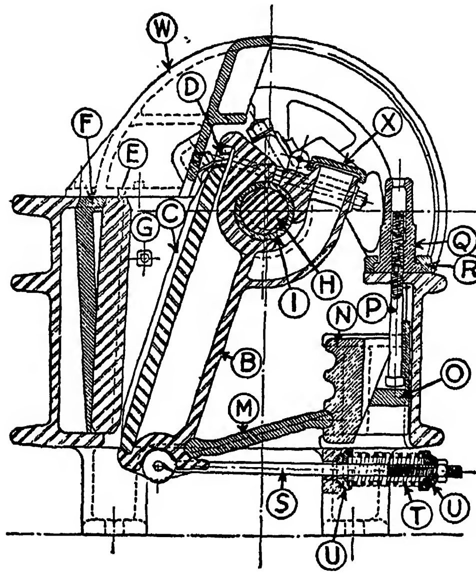


Fig. 2321 A Jaw Crusher
Courtesy Wheeling Mold and Foundry Co.

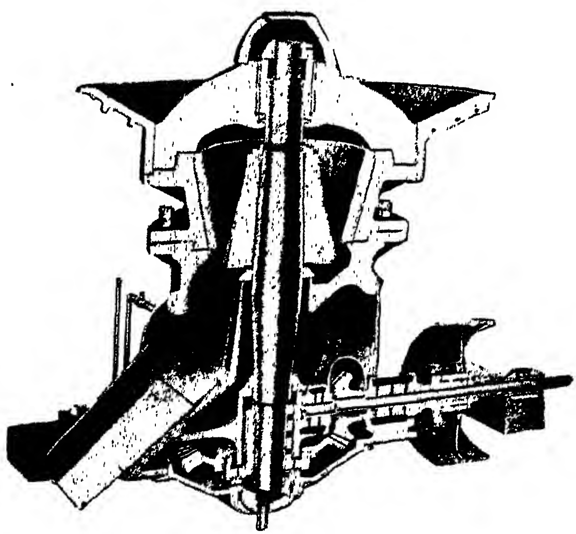


Fig. 2322 A Photo-Sectional View of Gyratory Crusher

and sustained by a yoke. The lower end of this vertical shaft is carried by a bearing and collar eccentrically mounted on a bevel gear. Power is supplied by a belt drive to the pulley on the horizontal shaft, thence to the horizontal bevel gear. Thus the vertical shaft, with a fixed center at its upper bearing and a rotating center at the lower, is given a gyratory motion. The "crushing head," shaped like the frustum of a cone, is carried on the shaft and rotates with it. The opening between the crushing head and the "concaves" of the crusher therefore changes from its maximum, and vice versa, with each gyratory revolution of the shaft. The crushing head is adjustable, thereby governing the size of the output. In the installation of the crusher, a platform is built on a level with the brim of the hopper, and a ramp, built up to this platform, allows laborers to wheel loads of uncrushed stones up to the crusher. (See Fig. 2320A.)

LIFTING AND HANDLING EQUIPMENT.

2323. Of the lifting and handling equipment used in engineering construction work, the following list includes the principal types:

Derricks,	Guy
	Stiff Leg
	Bull Wheel
Cranes,	Locomotive
	Crawler
	Bucket Handling
	Transfer
	Jib
	Pillar
	Bracket
	Electric Traveling
	Gantry
Lifting Attachments	
Hoists	

2324. **Derrick.**—The derrick is an apparatus whose purpose is the lifting and handling of heavy weights, such as machinery, equipment, supplies and engineering materials of all kinds. It consists of a central mast supported in a vertical position by braces or guy wires, and a boom, connected to the mast at the top by means of blocks and tackle and pin-connected at the bottom as shown in Fig. 2324A. The boom carries a second set of pulleys suspended from its free end, to which a hook, bucket, lifting magnet, or other lifting device may be attached.

The base of the mast may be fitted with a bull wheel as shown in Fig. 2324A, which is revolved by means of cables wound on a drum of the engine. In the stiff leg derrick, the boom and weight can obviously be swung only within the limited space between the legs. In the guy derrick, however, in which guy wires replace the braces of the stiff leg derrick, the boom may be swung through a complete circle provided only that its tip is dropped low enough to clear the guys. Thus the range of its utility exceeds that of the stiff leg derrick.

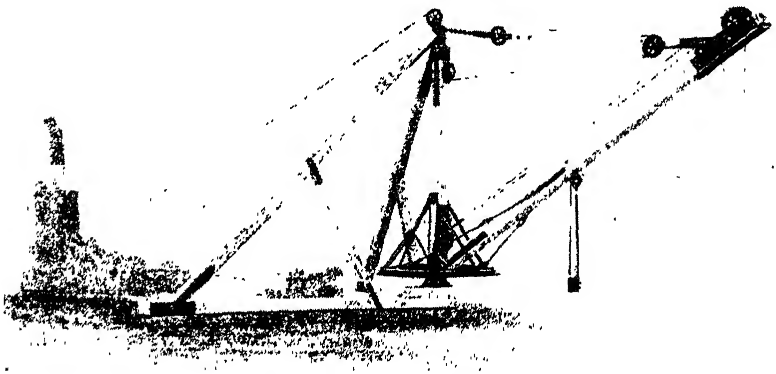


Fig. 2324 A Stiff Leg Derrick

2325. **Crane.**—The crane is a hoisting apparatus similar in principle to the derrick but different from the derrick in that it is mounted on a movable base usually carrying its own power. It is therefore capable of moving horizontally so that it may deposit its load at any point. It is of wider application, generally of greater lifting capacity, and of greater efficiency than the derrick.

Cranes will hoist or lower the load, rotate in either direction, and travel in any direction, and are also provided with mechanism for raising and lowering the boom. All these operations are controlled from the operator's platform, through levers, so that it is possible to secure very fast operation by properly coördinating the various motions. For instance, when using a grab bucket, for unloading from cars and delivering to storage, or for reclaiming from storage and delivering to cars, trucks, or other similar units, it is possible to so coördinate the hoisting and rotating motions as to be able to operate at as fast a rate as 2 to 3 complete trips per minute under favorable circumstances.

2326. *Locomotive Crane.*—The locomotive crane is a locomotive equipped as a crane built to travel on rails. Fig. 2326A gives a typical

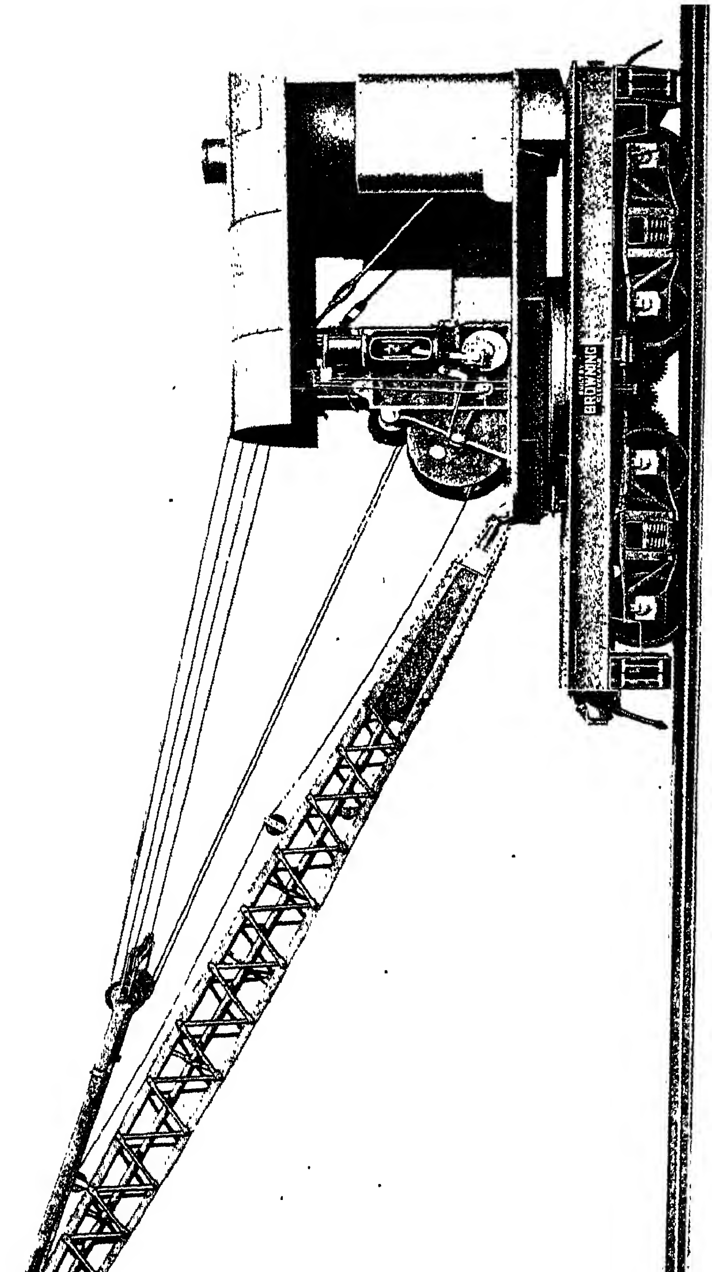


Fig. 2326 *A* Locomotive Crane

example of such a crane. The locomotive crane is usually furnished either of the eight wheel double track type or the four wheel type, for operating upon standard gauge railway track, but is also made with wider bases (requiring extra track), to increase the stability of the crane when conditions make this advisable. The standard gauge cranes are usually furnished with booms ranging in length from about 40′ to 75′ although this can be varied within certain limits to suit the work. These limits vary from 12′ to 40′ on small cranes, running up to 75′ on the largest cranes.

The lifting capacities of large cranes run from 7000 pounds at the maximum radius (when boom is horizontal) to 80,000 pounds or thereabouts at the minimum radius (when boom is vertical). The maximum capacities vary inversely with the radii of the booms. Thus, in the small cranes, the maximum lifting capacity runs from 5000 to 6000 pounds at the largest radius to 32,000 pounds at the smallest radius.

2327. *Crawler Crane.*—There is also an increasing demand for revolving cranes mounted upon crawler treads. These cranes, on account of operating upon all kinds of soft and hard ground, climbing over obstructions such as railroad tracks, etc., are necessarily made as light as possible, many parts of the crane, such as the crawler treads, being made of special alloys in order to provide the desired combination of light weight and ample strength. The crawler crane is usually furnished with gasoline engine drive, which has been found to be the most economical and flexible for most conditions under which this type of crane is operated.

2328. Either the locomotive crane or the crawler crane can be equipped with grab bucket for handling sand, gravel, coal, cement clinker, crushed stone, ashes, refuse, and other bulk materials; or they can be equipped with hook or hook block for handling miscellaneous light and heavy loads, as well as with lifting magnet for handling magnetic materials. They are also sometimes equipped with special attachments so that they can do the work of pile drivers, steam shovels, dragline buckets for stripping operations, etc.

2329. *Bucket Handling Crane.*—Bucket handling cranes (Fig. 2329A) are cranes adapted to the handling of material by some form of bucket.

2330. *Transfer Crane.*—Transfer cranes are those used in transferring articles from one carrier to another, such as overhead cranes for unloading freight cars and loading others.

2331. *Jib Crane.*—Jib cranes consist of a "jib," or projecting arm, connected to a pivoted vertical member around which it may be swung laterally. A type of jib crane is shown in Fig. 2331A.

2332. *Pillar Crane.*—In pillar cranes, the vertical member is a self-supported pillar which revolves upon its base through a complete circle. These are mainly used in docks and yards (Fig. 2332A).

Fig. 2329 A Bucket-Handling Electric Gantry
Courtesy Whiting Corporation

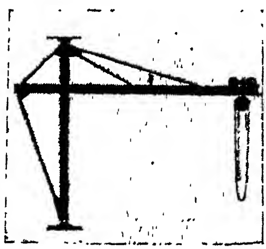


Fig. 2331 A Jib Crane

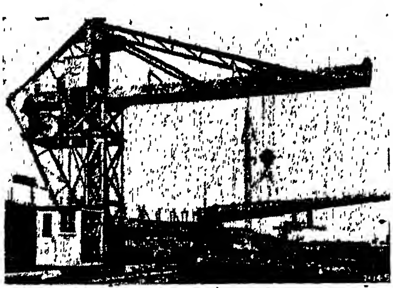


Fig. 2332 A Pillar Crane
Courtesy Whiting Corporation

2333. *Bracket Crane.*—Bracket cranes are supported by wall brackets. Their use is mainly indoors in shops.

2334. *Electric Traveling Crane.*—Where very heavy loads are dealt with, as in power plants, structural shops, ship-yards, etc., electric traveling cranes are usually employed (see Figs. 2334A and B). They consist of built up girders securely mounted at each end on bridge trucks running on rails which are carried on the side walls of the building. The

girders carry a track on which run one or more trolleys. From the trolleys, the lifting equipment is suspended by cables and pulleys. Power is furnished by an electric motor to a shaft geared to the wheels of the

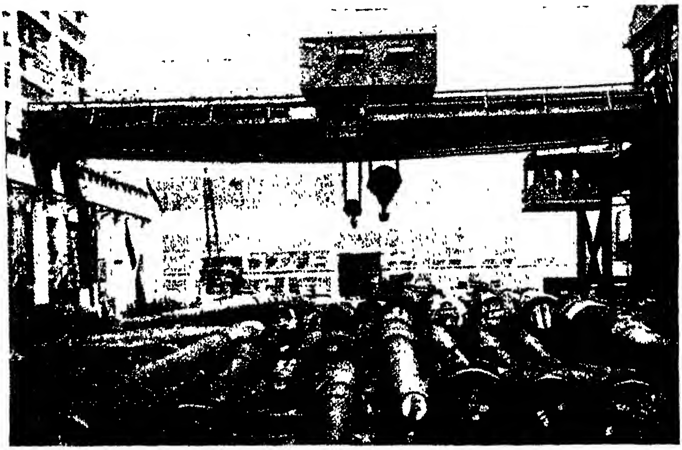
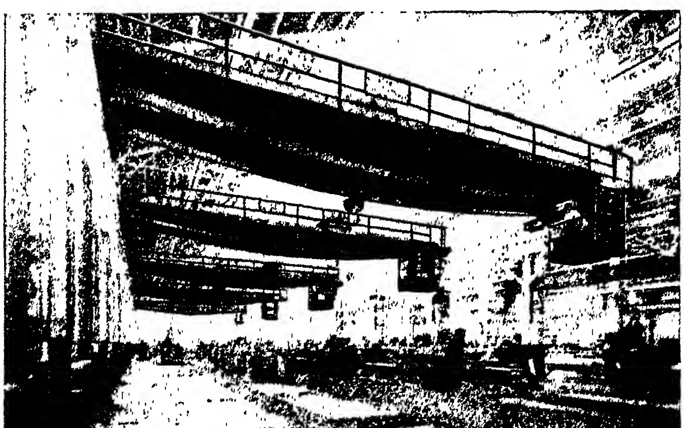


Fig. 2334 A
Electric Traveling Crane
Courtesy Whiting Corporation



Five 10-Ton Whiting Electric Traveling Cranes on One Runway, 50-Ft. Span Watertown Arsenal, Watertown, Mass.
Fig. 2334 B

bridge trucks. The crane is operated from a suspended cage containing all the controls for the entire mechanism. Suitable safety devices are installed to protect the operator and to guard against possible injury

to the crane due to failure to shut off the controls before their limits of operation are reached.

2335. In lifting and moving a load, the bridge drive is set into operation, moving the crane to a position in the shop directly over the load to be lifted. The trolleys are then run out to a suitable position and their lifting devices lowered and hooked up to the load. The motors controlling the lifting are then operated, raising the load to a suspended position. The bridge drive, again set into operation, moves the crane with its load to another portion of the shop, where the load is lowered to the desired position.

2336. The electric traveling crane is adaptable to a wide range of service and is the type most commonly used in foundries, steel mills, machine and structural shops, forge plants, power and coaling stations. Every engineer, in designing power plants, shops, etc., must include in his design adequate apparatus for lifting and handling all heavy machinery which is to be installed in, or which may be manufactured in, the building, whether such installations are permanent or temporary. Electric traveling cranes are very generally included as a part of this apparatus.

2337. *Gantry Cranes* are cranes operating on tracks and supported by a gantry; i.e., by a framework or scaffold built for the purpose of raising the crane runway to an elevation above material which the crane is designed to handle (Fig. 2337A). The gantry may be in many forms. In general construction it is similar to the overhead traveler, but runs on the ground and the travel mechanism is operated by means of bevel gears and vertical shafts on the legs. Often one leg may be omitted and one end carried on a runway along a building wall as shown in Fig. 2329A. Gantry cranes may be made to serve areas outside their tracks by having cantilevers at one or both ends (Fig. 2329A). Gantry cranes are particularly adapted to outdoor service, where overhead runways would be expensive or otherwise undesirable. The track for a gantry crane may be readily extended, and when necessary the location may be completely changed without much trouble or expense.

2338. Some of the American terminals in France were equipped with types of Gantry Wharf Cranes spanning three railway tracks as shown in Fig. 2337A. The cargo is unloaded by the cranes and loaded into freight cars, or placed at the rear of the docks whence it is moved to warehouses until shipment. These cranes proved to be rapid unloaders of the miscellaneous cargo used by the U. S. Army in France.

2339. **Lifting Attachments.**—Lifting attachments used in conjunction with cranes and derricks include buckets of several types, which sink into the material by their own weight, scoop it up when hoisted, and release it automatically (Figs. 2339A and B); slings, hooks, tongs, and

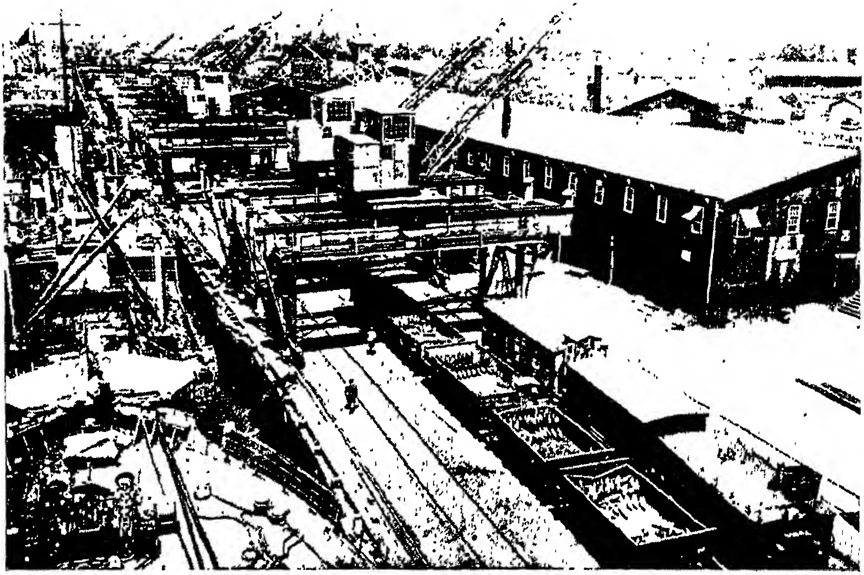


Fig. 2337 A Gantry Cranes
Courtesy Brown Hoisting Machinery Co.

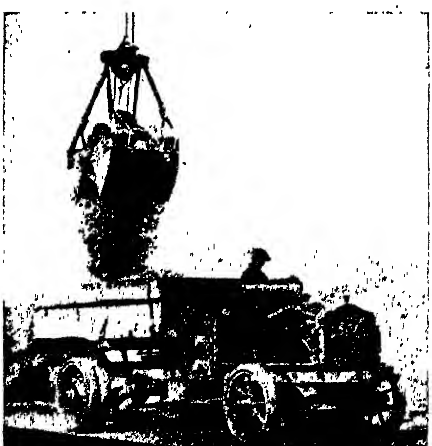


Fig. 2339 A
Clam Shell

grapples, each suitable for its own special kind of work; and lifting magnets, whose lifting force is due to the attraction induced by an electric current passing through a solenoid (Fig. 2339C). The distinguishing characteristics of these lifting attachments, as well as their method of operation, will be obvious from an examination of the figures.

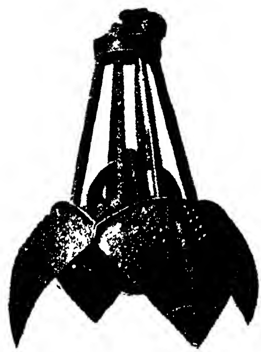


Fig. 2339 B. Orange Peel

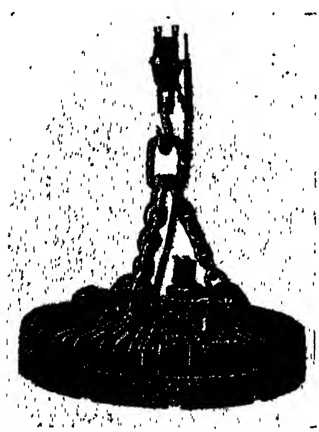


Fig. 2339 C Lifting Magnet
Courtesy Link-Belt Co.

2340. **Hoists.**—The principal types of hoists are Builders' Hoists, Elevators, and Jacks.

2341. *Builders' Hoists.*—Builders' hoists comprise a boiler, engine, winding drums, gearing, and controls, all mounted as a unit on the same base. Such a hoisting engine is shown in Fig. 2341A. It is a double

cylinder, friction drum, link motion, portable engine designed for small cableways, general contractors' work, etc. Its winding drums are controlled by levers fitted with thumb latches engaged with notched quadrants. The type of engine shown may be had in sizes developing up to 50 horsepower, with a lifting capacity of 6500 pounds on a single rope. Other engines develop over 100 horsepower and up to about 10,000 pounds lifting capacity.

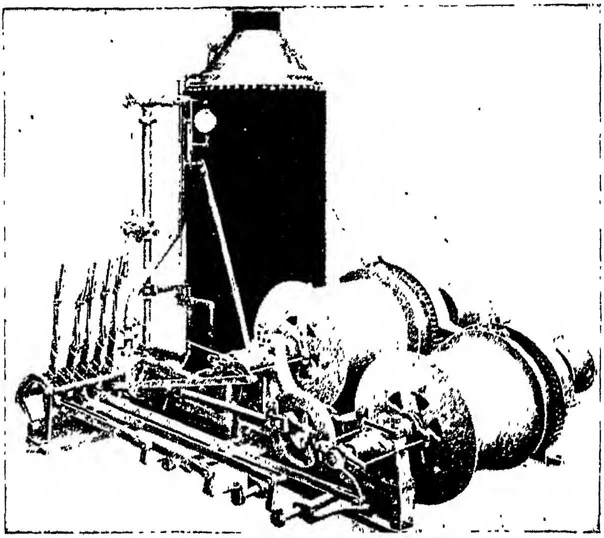


Fig. 2341 A Builders Hoist
Courtesy S. Flory Mfg. Co.

2342. *Material Elevators.*—Builders' material elevators are simply constructed elevators consisting of a platform reinforced with iron. They are used for hoisting materials from the ground to an elevated part of a structure in process of erection. Often these platforms are double, one descending as the other is raised, the lower being then loaded as the upper one is discharged. Elevators may be operated by several different sources of power, such as steam, compressed air, electricity, etc., but the type mentioned above, the builder's hoist, is most used in the lighter service of general contracting.

2343. *Screw Jack.*—The lifting jack is a screw jack operated by a bevel or worm gear similar to the familiar type of automobile jack. Fig. 2343A shows a jack obtainable in sizes lifting up to 100 tons. It is designed to be portable, and to sustain its load at any position without backing down. Jacks of this sort in the standard sizes are limited in their elevating ranges to about two feet.

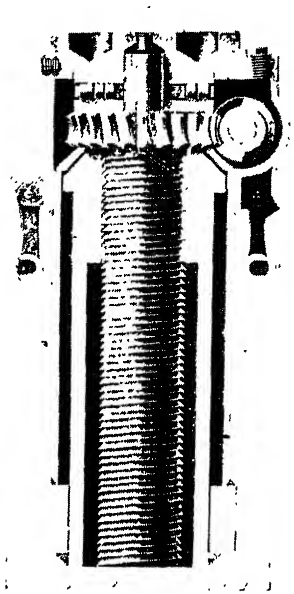


Fig. 2343 A Screw Jack
Courtesy McKiernan-Terry Co

2344. *Hydraulic Jack.*—In heavy work, hydraulic jacks are also used. The principle of their operation is the equal transmission of water pressure in all directions: so that a unit pressure over a small area, thus requiring a small total pressure, will cause the same unit pressure over a large area, thus providing a large total pressure. This is explained in paragraph 523.

HAULING, CONVEYING, AND DISTRIBUTING EQUIPMENT.

2345. The transportation of fuel, material, equipment, and supplies, may be effected by many methods; the selection of equipment therefore being, as with any operation involved in construction work, dependent upon conditions imposed by circumstances attending the particular project under consideration. Dump cars, automobile trucks, railway cars, wheel-barrows, carts, wagons, and tractors drawing trailers, are all useful agencies in systematically and economically effecting the operation of hauling. No exact distinction can be made between hauling, conveying, and distributing, nor between these and operations previously discussed in this chapter. Equipment used for one is very often essential in another operation. Thus, if hoisting equipment is combined with a

cableway on which an excavating bucket is carried, both together form a method of excavating, hoisting, conveying, handling, dumping, and distributing the excavated material.

2346. **Belt Conveyor.**—Belt conveyors may be utilized to accomplish the handling of some materials. These conveyors consist of wide belts carried on revolving rollers. The material to be conveyed is continuously loaded at one end and discharged at the other.

2347. **Bucket Elevator.**—Geared bucket elevators may properly be classified as elevating conveyors. They comprise a series of cups, attached to a belt or otherwise linked together, meshed with power driven gears which cause continued upward movement during which the cups are filled at the lower end and discharged above. This type of elevating conveyor is shown in the crusher plant in Fig. 2320A.

2348. **Cableways.**—Suspended cableways are used in many projects. They may cover large areas with slight variation in their position. It is usually expedient to erect, at each end of the cable, a tower support of steel or wood, the ends of the cable being anchored in the ground. The cable itself carries a runner supporting some form of lifting attachment controlled by a hoisting engine located at one of the supporting towers. Often one tower is mounted on a base moving on tracks laid on a curve whose center is the stationary tower. When construction work is completed on a sector between the temporary position of the movable tower and the center tower, the movable tower is run to another position on its tracks and the construction continued on the new sector between the two towers. Sometimes only one tower is used and the cable carriage and its suspended attachment run along the inclined cable by the force of gravity. Power is then applied only in operating it in the other direction.

CONCRETE CONSTRUCTION EQUIPMENT.

2349. In another part of the text is discussed the making of cement concrete, the proper proportioning of its ingredients, the various kinds of aggregates, and its wide usage in modern construction work. In the chapters on Foundations, Building Construction, Bridges, Railways, Roads and Pavements, Water Supply, and Sewers and Sewage Disposal, concrete is seen to enter as an important part of each of these types of construction project. Its use is therefore varied and extensive and demands construction equipment designed to meet all these special requirements.

2350. **Concrete Mixers.**—The use of concrete involves its preparation, its distribution to the proper point, and its treatment after application, which operations as one item are included in estimates under the

heading of cost of "mixing, placing, and finishing." Many types of concrete mixers perform merely the operation of mixing the ingredients. In road construction, however, in modern concrete paving machines, these operations are to a large degree accomplished by one machine alone.

2351. **Concrete Paver.**—This usually carries a platform from which one operator controls its movements and operations. The cement, sand, and stone are dumped in proper proportions onto an apron receiver which is then elevated, forming a chute from which the material slides into a revolving drum. This is fitted inside with revolving blades whose relative motion is contrary to the motion of the drum itself. Power is furnished by a gasoline motor which forms an integral part of the paver. Thorough mixing is accomplished in one or two minutes and the mixed material, now in the desired consistency by reason of a regulated addition of water during the mixing process, runs down a chute on the opposite side. This chute is swung back and forth at the will of the operator, thus distributing the concrete to the desired point of the road bed.

2352. **Template.**—A template is a long plank or metal plate with its lower edge shaped to the crown of the finished road surface. Its ends rest on steel forms placed on each side of the road and set to the grade of the road. This template is gradually oscillated from side to side by a workman at each end, and, at the same time, given motion parallel to the center line of the road, which carries the surplus concrete before it, thus extending the finished paving. The paver moves along on its tractor wheels keeping pace with the advance in construction, thus making the operations described practically continuous.

2353. **Forms.**—In all concrete construction work, forms for the concrete are of course essential. These may be of timber or of metal. Timber forms are temporary in their usefulness and, due to warping and damage in removal, can seldom be used in more than one construction. Steel forms are therefore used to some extent because of their permanency and availability for repeated use in many works. They are expensive, however, and their use is therefore hardly warranted by any but construction firms whose volume of business makes the use of steel forms eventually economical.

2354. In massive construction work, there becomes necessary a distribution system of wider range than any heretofore mentioned. Thus the ingenuity of the construction engineer enters into the problem, demanding a combination of the several types of construction equipment previously discussed which best suits the special conditions.

In the construction of large fortifications by the Corps of Engineers, U. S. Army, the following method has at times been employed in effecting a wide range in the distribution of the concrete:

A tower, mounted on a car platform, forms one support to a cable con-

nected to a stationary tower mast, whose base forms the center of a curved track on which the car runs. The construction takes place between these two towers. A bucket, carried on the cable suspended between the towers, is so rigged by pulleys and cables that it can be lowered to a concrete mixer at the base of the stationary tower. Here it receives its charge of concrete, is then hoisted, run out to the desired point above the work and is lowered, dumping its contents into the forms at the desired spot. These having been filled, the car carrying the movable tower is run out to another part of the track and the operation continued.

2355. **Gravity Distribution.**—In construction work involving the delivery of concrete to several units grouped in the same vicinity though

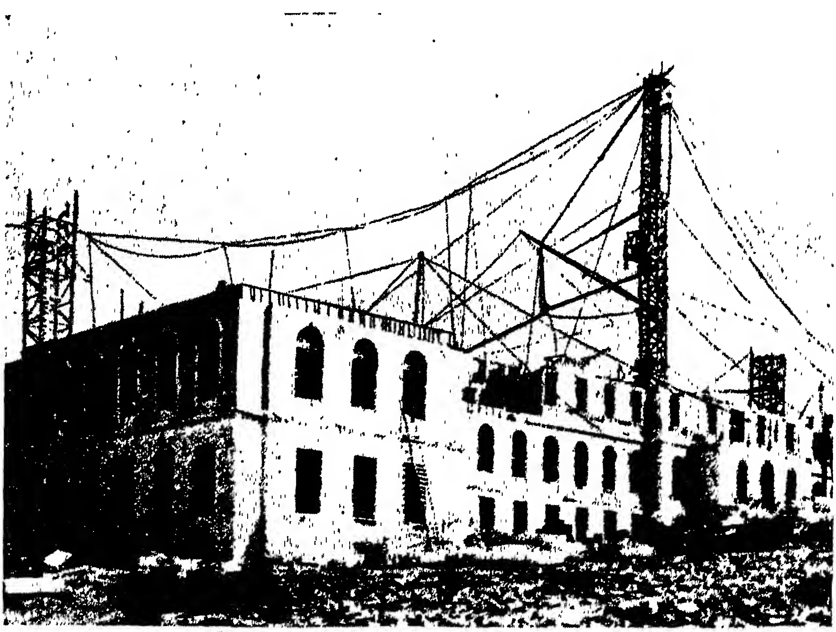
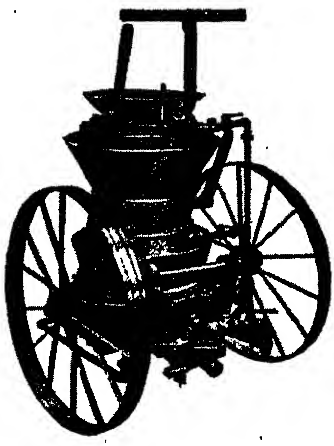


Fig. 2355 A. Gravity Distribution

somewhat distant from each other, a gravity distribution plant is largely used. This comprises a steel or wooden hoisting tower, centrally located with respect to the several construction units, and a system of inclined chutes radiating from this tower to the unit whose construction is then in progress (see Fig. 2355A). A concrete mixer, placed at the base of the tower, discharges its mixed concrete mortar into a receiving bucket which is hoisted to the top of the tower and there automatically dumps

its load into a hopper. From this the concrete flows by gravity down through the chutes to the forms. When the concrete has set and the forms are removed voids may be found to exist. These are filled by spraying that portion of the structure with concrete mortar under pressure, for which purpose a cement gun may be used.

2356. **Cement Gun.**—The cement gun is shown in Fig. 2356A. Its hopper is charged with dry sand and cement already thoroughly mixed.



2356 A Cement Gun
Courtesy Cement-Gun Co.

The dry mixture passes through a mechanism which feeds it continuously in small doses to a hose, through which it is forced by compressed air. At a nozzle on this hose water is combined with the dry mixture in a fine spray under regulated pressure. The material thus hydrated is therefore applied in a forced spray, which gives to the finished coating even a greater density than is obtained in poured concrete.

CONCLUSION.

2357. One needs only to glance at the many and varied types of construction equipment advertised in the current engineering periodicals to realize from how vast a field he must select certain machinery to suit his purpose. In this selection, of course, no definite rules can be laid down. The construction engineer or contractor learns from his own observation, from his own experience, and from the records of the ex-

periences of others, as found in articles and pamphlets and journals published by the engineering societies, just what types of equipment most efficiently and most economically meet his needs. To include herein a complete discussion of every different kind of equipment which is available for use in engineering operations is beyond the scope of this textbook. For the details of this equipment the student should consult handbooks of construction equipment, of which there are on the market several standard works of recognized value.

For such detailed information see

Handbook of Construction Equipment, Dana
Handbook of Reinforced Concrete Construction, Hool and Johnson
Current Engineering Periodicals
Catalogues of Construction Equipment

CHAPTER XXIV.

SPECIFICATIONS.

2401. **Definition.**—Every engineering structure originates as a mental vision in the mind of some person who sees the desire for or the need of the structure. In order to transform that vision into a reality the engineer must take the necessary steps so that the project may be clearly and definitely expressed to those whose labor is necessary in all the operations involved in the realization of that project.

2402. To thus express his thought, the engineer employs graphical symbols to as great an extent as possible. These comprise the plans, elevations, cross-sections, and all details of construction, consolidated into what are known as "plans." It is essential, however, to fully describe all details of construction which may not be adequately represented on the plans, and to give such specific directions as the designing engineer may see fit to give in order fully to carry out his ideas. These written directions constitute what are known as "Specifications."

2403. Specifications, then, may be simply defined as definite and specific descriptions of all the details of operation, construction and erection of a structure, and a complete statement of all directions governing the execution of the design for that structure.

EXECUTION OF WORK.

2404. There are two general methods of executing an engineering project after it is designed.

a. The execution of the design may be carried out wholly by the designing engineer and his associates, if any, by employing his own laborers and directing the work himself.

b. The actual construction of the work may be done by another party called the contractor.

2405. The first method is the one employed by many large engineering firms and by the United States Government in many cases. The Panama Canal was completed by the Officers of the Corps of Engineers, U. S. Army, following this method. It is also followed by large railway corporations who maintain their own engineers. The second method is more often used, however. The contractor enters into a contract with the owner to execute fully and faithfully the design of the engineer in

accordance with the engineer's plans and specifications. It is necessary, therefore, to write into the specifications many clauses intended to prescribe the duties of the contractor so specifically that he will properly execute his part of the agreement, and so that the owner will be relieved of liability for all contingencies which might arise during the progress of the work of the contractor, or as a result of that work. To insure proper work by the contractor, the owner has an inspector as his representative on the work. This kind of work is often the first experience of young engineers: and they should study the specifications over and over again so that they will be able to protect the interests of the owner.

CONTRACTS.

2406. A contract is an agreement creating between the parties thereto an obligation enforceable at law. It may be either "express" or "implied." The former is created by spoken or written words; the latter, by actions clearly indicating a mutual agreement. There are many classifications and sub-classifications of contracts. For such details the student should consult a textbook on contract law.

2407. **Necessary Elements of a Contract.**—In order to be valid, contracts must contain four essential elements as follows:

a. The parties to the contract must be competent to contract. The courts will not ordinarily enforce contracts made by lunatics, infants, etc.

b. The subject matter of the contract shall be legal. No contract is valid if the parties thereto are contracting to do some act contrary to law. Thus the winner of a bet could not legally enforce the payment of that bet where gambling is prohibited by law.

c. There must be "mutuality of consent"; that is, each party to the contract must have a clear understanding of the terms of the contract, without fraudulent misrepresentation by any party.

d. There must be a consideration; that is, something of value paid or relinquished by one party, in consideration of which the other party binds himself to do or omit to do certain acts.

Lacking any of these elements, the contract is void.

2408. **Default.**—Contracts are said to be "discharged by performance" when the work specified has actually been completed in accordance with the terms of the contract. They may also be discharged by mutual agreement of the contracting parties; by becoming impossible of performance; or by default — that is, by breach of contract by one of the contracting parties. In a case of default by the contractor, where the

contractor may have refused to carry out his part of the work either in part or whole, the owner has the single recourse of taking the case to the courts. It is impossible in most cases to compel a contractor to continue his work. Action taken in the courts, however, usually results in awarding the owner damages to an amount equivalent to such loss as he can prove himself to have sustained by reason of the contractor's default.

2409. **Bond.**—The possibility of such default is usually met by the requirement of a **bond** from the contractor. This is usually a surety bond issued by some responsible surety company. It consists of a statement that the company will meet, to the amount of the bond, any loss sustained by the owner in case of a breach of contract by the contractor. This bond is usually required of the contractor at the time of awarding him the contract.

AWARD OF CONTRACT.

2410. **Procedure.**—The procedure in the award of contracts is practically the same in all types of engineering enterprises and, in public works, it is prescribed by law.

2411. **Advertisement.**—Advertisements are published in technical papers which will reach contractors who may be interested. These contain a statement descriptive of the particular work, giving data indicating the magnitude of the work, stating where information, plans and specifications may be examined, inviting bids on this work, with also the necessary directions as to their submission, date of examination, etc.

2412. **Information for Bidders** is prepared, enlarging upon the data given in the advertisement. This contains in general the following data:

a. Reference to action authorizing the work, as a city ordinance or resolution adopted by a board of directors.

b. Restriction of bids to a certain prescribed form. This includes information as to whether bids will be received covering separate parts of the work, such as separate bids for heating installation or electrical installations.

c. Directions as to the submission of samples.

d. Instructions for the preparation of bids.

e. Such estimate of quantities of materials as have already been prepared by the engineer.

This information, together with plans, specifications, and a Form of Proposal are furnished the bidder usually upon the payment of a small

fee which is refunded upon the return of the documents furnished him. In large projects, involving elaborate specifications, the specifications, information for bidders, etc., are usually printed in pamphlet or loose-leaf form, often of pocket size for convenience.

2413. **Form of Proposal.**—The Form of Proposal is a definite offer to execute the work described in the plans and specifications, for a stipulated amount. Its principal requirements are as follows:

a. State the offer.

b. Stipulate the amount in words and figures.

c. Stipulate the terms of payment agreed upon.

d. State that no fraud or collusion exists between the bidder and any other.

e. State that plans and specifications have been understood, and that the bidder has made an examination of the construction site.

f. Specify the dates of commencement and completion.

g. State that the money required if any for deposit with the submission of the bid as proof of good faith accompanies the proposal.

h. Agree to furnish bond if awarded the contract, and state the nature of the security contemplated.

i. Sign the document.

2414. **Reception of Bids.**—Bids are received and opened (generally in public) at a stated time on a fixed date. In Government work, the law usually requires that the contract be awarded to the lowest responsible bidder. It is important, therefore, that bids be so prepared that they may be accurately and fairly compared.

2415. **Unbalanced Bids.**—Often contractors are left to make their own estimates of materials from their own examination of the site of the construction work. This obviously makes possible a wide divergence in the quantities on which the several contractors may have prepared their bids. For convenience in comparing bids, therefore, a preliminary estimate of quantities is prepared by the engineer and issued with the information for bidders. The contractors then fix their own unit prices based on the engineer's estimate. Contractors often submit what are known as "unbalanced bids"; that is they so arrange their bids per unit of work that the total amount will be about right if the estimate is correct, but so that they will receive excessive prices for extra work if the estimate is not correct in the items on which they bid excessive prices.

Let there be taken as an illustration an engineering project in which the calculated excavation is 5000 cubic yards; and on which two contractors A and B bid as follows:

Estimated Quantities	A's Bid		B's Bid	
Earth, 4000 cubic yards....................	$0.50	$2000	$0.05	$ 200
Rock, 1000 cubic yards	$1.50	$1500	$3.00	$3000
		$3500		$3200

The contract is awarded to B, the lower bidder. Yet, the actual construction of the project results as follows:

Actual Quantities	Cost if awarded to A	Cost as awarded to B
Earth, 3000 cubic yards....................	$1500	$ 150
Rock, 2000 cubic yards	$3000	$6000
	$4500	$6150

It will be seen that B in his bid lowered the unit price for earth excavation and raised that for rock above fair prices as represented by the unit prices in A's bid. In the work as actually performed, B received much more money than would have been received by A, because B submitted an unbalanced bid in the belief that the estimated quantities given out by the engineer were erroneous. Great care should be taken in the consideration of bids and awards of contract to prevent such practice. This can best be done by means of an accurate and exhaustive survey by the engineer as a basis for his estimate of quantities.

CLAUSES OF SPECIFICATIONS.

2416. In order also that bids may all be prepared on the same basis, the specifications which they are prepared to meet must be complete. Every factor which governs the contractor in the materials selected, in the manner of performance of all duties necessary in the execution of the work specified—in short, all conditions which might affect the contractor's estimate and raise or lower the cost, are necessarily included in the specifications.

2417. Specifications are therefore grouped into certain **General Clauses** and **Technical Clauses.** The General Clauses stipulate what shall be done in every case which might arise from the standpoint of the lawyer and business man; the Technical Clauses treat of the particular details of design in regard to materials, relative proportions, qualifications, etc.

2418. **Technical Clauses.**—The Technical Clauses, being descriptive of a special design, are therefore dependent on the project under consideration. No exhaustive treatise on what should or should not be included in them can therefore be made. It may be said, however, that they should be drawn up with a view to obtaining a desired quality of result, omitting such specifications as would curtail the contractor's free judgment as to the best method of operating his employees in producing that result.

2419. **General Clauses.**—The General Clauses provide for certain contingencies that might arise during the progress of the work, are often applicable to many types of construction work, and will therefore be more fully discussed. They include:

a. **Clauses defining the work.** These state precisely to what plans and within what limits the specifications are intended to apply. They also differentiate the particular work considered from any work covered by other contracts and specifications.

b. **Definitions of such terms used in the specifications as otherwise may be ambiguous and indefinite.** Thus it must be made clear exactly to whom the terms **owner** and **engineer** apply. Also, the several units of measurement must be specifically limited to the meaning they are intended to convey.

c. **Changes in specifications and plans after the contract has been made.** Under Contract Law, the contractor is bound only to stipulations in the original contract. Thus the contractor cannot be held to any changes in design made after signing the contract unless a clause in the contract specifically states the terms and conditions under which such changes may be made. It is thus necessary to reserve to the engineer the right to make changes in the design. A means of adjustment of the differences in cost resulting from such changes must consequently be made, and it must be stipulated that such changes will not affect the contract or bond.

d. **Ambiguity and inconsistencies in plans and specifications.** Care should be taken to avoid all inconsistencies between different parts of the specifications, and between the specifications and plans. This will be effected to a large extent by conciseness of expression and avoidance of repetition. It should be definitely stated, however, that the plans and specifications are to be considered mutually supplementary, that work shown on only one will have full force as if shown in both, and that, in case of inconsistencies, the decision of the engineer as to the correct interpretation shall be final.

It is often specified, furthermore, that the contractor shall do all work which, in good practice, is considered necessary to the proper execution of the design, whether or not specifically shown on the plans or men-

tioned in the specifications. Such a clause, however, can legally be held to apply only where such work is absolutely necessary and can be considered implied in the contract.

e. **Responsibility for the data included in the plans and specifications must be established.** It is usual to require the contractor to verify such data by personal inspection and examination of the site. If, however, the engineer has made a thorough examination and knows certain conditions to exist, the owner may quite properly assume responsibility for the data furnished. This will lower the risk taken by the contractor and consequently lower his bid.

f. **Miscellaneous provisions.** Although the contractor is always bound in the execution of his work by local and state laws, it is nevertheless customary to include in the general clauses of the specifications certain clauses requiring the contractor to conduct his operations in accordance with the law in regard to the treatment of employees, sanitation, sale of intoxicants, etc., and to require him to procure all permits necessary to comply with the law, and to assume all liability for infringement of patent rights, local laws, and the rights of all property owners adjoining the site of construction.

g. **Powers of the engineer.** In all specifications a clause of vital importance is that relating to the powers of the engineer. The engineer occupies a peculiar status. His duties are **legislative** in that he prepares the plans and specifications; **executive** in that he supervises the execution of the design; and **judicial** in that the specifications generally prescribe that he has the power of final decision and final interpretation of matters which may be contested, or claimed to be indefinite or ambiguous (though of course the contractor can always object to his final decision and take the matter to the courts). These powers must be given to the engineer in the specifications; and also he must be given the duties of laying out the work, of measuring the amounts of all material upon which payment is based, of rejecting materials which are not satisfactory to him, and of requiring rejected material to be removed from the site. It is therefore specified that the contractor shall at all times have some one on the job to represent him and carry out such directions as the engineer may give.

2420. It should further be specifically stated that any work found to be defective, even though unnoticed at the time of its construction, shall be made good by the contractor at his own expense at any time that it may be discovered prior to the final acceptance by the engineer.

2421. An attempt has been made to standardize specifications in so far as this may be possible. It is obviously impossible, however, to adopt for all kinds of work specifications which may be termed "standard."

The specifications given in various books of specifications are illustrative of the principles mentioned in this chapter; but they should be considered only as a basis for desired specifications and are in no sense satisfactory for every kind of work.

For more detailed information see

Engineering and Architectural Jurisprudence, Wait
Contracts in Engineering, Tucker
The Principles of American Law of Contracts, Lawson
Law of Contracts, Brantley
Good Engineering Literature, Frost
Theory and Practice of Technical Writing, Earle
Elements of Specification Writing, Kirby

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